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Intermittency Chaos of Two Coupled Maps with Delay Coupling

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Abstract

Generally, complex dynamical phenomena can be observed in networks formed by many elements with nonlinearity. Coupled Map Lattice has proposed by Kaneko, to represent the complex high-dimensional dynamics, for example biological systems, networks in DNA, and neural networks. In this study, we investigate the influence of the delay in two coupled cubic maps with intermittency chaos. Moreover, the relation between average length of laminar part and the combination of delay is investigated.

1. Introduction

Generally, complex dynamical phenomena can be observed in networks formed by many elements with nonlinearity. Coupled Map Lattice (CML) has proposed by Kaneko [1]-[4], to represent the complex high-dimensional dynamics, for example biological systems, networks in DNA, economic activities and neural networks. Furthermore, we focus on intermittency chaos and delay. The delay naturally occurs from information transmission and processing speeds in the realistic networks[5]. In Ref.[5], the study investigated the synchronization states of the coupled logistic maps with the delay. As a result, the synchronization state of coupled chaotic maps are induced by the delay. Therefore, the studies considered the delay in coupled chaotic maps are investigated actively. In addition, intermittency chaos has stability and mobility and gains good result for information processing. We consider that intermittency chaos is related to various phenomena[6][7], e.g, information processing of the brain. In order to make clear the mechanism of such phenomena in various fields, unveiling the roles of intermittency chaos is very important.

In this study, we focus on the influence of the delay in two coupled cubic maps with intermittency chaos. When we set a control parameter of two cubic maps to generate intermittency chaos near the six periodic window, various synchronization states are confirmed in laminar part. Moreover, the relation between average length of laminar part and combination of the delay is investigated. Thereby, we could consider two maps with six periodic solution easily to become to the synchronization states by delay.

2. Two coupled cubic maps

A cubic map is expressed as follows:

$$f(x) = ax^3 + x(1 + a), \tag{1}$$

where a represents a control parameter.

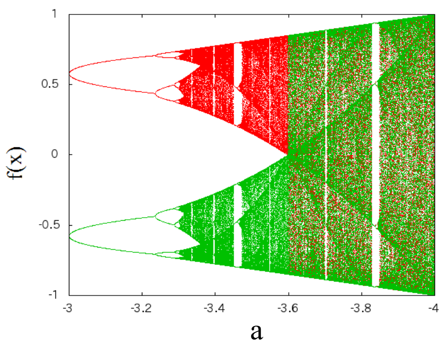


Figure 1: The bifurcation diagram of cubic map.

Figure 1 shows the bifurcation diagram of cubic map. This figure shows period-doubling bifurcations and periodic windows (near a = -3.69, -3.83). We focus on the boundary of periodic windows in the bifurcation diagram of cubic map. At the boundary of six periodic window intermittency chaos is observed in Fig. 2. Laminar represents the periodic state and burst represents the chaotic state. We define a = -3.69964153 representing the coexistence of laminar and burst in here. Figure 3 shows the time series of cubic map with intermittency chaos (a = -3.69964153). This figure shows intermittency chaos is switching between laminar and burst. Furthermore, in the case of a = -3.69964153, intermittency chaos including six periodic laminars are observed. In this study, we consider two coupled cubic maps with the delay.

The coupling system is expressed as follows:

$$\begin{cases} x_{(1,i+1)} = (1-g)f(x_{(1,i)}) + gf(x_{(2,i-\tau_1)}) \\ x_{(2,i+1)} = (1-g)f(x_{(2,i)}) + gf(x_{(1,i-\tau_2)}), \end{cases} \quad (2)$$

where g represents the coupling strength, τ_1 and τ_2 represents the delay between the maps.

