RISP
NCSP'16
International Workshop on
Nonlinear Circuits, Communications and Signal Processing
Honolulu, Hawaii, USA, March 6-9, 2016

Chaos Propagation and Synchronization in Coupled Chaotic Circuits as Double Ring Combination

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Abstract

In this study, we compare the differences between double and single ring model. Moreover, we investigate chaos propagation and how to propagate chaos by increasing the coupling strength. These modeles are coupled chaotic circuits when one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors.

1. Introduction

Synchronization is one of the fundamental phenomena in nature and one of the typical nonlinear phenomena. Synchronization observed on large-scale coupled chaotic systems have been attracted attention in various science field because it can be regarded as models of real physical system. Chaos synchronization has been interested by many researchers not only engineering but also the physical and biological fields [1]∼ [3]. In particular, it is important to investigate synchronization phenomena of coupled circuits under some difficult situations for the circuits. Additionally, it is applicable to the fields of medical science and biology and so on. As previous studies, synchronization and chaos propagation have been investigated in the ring or star-and ring of coupled chaotic circuits [4]∼ [5].

In this study, we investigate chaos propagation and synchronization phenomena in coupled chaotic circuits with double or single ring combination. We use the 11 chaotic circuits coupled by the resistors in each system. The double ring combination system, chaotic circuits are connected to only 5 circuits. Moreover, other 5 circuits are connected with the outside. On the other hand, the single ring combination system, chaotic circuits are connected to direct all circuits. In each model, one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. First, we observe chaos propagation by increasing the coupling strength. Moreover, by measuring the phase difference among all adjacent circuits in each combinations, we investigate synchronization in the entire system.

2. System model

The chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. We propose two system models. The double ring combination system, chaotic circuits are connected to only 5 circuits. Moreover, other 5 circuits are connected with the outside (see. Fig. 2). On the other hand, the single ring combination system, chaotic circuits are connected to direct all circuits. In this system, the circuit in the center of the system generates chaotic attractor and the other circuits generate three-periodic attractors. We use the 11 chaotic circuits coupled by the resistors in each system.

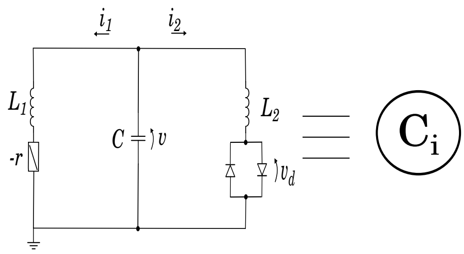

Figure 1: Chaotic circuit.

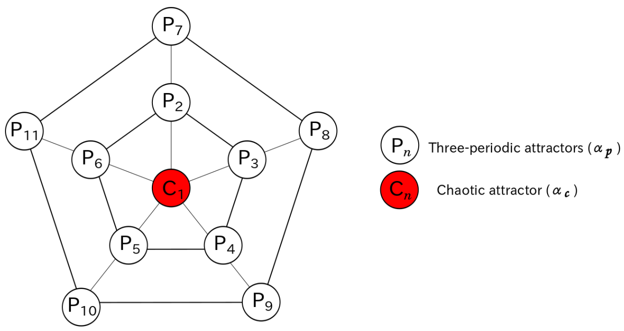

Figure 2: Double ring model.

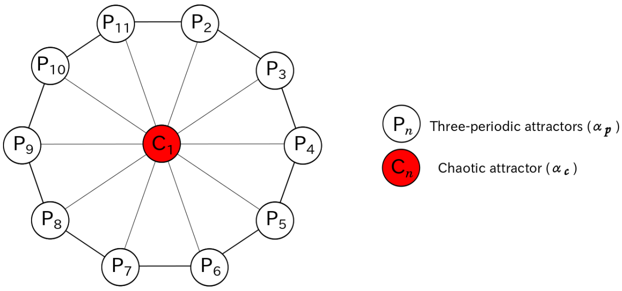

Figure 3: Single ring model.

