Synchronization in Complex Networks by Coupled Parametrically Excited Oscillators with Parameter Mismatch

Kosuke Oi, Yoko Uwate and Yoshifumi Nishio
Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770–8506, Japan
Email: {ooi,uwate,nishio}@ee.tokushima-u.ac.jp

***Abstract*—In this study, we investigate synchronization in complex network with two hubs by using parametrically excited van der Pol oscillators with parameter mismatch. By means of computer simulation, we confirm various change of synchronization probability in complex network, and observe effects on synchronization probability by changing structural metrics (degree and path length) in subset of the nodes including larger mismatch in complex network.**

I. Introduction

Synchronization is one of the fundamental phenomena in nature and it is observed over the various fields. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. Synchronization generated in the system can model certain synchronization of natural rhythm phenomena. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena of the parametric excitation circuits for future engineering applications. In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Refs. [4] and [5].

In our research group, we have investigated synchronization of parametrically excited van der Pol oscillators [6]. By carrying out computer calculations for two or three subcircuits cases, we have confirmed that various kinds of synchronization phenomena of chaos are observed. In the case of two subcircuits, the anti-phase synchronization is observed. In the case of three subcircuits, self-switching phenomenon of synchronization states is observed.

However, we have investigated the only simple network models. It is important to investigate more complex network for the broad-ranging future engineering applications. In our previous study, we have challenged to investigate the synchronization and clustering in more complex network modified from "Dolphin social network" [7] by using parametrically excited van der Pol oscillators with small mismatch [8]. We have confirmed that the network with hubs can induce synchronization. Though, we have only investigated the effects on synchronization offered by the location and number of pieces of the hubs, and we have only added small mismatch (dispersion) in whole.

In this study, in order to more particular investigation of synchronization in complex network, we focus on relationship between the structural metrics in subset of the nodes including larger mismatch and synchronization in complex network. In order to research this relationship, we investigate synchronization in complex network by changing structural metrics (degree and path length) corresponding to the subset of the nodes adding larger mismatch than other nodes. First, we investigate the synchronization probability in the network by changing the degree of the three nodes added larger mismatch. Next, we investigate the synchronization probability in the network by changing the path length among hubs, three nodes with larger mismatch.

II. Proposed network model

Topological structures in complex networks of N nodes and E edges can be evaluated by the typical structural metrics (degree, clustering coefficient and path length). First, degree is the number of edges on a node. Second, clustering coefficient is the number of actual links between neighbors of a node divided by the number of possible links between those neighbors. Third, path length (L) shows the average shortest path in the network between two nodes. This is given as follows:

$$L_i = \frac{1}{N-1}\sum_{i \neq j}^{N} l_{ij}. \tag{1}$$

$$L = \frac{1}{N}\sum_{i=1}^{N} L_i. \tag{2}$$

Where L_i is the shortest path in the network between node i and node j.

In this research, we consider coupled parametrically excited oscillators complex network in Fig. 1. In this system, parametrically excited van der Pol oscillators are coupled by one resistor R. In this network, the number of nodes which is parametrically excited van der Pol oscillator is 62 and the number of edges which is one resister is 100. The 15th and

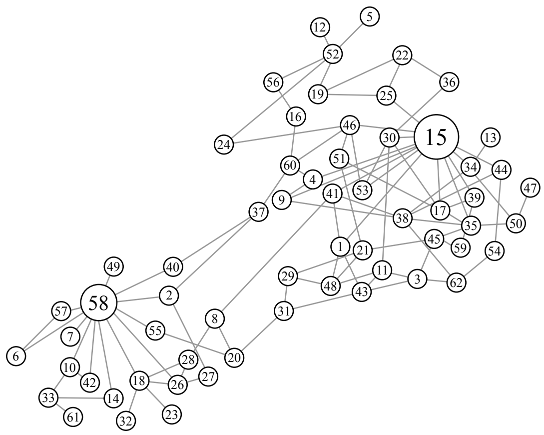


Fig. 1. Proposed network.

58th nodes are hubs in this network. The bond number of 58th node is 11, and the bond number of 15th node is 13.

Table 1 shows the properties of the proposed network as shown in Fig. 1.

TABLE I
PROPERTIES OF PROPOSED NETWORK AS SHOWN IN FIG. 1.