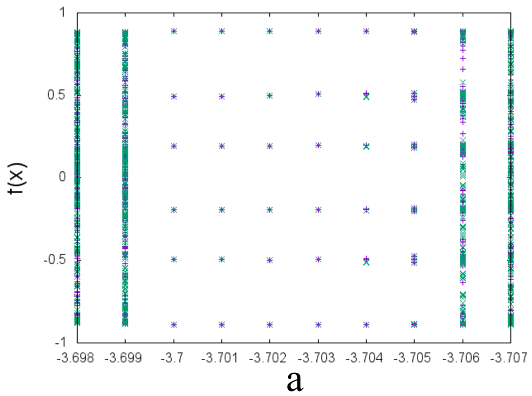


Figure 2: The boundary of six periodic window.

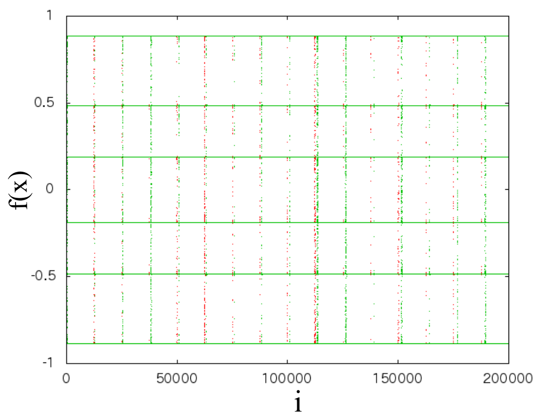


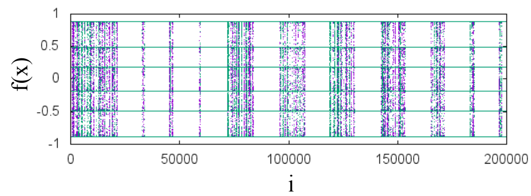
Figure 3: Time series of cubic map with intermittency chaos (a = -3.69964153).

3. Simulation Results

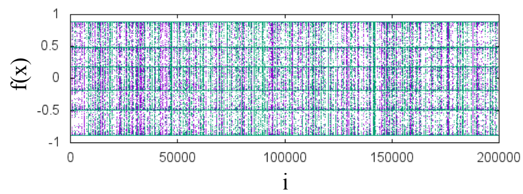
3.1 Average length of laminar part

In this study, the initial conditions and the parameters of two cubic maps are fixed with $x_{(1,0)}$ = 0.1, $x_{(2,0)}$ = -0.2, g = 0.0000001, 0.00001, respectively. The iteration time is fixed with i = 200000. And range of the delay is τ_1 and τ_2 = 0, 1, $\cdots$, 12. Figures 4 and 5 show the time series of coupled cubic maps with the delay. Laminar during the iteration time is longer than the other, e.g, Figs. 4 (a), (c) and Figs. 5 (a), (c). The other almost aren't observed the laminar part, e.g, Figs. 4 (b) and Figs. 5 (b). As a result, increasing the value of the coupling strength, the difference of laminar and burst can be clearly seen.

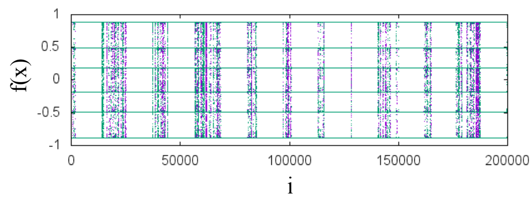
Next, we investigate the length of laminar part in coupled cubic maps with the delay. In order to investigate quantitative average length of laminar part, we define laminar part by $|x - \pm 0.1895$ or ± 0.4865 or $\pm 0.8873| < 0.0001$. Figures 6 and 7 show the average length of laminar part during the iteration time. From Fig. 6, if the case of non-delay (τ_1 =0, τ_2 = 0), average length of laminar part is 1547. In the case of τ_1 + τ_2 = 6n (n = 1, 2, 3, 4), the average length of laminar parts are longer. The other has become less than 200. Among the τ_1 + τ_2 = 6n, than when the case of non-delay, combinations of τ_1 + τ_2 = 6n that the synchronization state is longer than non-delay is confirmed thirteen locations. From Fig. 7, if the case of non-delay (τ_1 =0, τ_2 = 0), the average length of laminar part is 10366, because the density of burst part is increased when the coupling strength is high. In the case of τ_1 + τ_2 = 6n, the average length of laminar parts are longer than Fig. 6. The other has become less than 30. Among the τ_1 + τ_2 = 6n, than when the case of non-delay, combinations of τ_1 + τ_2 = 6n that the synchronization state is longer than non-delay is confirmed six locations.



