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Synchronization Phenomena between Triangle System and Two-Triangles System of FitzHugh-Nagumo Neurons with Diffusive Coupling

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Abstract

We study synchronization phenomena on FitzHugh-Nagumo (FHN) model. We investigate a three-neuronal system and a four-neuronal system. We focus on the coexistence of the periodic solutions on the each system by changing the initial value and the coupling strength. Some periodic solutions depend on the initial value. In addition, we investigate firing patterns on the both systems. The only one firing pattern which exist on the four-neuronal system is found.

1. Introduction

Many chemical and biological phenomena can be modeled in reaction-diffusion systems. The reaction-diffusion systems have various dynamical behavior. Excitability is the most important characteristic and the foundation on numerous physical biological systems.

Creature's periodic motion is controlled by the neural circuit called Central Pattern Generator (CPG). Neural oscillator is what has modeled excitation and inhibitory mechanism between the neurons in the neural circuit. FHN model is one of neural oscillator models. Concretely, for example, we can apply neural oscillators to robot motion control and nervous disease. The network of neurons in the brain exhibits a subtle balance of dynamic chaos and self-organized order[1]. A number of neurological disease are characterized by a disturbance of this balance. For instance, it is synchronized firing of electrical pluses of the neurons. Time-delayed feedback control has recently been applied to suppress this undesired synchrony.

Periodic neural firing is produced by CPG and has great significance for the control of dynamic functions of the body[2]. It is worth to investigate the mechanism of neural networks which cause such a variety of periodic activities[3]. These complex firing patterns which include chaotic firings are modeled through the effect of high dimensional dynamics of an individual element, interaction of many neurons with spatial degrees of freedom, and time delayed coupling[4]. There are many studies for excitable elements under external periodic stimuli, which show various behavior including phase-locking and chaotic behavior. It has been also studies for an ensemble of oscillatory or excitatory elements interacting each other in the context of synchronization and chaos[5].

Creatures have the neural circuit in their bodies. It is a very complex structure. For the reason, we would like to investigate more complex networks. However, we need to investigate fundamental networks. In this study, we investigate synchronization phenomena of a three-neuronal system and a four-neuronal system on FHN model. We focus on two points. The one is the difference of synchronization phenomena by changing the initial value on the each system. The another one is the difference of periodic solutions including firing patterns between the three-neuronal system and the four-neuronal system. We observe the coexistence of the periodic solutions. In addition to that, we find the only firing pattern which exist on the four-neuronal system.

2. Fitzhugh-Nagumo model

In the following, we study the system which is modeled by the excitable FHN models. The FHN model is a two-dimensional simplification of the Hodgkin-Huxley model of spike generation in squid giant axons. This model was suggested by FitzHugh and the equivalent circuit was proposed by Nagumo et al. FHN model is a very simple model for excitable system. The model contains the only two variables, however, does not describe a specific biochemical reaction. It can be used to describe neural and cardiac dynamics.

FHN model is given by the following equations:

$$\frac{du_i}{dt} = u_i(u_i - \alpha)(1 - u_i) - v_i + \frac{K}{N}\sum_{i \neq j}(u_j - u_i) \tag{1}$$

$$\frac{dv_i}{dt} = \tau(u_i - \gamma v_i) \tag{2}$$

where u_{ij} is the activator, v_{ij} is the inhibitor, α, τ and γ are parameters, K is the coupling strength and N is the number of elements. α, τ and γ are fixed at $\alpha = 0.01$, $\tau = 0.001$ and $\gamma = 0.0$ because these values are used in previous work of a pair of excitable FHN elements. The time series are generated by the fourth-order Runge-Kutta method with step size $h = 0.05$.

3. A pair of excitable neurons

As a previous study, there is a study which focuses on a minimal model that consists of repulsively coupled two excitable neurons. This model produces and sustains various firing patterns. There are many types of sequence of firing. The diffusive coupling with a negative coefficient has been considered as the effect of phase-repulsive coupling on two dimensional coupled FHN arrays. The periodic firings are found in two neurons with excitatory and inhibitory synaptic couplings.

