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Synchronization State of Chaotic Circuit Containing Time Delay in One Direction

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Abstract

Synchronization state can be observed in coupled circuits. Further, interesting synchronization state was confirmed in coupled time delayed chaotic circuits. In this study, we propose novel coupled systems and investigate synchronization state in coupled time delayed chaotic circuits. The proposed coupling methods of time delayed chaotic circuits depend on attractor types. We focus on the relationships between synchronization state and the coupling methods. Moreover, we investigate the special coupling methods of time delayed circuit in this study.

1. Introduction

There are many nonlinear systems containing time delay, such as neural networks, control systems, meteorological systems, biological systems and so on in the natural world. Namely, it is considered that investigation of stability in such time-delay systems is significant [1]. Generation of chaos of them all is reported self excited oscillation system containing time delay [2]. This chaotic circuit can be easily realized by using simple electric circuit element and analyzed exactly. On the other hand, there are examples of nonlinear phenomena, chaotic synchronization and so on [3]. In particular, many studies on synchronization of coupled chaotic circuits have been reported [4].

In this study, we devise coupled systems that takes advantage of features of the time delayed chaotic circuit. The novel coupled systems are utilizing the characteristics of the circuit having time delayed feedback. This circuit is auto gain controlled chaotic oscillator containing time delay. The circuit has feedback systems which control the gain. We investigate synchronization state in coupled time delayed chaotic circuits. By carrying out computer simulations, time delay of subcircuits effects a change of synchronization state.

2. Time Delayed Chaotic Circuit

Figure 1 shows the time delayed chaotic circuit. This circuit consists of one inductor L , one capacitor C , one linear negative resistor $-g$ and one linear positive resistor G of which amplitude is controlled by the switch containing time delay. The current flowing through the inductor L is i, and the voltage between the capacitor C is v. The circuit equations are normalized as Eqs. (1) and (2) by changing the variables as below.

(A) In case of switch connected to $-g$,

$$\begin{cases} \dot{x} = y \\ \dot{y} = 2\alpha y - x, \end{cases} \quad (1)$$

(B) In case of switch connected to G,

$$\begin{cases} \dot{x} = y \\ \dot{y} = -2\beta y - x. \end{cases} \quad (2)$$

By changing the parameters and variable as follow:

$$i = \sqrt{\frac{C}{L}} V_{th} x, v = V_{th} y, t = \sqrt{LC}\tau,$$
$$g\sqrt{\frac{C}{L}} = 2\alpha \text{ and } G\sqrt{\frac{C}{L}} = 2\beta.$$

The switching operation is shown in Fig. 2, it controls the amplitude of the oscillator. This switching operation is included time delay. T_d denotes the time delay. First, the switch is connected to a negative resistor. In state of that, the voltage v is amplified up to while v is oscillating, the amplitude exceeds the threshold voltage V_{th} which is the threshold control

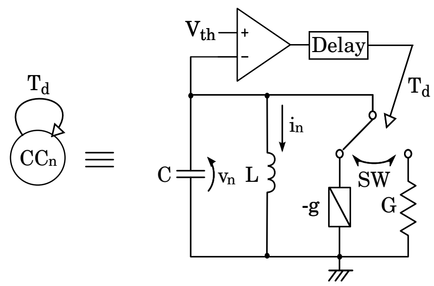


Figure 1: Time delayed chaotic circuit.

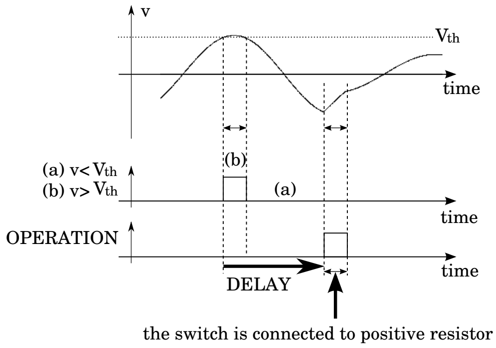


Figure 2: Switching operation.