(a) τ_1=0, τ_2=0.

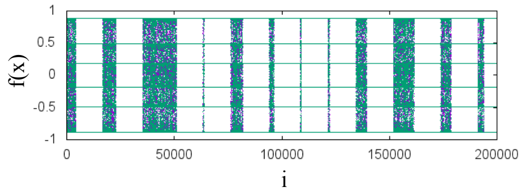


(b) τ_1=0, τ_2=3.

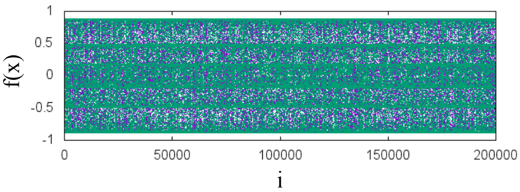


(c) τ_1=1, τ_2=5.

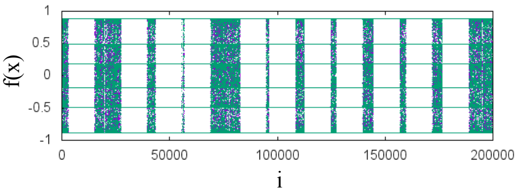
Figure 4: Time series of coupled cubic maps (g = 0.0000001).



(a) τ_1=0, τ_2=0.



(b) τ_1=0, τ_2=3.



(c) τ_1=1, τ_2=5.

Figure 5: Time series of coupled cubic maps (g = 0.00001).

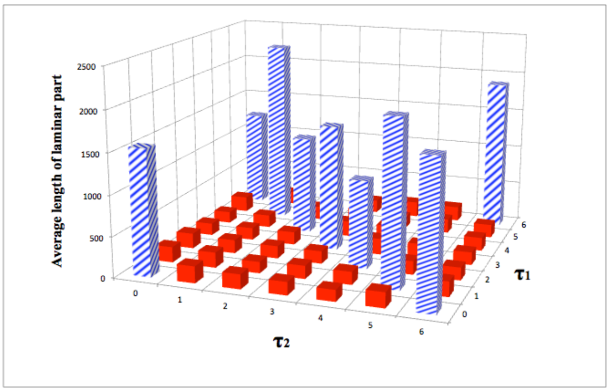


Figure 6: Average length of laminar part (g = 0.0000001).