Five firing patterns have found on two neurons by changing the coupling strength K from -1 to 0. As an example, we show time waveforms of two neurons in case of $K = -0.012$ in Fig. 1. We explain two-phase firing pattern in detail. After N_1 excites, N_2 excites soon. After these successive excitations, both neurons stay quiescent state for a while. On the next successive excitations, N_2 excites first. After that, N_1 excites soon. These states are repeated. Accordingly, we can symbolize this firing pattern as $N_1N_2 - N_2N_1-$. In the same way, we can symbolize four other firing patterns as N_1N_2-, $N_1N_2N_1 - N_2N_1N_2-$, $N_1N_2 - N_1N_2 - N_2N_1 - N_2N_1-$ and $N_1N_2 - N_1N_2 - N_1N_2 - N_2N_1 - N_2N_1 - N_2N_1-$.

Here, we focus on the three-neuronal system and the four-neuronal system. In this study, we consider analysis of these systems by changing the initial value. We find that these systems show a various of firing patterns including chaotic firing.

4. Network Systems

We introduce the network systems used in this study. We investigate two network systems. The three-neuronal system is shown in Fig. 2. This system is a very simple system that a neuron is just connected to a system of two neuron as a ring topology. The each neuron connects with the other neurons. We name this three-neuronal system triangle system. The four-neuronal system is shown in Fig. 3. This system is a system that a neuron is connected to a triangle system. We put the neuron on the place that N_2 and the neuron contrast with a side between N_1 and N_3 on the triangle system as a symmetry axis. This system has axial symmetry and include two triangle. Therefore, we name this four-neuronal system two-triangle system. The each neuron connects with the other neurons, except that N_2 does not connect with N_4.

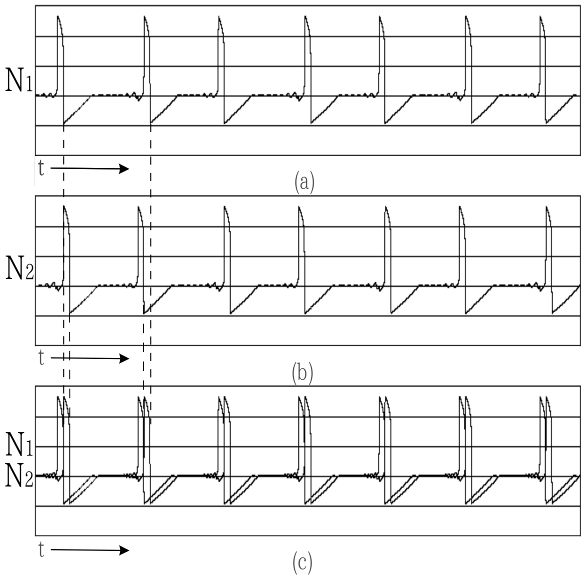


Figure 1: Time evolution of two neurons on FHN model for the coupling strength $K = -0.012$. The solid lines of (a) and (b) correspond to time waveforms of N_1 and N_2, respectively. The solid lines of (c) correspond to both neuronal time waveforms.

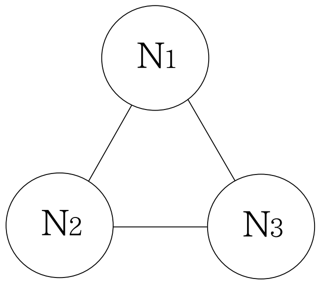


Figure 2: The triangle system.

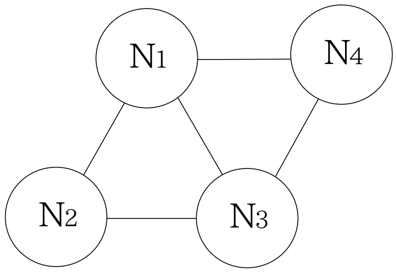


Figure 3: The two-triangle system.

5. Numerical Experiments

5.1 Coexistence of Periodic Solutions

We investigate synchronization phenomena on triangle system and two-triangle system by changing the coupling strength K from -1 to 0. We also focus on the coexistence of the periodic solutions by changing the initial value and various firing patterns. In this section, we consider the coexistence of the periodic solutions. We decide the conditions of the initial value. We define some rules. We use the same absolute value on all neurons and investigate the systems by means of five types of the absolute values. The absolute values are $0.25, 0.5, 1, 2$ and 4. The negative sign is set to odd-numbered neurons. Likewise, the positive sign is set to even-numbered neurons.

In the beginning, we consider the triangle system. Figure 4 shows the number of solutions by changing the coupling strength K from -1 to 0 on the triangle system. In this figure, "1" of solutions mean that the system has the same solution at all five types of the initial value. "0" of solutions correspond to chaotic firing patterns. These patterns are observed in the -0.7 to -0.61 range. "2" of solutions are observed, which show the coexistence of the periodic solutions.