Average degree	3.226
Average clustering coefficient	0.145
Average path length	4.357

III. SYSTEM MODEL

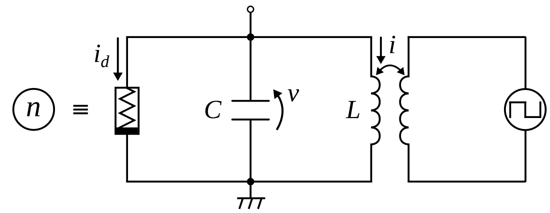
The circuit model of van der Pol oscillator under parametric excitation is shown in Fig. 2 (a). The circuit includes a time-varying inductor L whose characteristics are given as the following equation. The time-varying inductor is shown as Fig. 2 (b). $\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 3, and its amplitude and angular frequency are termed α and ω, respectively. By changing the value of α, the amplitude of parametric excitation can be changed. The $v-i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$L = L_0\gamma(\tau). \tag{3}$$

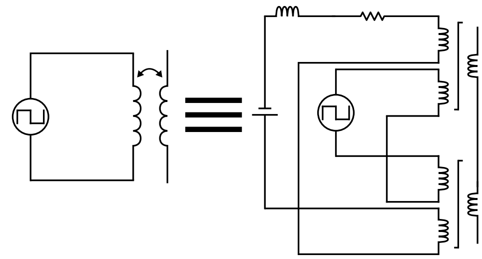
$$i_d = -g_1 v_k + g_3 v_k. \tag{4}$$

By changing the variables and the parameters,

$$\begin{cases} t = \sqrt{L_0 C}\tau,\ v_n = \sqrt{\dfrac{g_1}{g_3}}x_n \\ \omega = \omega_0\sqrt{L_0 C} \\ i_n = \sqrt{\dfrac{g_1}{g_3}}\sqrt{\dfrac{C}{L_0}}y_n \\ \varepsilon = g_1\sqrt{\dfrac{L_0}{C}},\ \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} \end{cases} \tag{5}$$



(a) Parametrically excited van der Pol oscillator.



(b) Time-varying inductor.

Fig. 2. Circuit model.

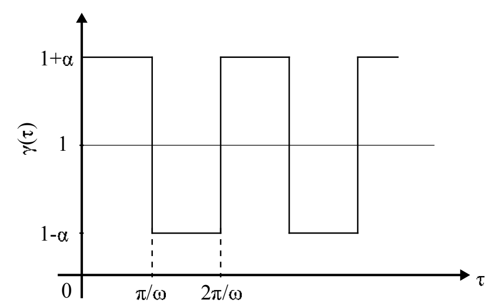


Fig. 3. Function relating to parametrically excitation.

The normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon(x_n - x_n^3) - y_n - \delta \displaystyle\sum_{k\in S_n}(x_n - x_k) \\ \dfrac{dy_n}{d\tau} = \dfrac{1}{\gamma(\tau)}x_n \end{cases} \tag{6}$$

where $n = 1, 2, 3, ..., 10$. S_n is the set of nodes which are directly connected to the node n.

IV. INVESTIGATION OF SYNCHRONIZATION

A. Simulation method

In this research, we investigate the synchronization by using computer simulation. We fix the the circuit parameters as $\varepsilon = 1.00$ and $\omega = 1.00$ for all circuits. Each circuit is given different initial values for computer simulations. In this simulation, all of the nodes involve small mismatch m_n in α (in Fig. 2) which is corresponding to the amplitude of the function relating parametric excitation within the compass of $-0.001 \leq m_n \leq 0.001$.

Additionally, in order to analyze synchronous state, we define the synchronization by the following equation:

$$|x_n - x_k| < 0.10 \quad (k \in S_n). \tag{7}$$

We fix the count of calculation as 100,000.

B. The results of simulation

In these simulations, first, we fixed the coupling strength δ into 1.70 for this network is synchronized as completely. Next, we investigate change of the synchronization probability by adding larger mismatch M from 0.002 to 0.01 in three nodes. These nodes have a certain characteristic corresponding to the structural metrics in order to investigate the relationship between the structural metrics in subset of the nodes including larger mismatch M and synchronization in complex network.

1) Effect of the degree including larger mismatch: In this simulation, we investigate change of synchronous state by adding larger mismatch M from 0.002 to 0.01 in 5 types node. These nodes have the degrees which is 1 to 5. We add the M in the amplitude of the function relating to parametric excitation $(1 + \alpha + m_n + M)$ in three nodes which have the 5 types degrees (1 to 5). Figure 4 shows degree distribution of this network. In this figure, vertical axis denotes the number of nodes, and horizontal axis denotes the value of degree.

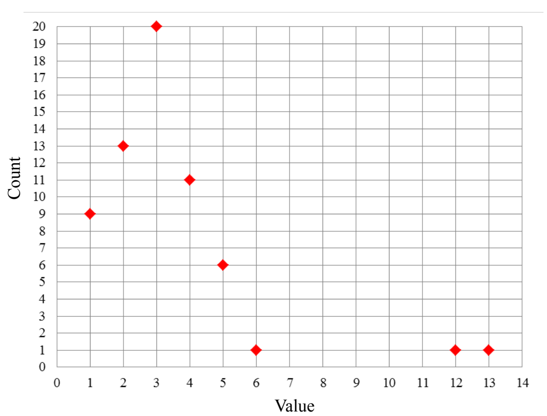


Fig. 4. Degree distribution of proposed network.