The circuit equations of this circuit are described as follows:

$$\begin{cases} L_1 \dfrac{di}{dt} = v + ri \\ L_2 \dfrac{di}{dt} = v - v_d \\ C \dfrac{dv}{dt} = -i_1 - i_2. \end{cases} \quad (1)$$

The characteristic of nonlinear resistance is described as follows:

$$v_d = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \quad (2)$$

By changing the variables and parameters,

$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}} V x_n, \ \ i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n, \ \ v = V z_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}}, \ \ \beta = \dfrac{L_1}{L_2}, \ \ \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \dfrac{1}{R}\sqrt{\dfrac{L_1}{C}}, \ \ t = \sqrt{L_1 C_2}\tau, \end{cases} \quad (3)$$

The normalized circuit equations are given as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha x_n + z_n \\ \dfrac{dy_n}{d\tau} = z_n - f(y_n) \\ \dfrac{dz_n}{d\tau} = -x_n - \beta y_n - \displaystyle\sum_{n=1}^{N} \gamma_{ij}(z_n - z_{n+1}) \\ \qquad (n = 1, 2, \cdots, N). \end{cases} \quad (4)$$

In Eq. (1), N is the number of coupled chaotic circuits and γ is the coupling strength. $f(y_i)$ is described as follows:

$$f(y_n) = \frac{1}{2}\left(\left|y_n + \frac{1}{\delta}\right| - \left|y_n - \frac{1}{\delta}\right|\right). \quad (5)$$

We define α_c to generate the chaotic attractor, and α_p is defined to generate the three-periodic attractors.

3. Simulation Result

We observe chaos propagation by increasing the coupling strength. By measuring the phase difference among all adjacent circuits in each combinations, we investigate synchronization in the entire system.

Moreover, we change the number of edges between 1st circuit and the other circuits. Chaotic ciricuit connects to all circuits in each system model (see. Fig. 4). Chaotic ciricuit connects to onlly one circuit in each system model (see. Fig. 5).

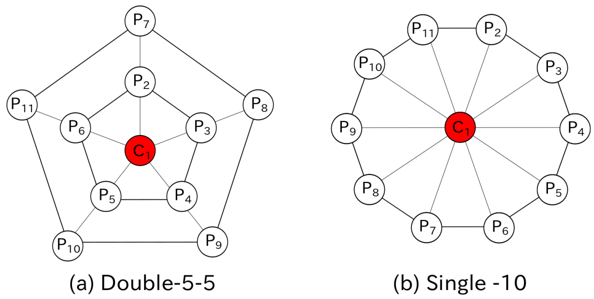

Figure 4: System model.

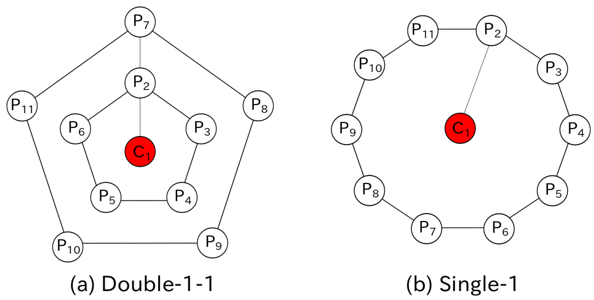

Figure 5: System model.

3.1 Convergence Time

In this section, we investigate convergence time from three-periodic attractors to chaos attractor. When all circuits are connected, we compare single-10 model and double-5-5 model. The simulation results are summarized in Table 1. Convergence time of single ring model is faster than double ring model. On the other hand, when chaotic ciricuit conected to onlly one circuit, we compare single-1 model and double-1-1 model. Convergence time of double ring model is faster than single ring model. Accordingly, when the number of edges is reduced, convergence time of the double ring model becames faster than single ring model.

Table 1: convergence time to chaos.

model	single-10	double-5-5	single-1	double-1-1
1st	194654	269522	703099	395966
2nd	181786	280897	662670	395175
3rd	219974	290051	697170	463205
4th	202727	269123	683756	400016
5th	186565	276105	663768	451242
Average	196959	277140	682093	421130

3.2 Chaos Propagation

Figure 6, shows chaos propagation attractors in the double ring combination system by increasing the coupling strength γ. At this time, we fix the coupling strength as $\gamma = 0.0000$, all circuits is not propageted the chaotic attractor of 1st chaotic circuit (see. Fig. 6(a)). When the coupling strength γ increase to $\gamma = 0.0010$, all circuits is propageted the chaotic attractor of 1st chaotic circuit (see. Fig. 6(b)).

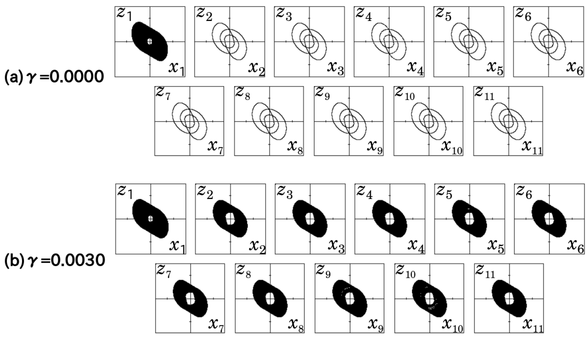

Figure 6: Chaos propagetion (double ring).

