Chaos Propagation in Coupled Chaotic Circuits with Multi-Ring Combination

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***Abstract*—In this study, we investigate chaos propagation in coupled chaotic circuits with multi-ring combination. We compare the different coupling combination. These models are coupled chaotic circuits when one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. By using computer simulations, we have observed that the chaotic attractor is propagated to the other circuits. The three-periodic attractors are affected from the chaotic attractors by changing the coupling combination and increasing the coupling strength. Moreover, we confirm that chaos propagation of the network without hub is faster than the network with hub.**

I. Introduction

Propagation in the network have attracted a great deal of attention from various fields. It is important to investigate chaos propagation under some difficult situations for the circuits. For example of some difficult situations, network is briefly given external stimulation and frustration is occurred in the network. Furthermore, viral infection and the traffic jam of the transportation network are mentioned as an example of propagation in the network. Therefore we consider that it is necessary to investigate that behavior of unlike the others influence the whole system. In the biology, we can prevent the unknown virus spreading if we comprehend the way of viral infections. Additionally, it is applicable to the fields of medical science and biology and so on. However, there are not many studies of large-scale network of continuous-time real physical systems such as electrical circuits. [1]-[5]

In the previous studies, chaos propagation have been investigated in ladder, ring or simple network system of coupled chaotic circuits [6]-[8]. In this simple network, the three-periodic attractors are affected from the chaotic attractors when the coupling strength and the number of edge are increasing. Moreover, we have observed that the process to chaos is changed in each model.

In this study, we investigate chaos propagation in coupled chaotic circuits with multi-ring combination. We propose a multi-ring combination system using of chaotic circuits coupled by the resistors. In this model, one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. First, we observe how to chaos propagation by increasing the coupling strength. Moreover, by changing the method of connecting the edges, we investigate chaos propagation in the entire system.

II. System model

The chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes.

We propose the triple-ring and quad-ring combination system (see Fig. 2). Each model is used chaotic circuit coupled by the resistors. In each model, central circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. One ring is consisted by five circuits. Additionally, multi-ring in this study is consisted by three or four rings.

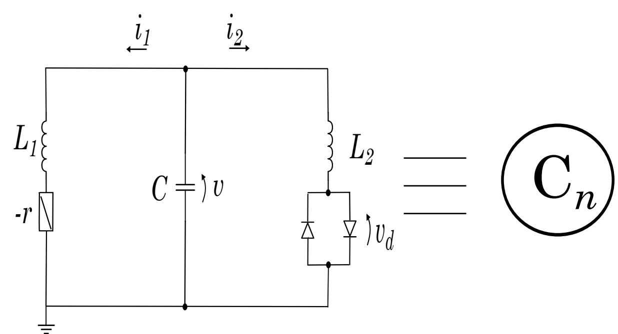

Fig. 1. Chaotic circuit.

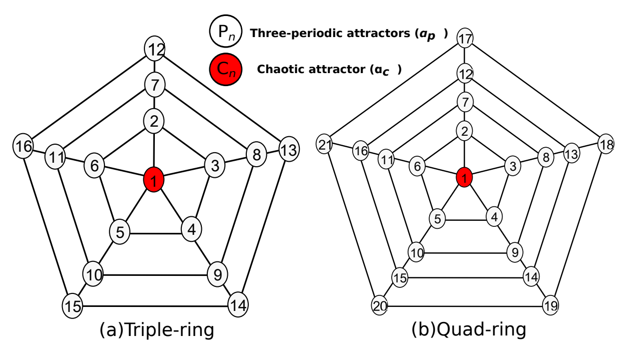

Fig. 2. Multi-ring combination.

The circuit equations of this circuit are described as follows:

$$
\begin{cases}
L_1 \dfrac{di}{dt} = v + ri \\[2ex]
L_2 \dfrac{di}{dt} = v - v_d \\[2ex]
C \dfrac{dv}{dt} = -i_1 - i_2.
\end{cases}
\tag{1}
$$

The characteristic of nonlinear resistance is described as follows:

$$
v_d = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \tag{2}
$$

By changing the variables and parameters,

$$
\begin{cases}
i_1 = \sqrt{\dfrac{C}{L_1}} V x_n, \quad i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n, \quad v = V z_n \\[2ex]
\alpha = r\sqrt{\dfrac{C}{L_1}}, \quad \beta = \dfrac{L_1}{L_2}, \quad \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\[2ex]
\gamma = \dfrac{1}{R}\sqrt{\dfrac{L_1}{C}}, \quad t = \sqrt{L_1 C_2}\tau,
\end{cases}
\tag{3}
$$