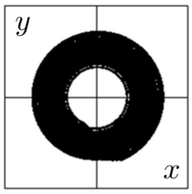


Figure 3: Chaotic attractor obtained by computer simulation.

loop. Second, the system memorize the time as T_{th} while v is exceeding the threshold voltage V_{th} and that state is remained for T_{th}. In subsequent the instant of exceeding threshold V_{th}, the switch stays the state for T_d. After that switch is connected to positive resistor during T_{th}. The switch does not immediately connect in the positive resistor however the switch is connected after T_d. A set of switching operations control the amplitude of v. Figure 3 shows chaotic attractor of time delayed chaotic circuit. By using mapping method to this circuit, we could derive the 1-dimensional Poincard map explicitly from each circuit, and the Poincark map was proved to have a positive Liapunov number with computer assistances [3].

3. Ring Coupled Time Delayed Chaotic Circuit

In this section, we investigate synchronization state of the coupling methods of three coupled time delayed chaotic circuits. Figure 4 shows the schematic of coupled three time delayed chaotic circuits. Two cases of interest are considered: coupling elements are resistors R_0 or inductors L_0. By changing the parameters and variables as follows:

$$i_n = \sqrt{\frac{C}{L}} V_{th} x_n,\, v_n = V_{th} y_n,\, t = \sqrt{LC}\tau,$$

$$g\sqrt{\frac{C}{L}} = 2\alpha,\, G\sqrt{\frac{C}{L}} = 2\beta \text{ and } \gamma = R_0\sqrt{\frac{C}{L}}.$$

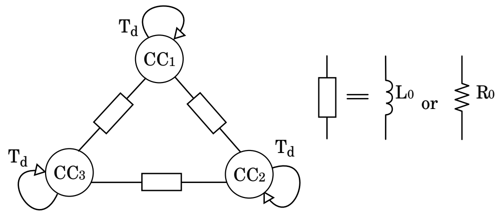


Figure 4: Ring coupled system by resistors.

3.1 Coupled by resistors R_0

Here is case of coupled system by resistors R_0. The normalized circuit equations of the system are given as follows:

(A) In case of that switch is connected to $-g$,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -x_n + 2\alpha y_n + \gamma(y_{n-1} - 2y_n + y_{n+1}), \end{cases} \quad (3)$$

(B) In case of that switch is connected to G,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -x_n - 2\beta y_n + \gamma(y_{n-1} - 2y_n + y_{n+1}), \end{cases} \quad (4)$$

where $(n = 1, 2, 3)$ and $x_0 = x_3$, $x_4 = x_1$. Figure 5 shows some of simulation results. In calculation result, in-phase synchronization state can be observed. When the coupling strength γ is large, full in-phase synchronization can be observed. However full in-phase synchronization can not be observed or synchronization is lost in case of small coupling strength γ.

3.2 Coupled by resistors L_0

In case of time delayed chaotic circuits coupled by inductors L_0, Fig 6 shows some of simulation results. By changing the parameters and variables when ring coupled system is connected by the inductor as follows:

$$i_n = \sqrt{\frac{C}{L}} V_{th} x_n,\, v_n = V_{th} y_n,\, t = \sqrt{LC}\tau,$$

$$g\sqrt{\frac{C}{L}} = 2\alpha,\, G\sqrt{\frac{C}{L}} = 2\beta \text{ and } \gamma' = \frac{L}{L_0}.$$

The normalized circuit equations of the system are given as follows:

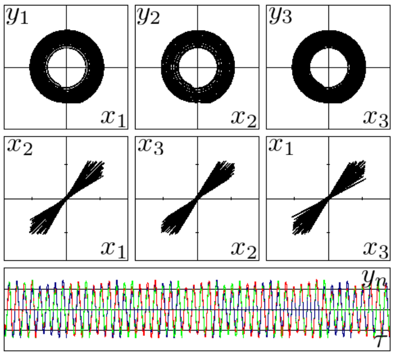
(A) In case of that switch is connected to $-g$,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -x_n + 2\alpha y_n + \gamma'(x_{n-1} - 2x_n + x_{n+1}), \end{cases} \quad (5)$$

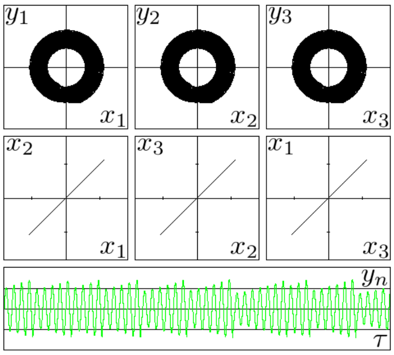
(B) In case of that switch is connected to G,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -x_n - 2\beta y_n + \gamma'(x_{n-1} - 2x_n + x_{n+1}), \end{cases} \quad (6)$$

where $(n = 1, 2, 3)$ and $x_0 = x_3$, $x_4 = x_1$. In-phase synchro-



(a) $\alpha = 0.015$, $\beta = 0.5$, $\gamma = 0.01$ and $T_d = \pi$.