Figure 7 shows chaos propagation attractors in the double ring combination system by increasing the coupling strength γ. At this time, we fix the coupling strength as $\gamma = 0.0000$, all circuits is not propageted the chaotic attractor of 1st chaotic circuit (see. Fig. 7(a)). When the coupling strength γ increase to $\gamma = 0.0010$, all circuits is propageted the chaotic attractor of 1st chaotic circuit (see. Fig. 7(b)). Figure 5 shows chaos propagation attractors in the single ring combination system. In this model, all circuits is s propageted the chaotic attractor of 1st chaotic circuit as well as double ring combination system.

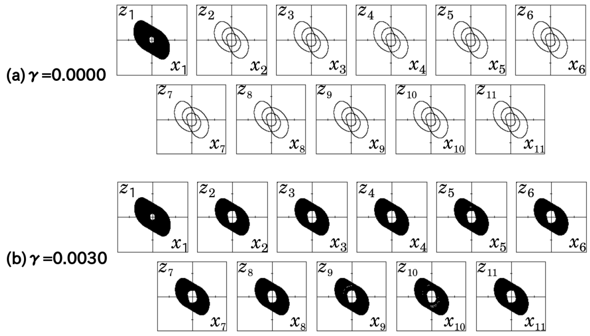

Figure 7: Chaos propagetion (single ring).

As a result, when the coupling strength increase, all three-periodic attractors are propageted the chaotic attractor of 1st chaotic circuit in each system model.

3.3 Synchronization Phenomena

In this section, we investigate the relation between the phase difference and the coupling strength in two models. The phase difference shows the average among all adjacence circuits.

First, we investigated the phase difference when chaotic ciricuit conected to all circuits in each system model (see. Fig. 4). If all circuits are not synchronized, the phase difference shows 90° . We can confirm that the phase difference is smaller and comes close to 0° by increasing the coupling strength.

The next, we investigated the phase difference when chaotic ciricuit conected to onlly one circuit in each system model (see. Fig. 5). If all circuits are not synchronized, the phase difference shows 90° . We can confirm that the phase difference is smaller and comes close to 00° by increasing the coupling strength.

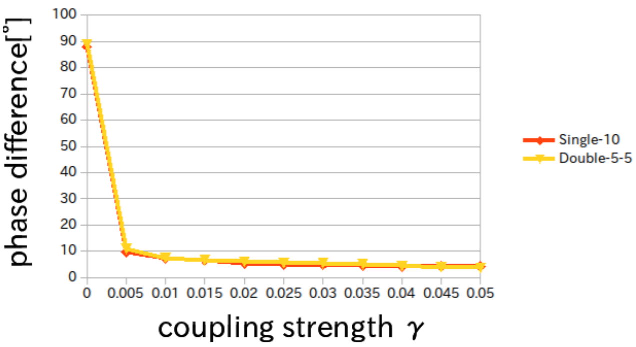

Figure 8: Relation between the phase difference and the coupling strength in double or single combination.

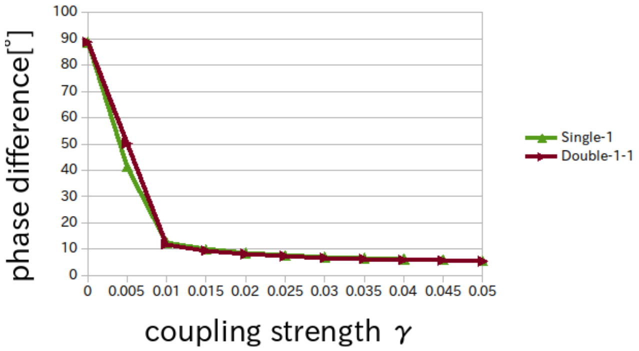

Figure 9: Relation between the phase difference and the coupling strength in double or single combination.

As a result, when the coupling strength increase, phase difference is smaller and comes close to 0° in each system. On the other hand, phase difference reach the synchronous state in double ring combination slightly than in single ring combination.

4. Conclusion

In this study, we have investigated chaos propagation and synchronization phenomena in coupled chaotic circuits as our proposed two system models. By the computer simulations, We have observed that the chaotic attractor is propagated to the other circuits. The three-periodic attractors are affected from the chaotic attractors when the coupling strength increases in each system model. Moreover we consider that the phase difference close synchronous state by increasing coupling strength in each system model.

Furthermore, we propose two difference system models, nevertheless chaos propagation and phase difference has become a similar result. In consequence, chaos propagation and phase difference are not affected by connected each circuits.

References

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