The normalized circuit equations are given as follows:

$$
\begin{cases}
\dfrac{dx_n}{d\tau} = \alpha x_n + z_n \\
\dfrac{dy_n}{d\tau} = z_n - f(y_n) \\
\dfrac{dz_n}{d\tau} = -x_n - \beta y_n.
\end{cases}
\tag{4}
$$

where α represents the chaos degree. $f(y_i)$ is described as follows:

$$
f(y_n) = \frac{1}{2}\left(\left|y_n + \frac{1}{\delta}\right| - \left|y_n - \frac{1}{\delta}\right|\right). \tag{5}
$$

In the proposed multi-ring system, the circuits are connected to only adjacent circuits by the resistors. The normalized circuit equations of the system are given as follows:

$$
\begin{cases}
\dfrac{dx_n}{d\tau} = \alpha x_n + z_n \\
\dfrac{dy_n}{d\tau} = z_n - f(y_n) \\
\dfrac{dz_n}{d\tau} = -x_n - \beta y_n - \displaystyle\sum_{j \in S_n} \gamma(z_i - z - j) \\
\qquad\qquad\qquad\qquad (n = 1, 2, \cdots, N).
\end{cases}
\tag{6}
$$

In Eq. (6), N is the number of coupled chaotic circuits and γ is the coupling strength. We define α_c to generate the chaotic attractor and α_p is defined to generate the three-periodic attractors. For the computer simulations, we calculate Eq. (6) using the fourth-order Runge-Kutta method with the step size h = 0.01. In this study, we set the parameters of the system as $\alpha_c = 0.460$, $\alpha_p = 0.412$, $\beta = 3.0$ and $\delta = 470.0$.

III. Triple Ring

A. System patterns

In this section, we explain the method of connecting the edges from ring to ring in triple ring model. We define the number of edge from central circuit to ring or from ring to ring is fixed to only one. In this triple ring model, we use 16 circuits. Central circuit is set to generate chaotic attractor and the other 15 circuits are set to generate three-periodic attractors. Furthermore, we change the method of connecting the edges. Figure 3 shows all system patterns in triple ring model under this condition.

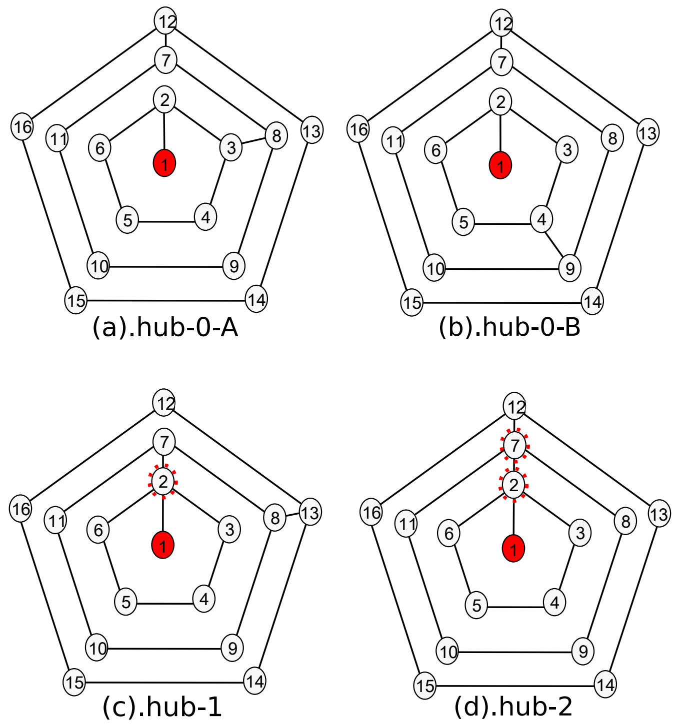

Fig. 3. Triple-ring combination.

We divide the coupling combination into four patterns (see Fig. 3). Each pattern is different of the connection edge or the number of hubs. We define hub as the circuit which connected by four edges. In our proposed system, there are some hubs in each system. The hub-1 model has only one hub such as 2nd circuit (see Fig. 3(c)). The the 2nd circuit is connected from 1st, 3rd, 6th, 7th circuit. The hub-2 model has two hubs such as the 2nd circuit and the 7th circuit (see Fig. 3(d)). The 2nd circuit is connected from 1st, 3rd, 6th, 7th circuit and the 7th circuits connected from 2nd, 8th, 11th and 12th circuit.