Secondly, we investigate the two-triangle system. Figure 5 shows the number of solutions by changing the coupling strength K from -1 to 0 on the two-triangle system. "0" of solutions correspond to chaotic firing patterns likewise. These patterns are observed in the -0.98 to -0.65 range. By connecting a neuron to the triangle system, the range of chaotic firing patterns on the triangle system is larger than that on the two-triangle system. On the two-triangle system, "2" of solutions are also observed, which show the coexistence of the periodic solutions. There are more cases which have the coexistence of the periodic solutions than on the triangle system.

5.2 Comparison of Firing Patterns

In this section, we focus on firing patterns. First of all, we investigate the periodic solutions on triangle system and two-triangle system by changing the coupling strength K from -1 to 0. In almost every cases, the periodic solutions do not depend on the initial value on the both systems. That is why we consider the only case that the absolute value of the initial value is 0.25 as an example. Figures 6 (a) and 6 (b) show the periodic solutions by changing the coupling strength K from -1 to 0 by one hundredth on the triangle system and on the two-triangle system, respectively. There are the existence of three periodic solutions in Fig. 6 (b) compared with Fig. 6 (a). That is clear at a glance. The triangle system is different from the two-triangle system in that it has the existence of three periodic solutions.

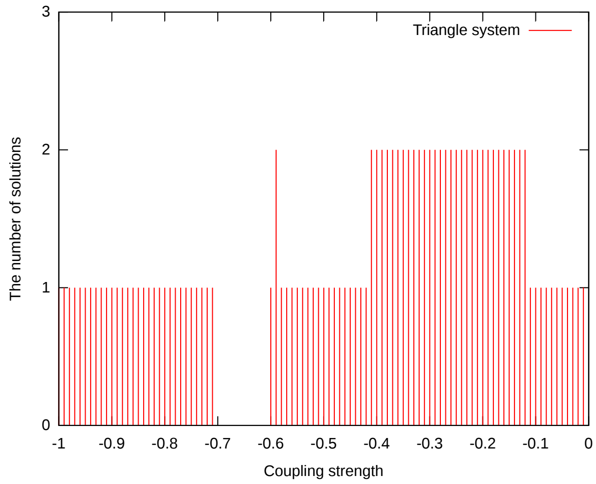


Figure 4: The number of solutions by changing the coupling strength K from -1 to 0 on the triangle system.

If we look at types of firing patterns, there are four types of firing patterns on the triangle system. We can symbolize the four firing patterns on the triangle system as $N_1(N_3)N_2-$, $N_2N_1(N_3)-$, $N_2N_1(N_3) - N_2N_1(N_3)-$ and $N_1(N_3)N_2 - N_2N_1(N_3)-$. Here, $N_1(N_3)$ means that N_1 is perfectly synchronized with N_3. On the other hand, there are six types of firing patterns on the two-triangle system. We can symbolize the six firing patterns on the two-triangle system as $N_1(N_3)N_2(N_4)-$, $N_2(N_4)N_1(N_3)-$, $N_1(N_3)N_2(N_4) - N_1(N_3)N_2(N_4)-$, $N_2(N_4)N_1(N_3) - N_2(N_4)N_1(N_3)-$, $N_1(N_3)N_2(N_4) - N_2(N_4)N_1(N_3)-$ and $N_1(N_3)N_2(N_4)N_1(N_3) - N_2(N_4)N_1(N_3)N_2(N_4)-$. $N_1(N_3)N_2(N_4) - N_1(N_3)N_2(N_4)-$ is the same with $N_2(N_4)N_1(N_3) - N_2(N_4)N_1(N_3)-$ as a type. However, $N_1(N_3)N_2(N_4)N_1(N_3) - N_2(N_4)N_1(N_3)N_2(N_4)-$ is observed for the first time by connecting a neuron to the triangle system. For example, this firing pattern is observed in case of $K = -0.990$. The observed time waveforms are shown in Fig. 7. Time waveforms N_1 and N_3 are synchronized at in-phase with N_2 and N_4. On the first successive excitations, the order of firing is $N_1N_2N_1-$. On the next successive excitations, the order of firing is $N_2N_1N_2-$. These states are repeated. A whole new firing pattern which is not found on the triangle system is observed by connecting a neuron to the triangle system. Three successive firings on one successive excitations were not observed until on the two-triangle system.