(b) $\alpha = 0.015$, $\beta = 0.5$, $\gamma = 0.1$ and $T_d = \pi$.

Figure 5: Simulation results of ring coupled system by resistors. Attractor. Lissajous figure. Time waveform. Red, blue and green colors denote y_1, y_2 and y_3 respectively.

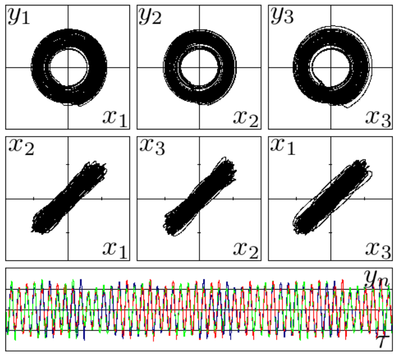
nization and three-phase synchronization can be observed in the ring coupled system by inductors L_0. When the coupling strength γ' is equal to 0.01, synchronization is lost. Consequently, the certain level of coupling strength is required for synchronization.

4. System Including Time Delay in One Direction

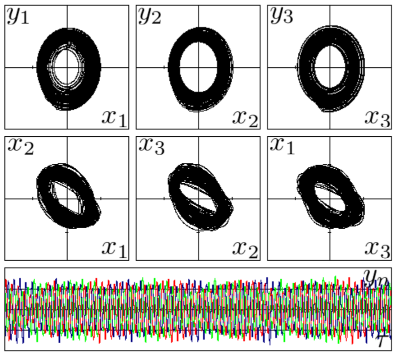
The circuit in this study have characteristic time delays methods. We have devised coupled systems as shown in Fig. 7. This system is coupled by resistors R_0 or inductors L_0. It is called coupled systems and "system including time delay in one direction"

4.1 Coupled by resistors R_0

Now, we use resistors R_0 to coupling elements. The normalized circuit equations of this system are same to Eqs.(3) and (4). The result as shown in the Fig. 8 can be obtained by difference of coupling strength γ. The time waveform of Fig. 8 (a) is in-phase synchronization and the amplitude of y_n is switching sequentially. However when the coupling strength γ is bigger then 0.1, switching synchronization state is lost and full in-phase synchronization state can



(a) $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.1$ and $T_d = \pi$.



(b) $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.2$ and $T_d = \pi$.

Figure 6: Simulation results of ring coupled system by inductors. Attractor. Lissajous figure. Time waveform. Red, blue and green colors denote y_1, y_2 and y_3 respectively.

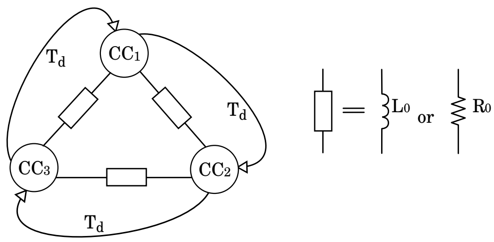
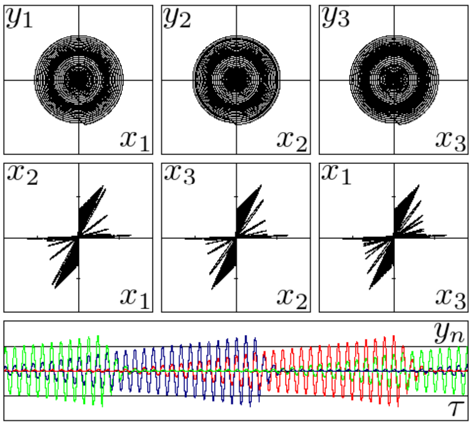


Figure 7: System including time delay in one direction.