Additionally, we divide the hub-0 model into two patterns according to distance from central circuit to the farthest circuit. For example, the shortest route is passed from 1st, 2nd, 3rd, 4th, 9th, 8th, 7th, 12th, 13th to 14th in the hub-0-A model (see Fig. 4(a)). The hub-0-A model is passed 9 circuits. In contrast, the shortest route is passed from 1st, 2nd, 3rd, 8th, 7th, 12th, 13th to 14th in the hub-0-B model (see Fig. 4(b)). The hub-0-B model is passed 7 circuits.

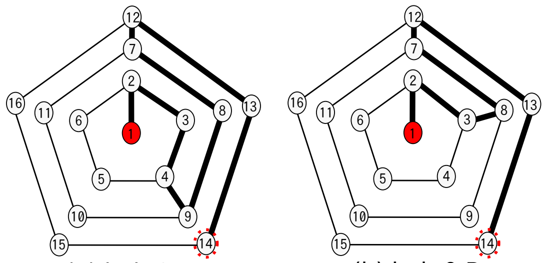

Fig. 4. Route from central circuit.

B. Minimum coupling strength

In this section, we investigate the minimum coupling strength when three-periodic attractors are changed to chaos attractor. When we increasing the coupling strength, we compare the each model. The simulation results are summarized in Fig 5. We define the attractor as a chaos attractor, if the attractor becames over three-periodic attractor. First, we compare the model with hub and without hub. The coupling strength of the model without hub is smaller than the model with hub. Next, we compare the hub-0-A model and the hub-0-B model. The distance form central circuit to the farthest circuit is different in each model. The Coupling strength of the hub-0-B model is smaller than the hub-0-B model.

Accordingly, we confirm that the model without hub is that chaos attaractor propagates faster than the model with hub. From these results, we consider that the hub weakens weight of chaos from central circuit. Because the hub circuit connects to many circuit than other circuit, we consider that the influence of chaos is slacked off by hub circuit.

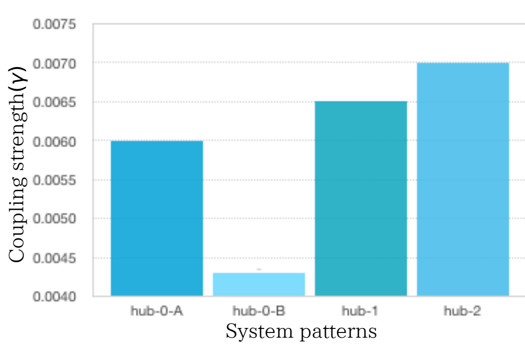

Fig. 5. Minimum coupling strength (triple-ring).

C. Chaos propagation

Figures 6 and 7 show chaos propagation attractors in the double ring combination system by increasing the coupling strength γ_c. At this time, we fix the coupling strength as $\gamma = 0.0000$, all circuits is not propagated the chaotic attractor of 1st chaotic circuit (see Fig. 6). The 1st circuit is chaotic attractor and from 2nd to 16th circuits are three-periodic attractor.

Moreover, from the result of the minimum coupling strength, we investigate the chaos propagation. When we fixed the coupling strength as $\gamma = 0.0060$, all circuits are propagated the chaotic attractor of 1st chaotic circuit in the hub-0-A and the hub-0-B model (see Fig. 7(a), (b)). On the other hand, all circuits are not propagated chaotic attractor in the hub-1 and the hub-2 model (see Fig. 7 (c), (d)). In the model with hub, when we fixed the coupling strength as $\gamma = 0.0060$, some circuit are not propagated the chaotic attractor. For example, the 10th and 11th circuits are not propagated in hub-1 model. we consider that the the result is affected by initial value. If the initial value change, the 10th and 11th circuits are propagated and some other circuits are not propagated in hub-1 model.

IV. Quad Ring

A. System patterns

We investigate quad ring model with hub or without hub. In quad ring model, we use 21 circuits. Central circuit is set to generate chaotic attractor and the other 20 circuits are set to generate three-periodic attractors. We define the number of edge from central circuit to ring or from ring to ring is fixed to only one. In quad-ring model, we define hub as the circuit which connected by four edges. There are some hubs in each system. We divide the coupling combination into 4 patterns(see Fig. 8). Each pattern is different of the connection edge and the number of hub. For example, the hub-3 model has 3 hubs such as the 2nd, 7th and 12th circuit.