Figure 5 shows the 5 types nodes with the value of mismatch M. In this figure, the red nodes express the node of 1 degree, the green nodes nodes express the node of 2 degrees, the purple nodes express the node of 3 degrees, the watery nodes express the node 4 degrees, and the blue nodes express the node of 5 degrees. Figure 6 shows the change of synchronization probability when we add the mismatch as Fig. 5. The colors of each line is corresponding to the colors of each node in Fig. 5. In Fig. 6, the vertical axis denotes synchronization probability, and the horizontal axis denotes the value of mismatch (M). 5 colors lines express degree-by-degree passage of the synchronization probability.

When three nodes have 4 degrees, we can confirm the synchronization probability maintains the highest condition.

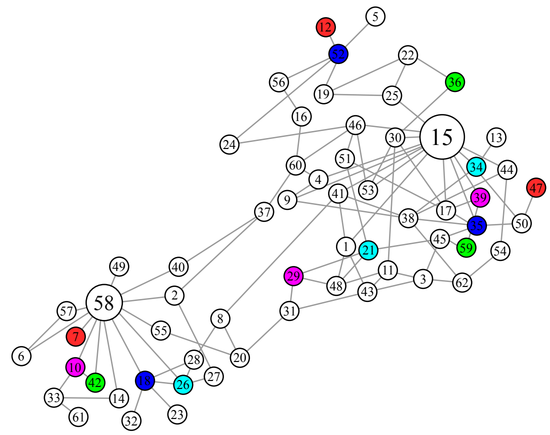


Fig. 5. The location of the 5-color nodes with larger mismatch M.

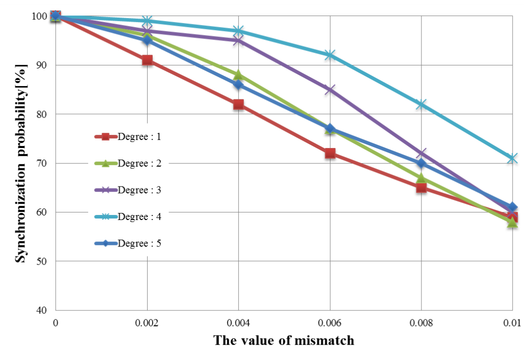


Fig. 6. Synchronization probability.

On the other hand, when three nodes have 1, we can confirm the synchronization probability maintains the lowest condition.

From this result, we can confirm degree of the node including larger mismatch affects the synchronization probability. Additionally, the synchronization probability is affected from not only the size of degrees but also the location of the nodes with larger mismatch M. Following this result, we consider it is important to investigate about the effect of the location of the nodes with larger mismatch M on the synchronization probability in the network.

2) Effect of the path length among nodes which have M: In this simulation, we investigate in what way the synchronization probability in complex network draw influence from the path length among hubs and three nodes with larger mismatch M. We investigate this influence in 3 cases. In the case 1, we investigate the passage of the synchronization probability by changing larger mismatch M and the path length between hubs and each of three nodes with M. In the case 2, the hubs (15th and 58th node) are not connected to any of three nodes with M. Based on the acknowledgment, we investigate the passage of the synchronization probability by changing larger mismatch M and the path length among each of three nodes with M. In the case 3, the hubs (15th or 58th node) are connected to any of three nodes with M. On that basis, we investigate the passage of the synchronization probability by changing larger mismatch M and the path length among each of three nodes with M.

First of all, in the case 1, synchronization probability changes as shown in Fig. 7. Three lines are corresponding to the value of the path length from the hubs to the three nodes with M. From this result, the synchronization probability of each line increase with decreasing the path length from the hubs to the three nodes with M.

Next, in the case 2, synchronization probability changes as shown in Fig. 8. Four lines are corresponding to the value of the path length among each of three nodes with M. In this figure, the passage of the synchronization probability does not have that large a change, however we can confirm the synchronization probability maintains the lowest condition when the path length among each of three nodes with M is 1.

Finally, in the case 3, synchronization probability changes as shown in Fig. 9. Four lines are corresponding to the value of the path length among each of three nodes with M. As compared with the result of Fig. 8, the difference of each line broaden out. Following this result, we can confirm the hubs exercise a much further effect in the synchronization probability in the network.