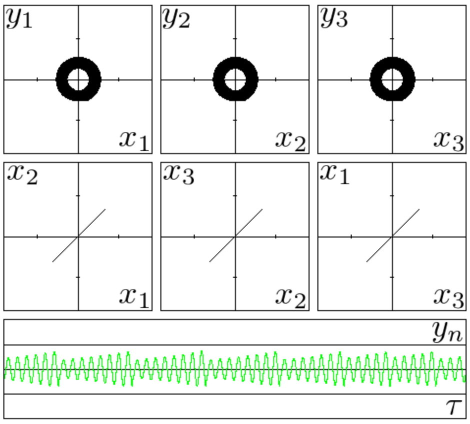
be observed. When the coupling strength γ is bigger then 0.1, switching synchronization state is lost and full in-phase synchronization state can be observed.

4.2 Coupled by inductors L_0

when we use inductors L_0 to coupling elements , the result as shown in the Fig. 9 can be obtained by difference of coupling strength γ'. Eqs.(5) and (6) are same to the normalized circuit equations of coupled by inductors L_0. The time waveform of Fig. 9 (a) has the phase difference and the amplitude of y_n is switching sequentially. However when the coupling



(a) $\alpha = 0.015$, $\beta = 0.5$, $\gamma = 0.01$ and $T_d = \pi$.



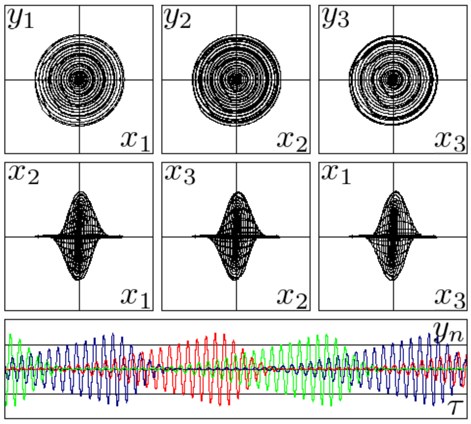
(b) $\alpha = 0.015$, $\beta = 0.5$, $\gamma = 0.1$ and $T_d = \pi$.

Figure 8: Simulation results of system including time delay in one direction coupled by resistor. Attractor. Lissajous figure. Time waveform. Red, blue and green colors denote y_1, y_2 and y_3 respectively.

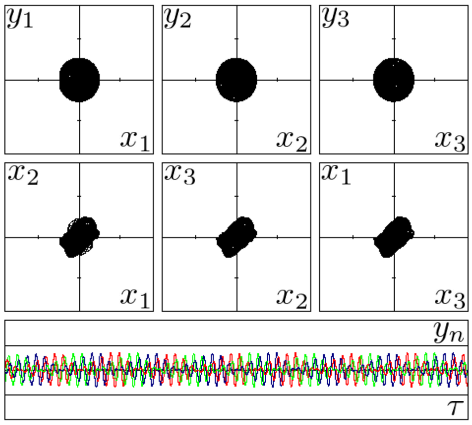
strength γ' is bigger then 0.1, switching synchronization state is lost and synchronization can be observed by initial values. Generally switching synchronization can be observed when system including time delay in one direction is coupled by resistors R_0 or inductors L_0. The amplitude is going divergence and convergence. Additionally the time of divergence and convergence is different.

5. Conclusion

In this study, we investigated synchronization state of novel coupled systems observed from some coupling methods of ring coupled by time delayed chaotic circuits. All the investigated coupling systems 4 types. In case of ring coupled by resistors, we observed in-phase synchronization state. The other case of ring coupled by inductors, in-phase synchronization and three-phase synchronization state can be observed. We devised coupled systems that takes advantage of features of the time delayed chaotic circuit. As a result, some



(a) $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.01$ and $T_d = \pi$.



(b) $\alpha = 0.015$, $\beta = 0.5$, $\gamma' = 0.1$ and $T_d = \pi$.

Figure 9: Simulation results of system that combines switching including ime delay in one direction coupled by inductor. Attractor. Lissajous figure. Time waveform. Red, blue and green colors denote y_1, y_2 and y_3 respectively.

special synchronization state can be observed. The switching of the amplitude of voltage in addition to the in-phase synchronization state can be observed by difference of coupling strength.

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