6. Conclusions

We investigated synchronization phenomena of the triangle system and the two-triangle system on FHN model by changing the initial value and the coupling strength. Some solutions

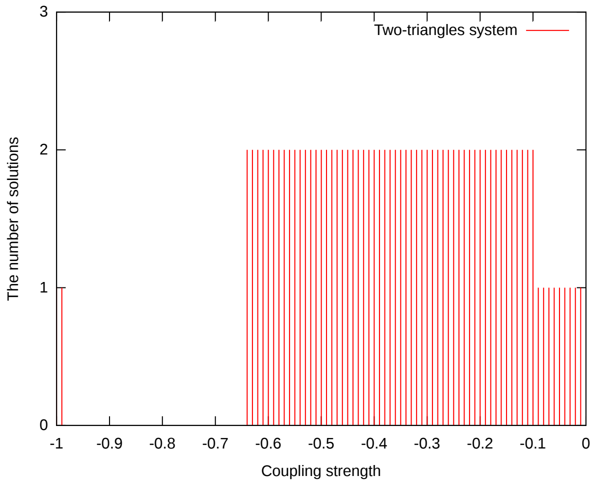
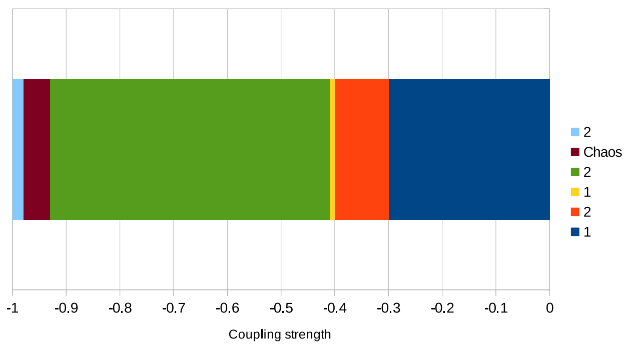
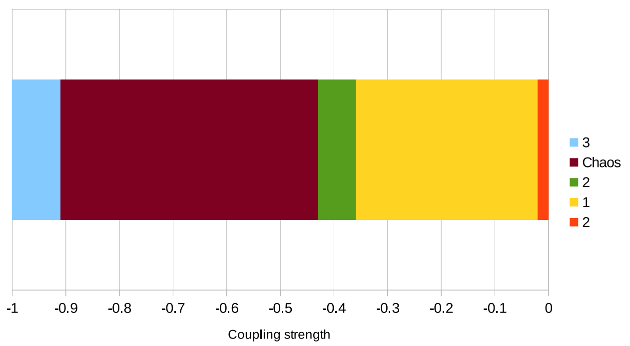


Figure 5: The number of solutions by changing the coupling strength K from -1 to 0 on the two-triangle system.



(a) The triangle system.



(b) The two-triangle system.

Figure 6: Periodic solutions by changing the coupling strength K from -1 to 0.

depended on the initial value, and others did not. There were more cases which have the coexistence of the periodic solu-

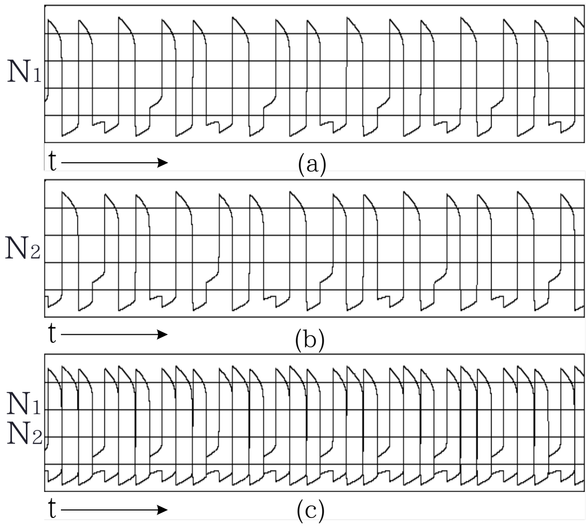


Figure 7: Time evolution of the two-triangle system on FHN model for the coupling strength $K = -0.990$. The solid lines of (a) and (b) correspond to time waveforms of $N_1(N_3)$ and $N_2(N_4)$, respectively. The solid lines of (c) correspond to all neuronal time waveforms.

tions as with chaotic firing pattern on the two-triangle system than those on the triangle system. As well as on a pair of excitable neurons, some firing patterns were observed on the triangle system and the two-triangle system. A whole new firing pattern was observed by connecting a neuron to the triangle system. The firing pattern has three periodic solutions.

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