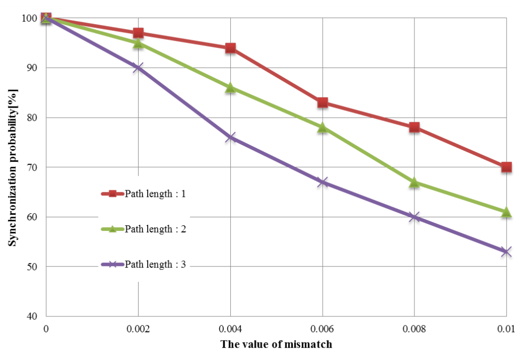


Fig. 7. Synchronization probability (case 1).

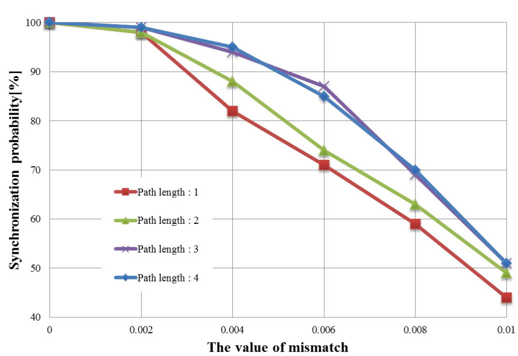


Fig. 8. Synchronization probability (case 2).

V. Conclusions

In this study, in order to more particular investigation of synchronization in complex network, we focus on relationship

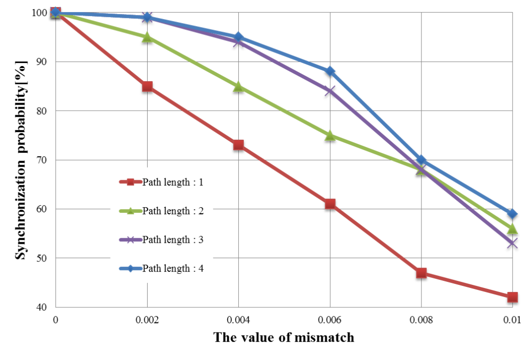


Fig. 9. Synchronization probability (case 3).

between the structural metrics in subset of the nodes including larger mismatch and synchronization in complex network. In order to research this relationship, we investigate synchronization in complex network by changing structural metrics (degree and path length) of the nodes corresponding to the subset of the nodes adding larger mismatch than other nodes. First, we investigate the synchronization probability in the network by changing the degree of the three nodes added larger mismatch. Next, we investigate the synchronization probability in the network by changing the path length among hubs, three nodes with larger mismatch.

We could observe the various change of the synchronization probability by changing structural metrics (degree and path length) of the nodes corresponding to the subset of the nodes adding larger mismatch than other nodes.

References

[1] I. Belykh, M. Hasler, M. Lauret and H. Nijmeijer, “Synchronization and graph topology,” Int. J. Bifurcation and Chaos, vol.15, no.11, pp.3423-3433, Nov. 2005.

[2] J. Cosp, J. Madrenas, E. Alarcon, E. Vidal and G. Villar, “Synchronization of nonlinear electronic oscillators for neural computation,” IEEE Trans. Neural Networks, vol.15, no.5, pp.1315-1327, Sep. 2004.

[3] C. Hayashi, “Nonlinear Oscillations in Physical Systems,” Chap. 11, McGraw-Hill, New York (1964).

[4] C. Hayashi, M. Abe, K. Oshima and H. Kawakami, “The method of mapping as applied to the solution for certain types of nonlinear differential equations,” Ninth International Conference on Nonlinear Oscillations, Kiev (Aug.-Sept.1981).

[5] M. Inoue, “A Method of Analysis for the Bifurcation of the Almost Periodic Oscillation and the Generation of Chaos in a Parametric Excitation Circuit,” Trans. of IEICE, vol. J68-A, no. 7, pp. 621-626, 1985.

[6] H. Kumeno, Y. Nishio, “Synchronization Phenomena in Coupled Parametrically Excited van der Pol Oscillators,” *Proc. NOLTA’08*, pp. 128-131, Sep. 2008.

[7] D. Lusseau, K. Schneider, O. J. Boisseau, P. Haase, E. Slooten and S. M. Dawson, “The bottlenose dolphin community of Doubtful Sound features a large population of long-lasting associations. Can geographic isolation explain this unique trait?”, *Proc. Behavioral Ecology and Sociobiology*, vol.54, pp. 396-405, 2003

[8] K. Oi, K. Ago, Y. Uwate, Y. Nishio, “Synchronization in Complex Networks with Dispersion by Coupled Parametrically Excited Oscillators”, *Proc. NCSP’15*, pp. 33-36, March, 2016.