B. Minimum coupling strength

In this section, we investigate the minimum coupling strength when three-periodic attractors are changed to chaos attractor. When we changing the coupling strength, we compare the each model. The simulation results are summarized in Fig 9. We define the attractor as a chaos attractor, if the attractor becames over three-periodic attractor. The minimum coupling strength of hub-0 model is smaller than the model with hub. Additionally, as the number of the hub increase, so minimum coupling strength is larger.

From the result, when we increasing the number of hub, chaos propagation of the model with many hubs is more difficult than the model without hub.

V. Conclusions

In this study, we have investigated chaos propagation in coupled chaotic circuits as our proposed system. By the computer simulations, we have observed that the chaotic attractor is propagated to the other circuits. The three-periodic attractors are affected from the chaotic attractors. Moreover, we confirm that the model with hub weakens weight of chaos from central circuit. From the result, chaos propagation has been changed over time from three-periodic attractor to chaotic attractor or from chaotic attractor to three-periodic attractor.

For the future works, we define the convergence time from three-periodic attractors to chaos in order to investigate the obtained phenomena of multi-ring system in detail.

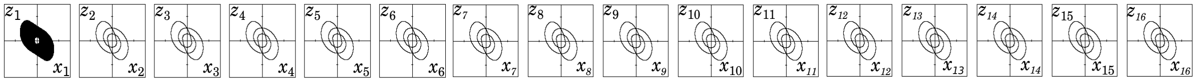

Fig. 6. Chaos propagation ($\gamma = 0.0000$).

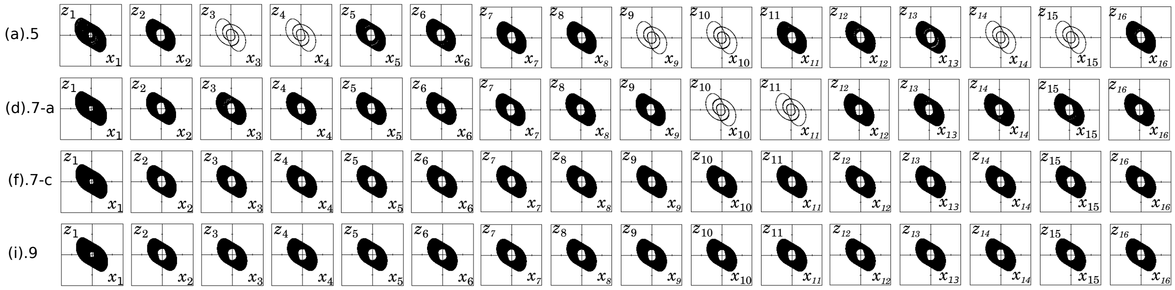

Fig. 7. Chaos propagation ($\gamma = 0.0060$).

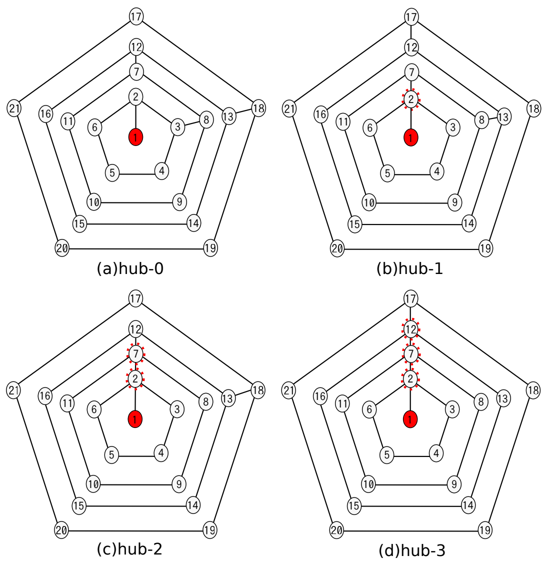

Fig. 8. Quad-ring combination.

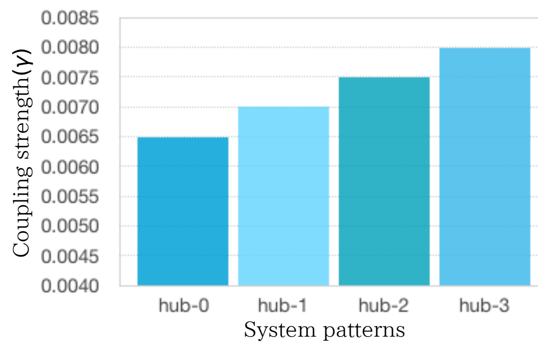

Fig. 9. Minimum coupling strength (quad-ring).

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