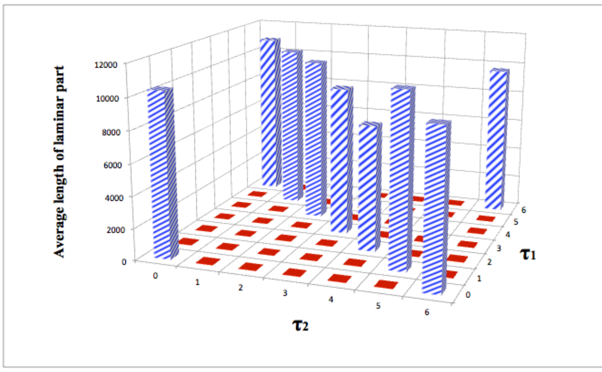


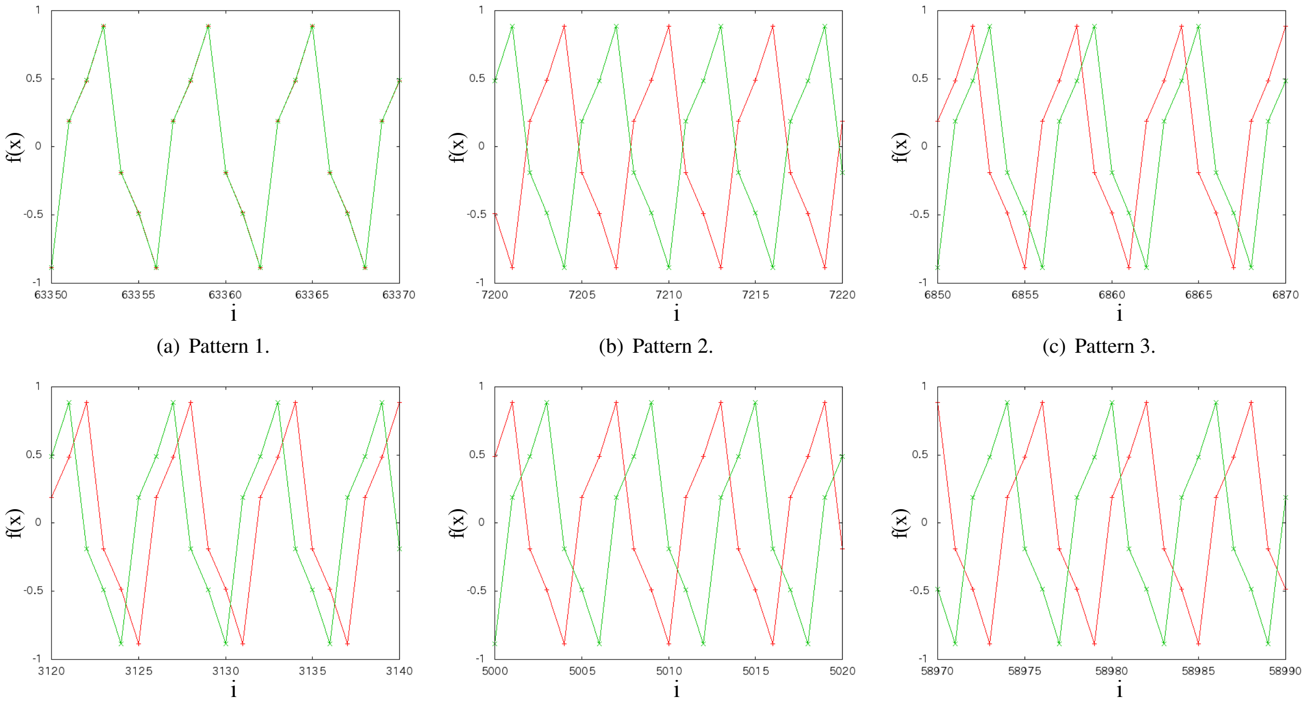
Figure 7: Average length of laminar part (g = 0.00001).

3.2 Synchronization Pattern

The synchronization states have six synchronization patterns in Fig. 8. Laminar part represents one of the synchronization states. There are all the patterns within each the time series. It does not have variation in the synchronization state until switch to burst part. We investigate whether variation of the synchronization state by the combination of the delay is observed or not. In this study, we focus on the synchronization states of each of the length of laminar part is more than 3,000. All synchronization states of this laminar part in the time series is the same synchronization pattern. If the case of non-delay, we confirm only in-phase synchronous, e.g, Fig. 8 (a). However, Fig. 9 shows variation of the synchronization states in the combination of $\tau_1 + \tau_2 = 6n$ ($n = 1, 2, 3, 4$). In addition, we search the synchronization state that consists of in-phase synchronous to anti-phase synchronous, it returns to the in-phase synchronous regularity. We have not searched variation to the coupling strength in the synchronization states.

4. Conclusions

In this study, we have investigated the influence of the delay in two coupled cubic maps with intermittency chaos. First, we observed the time series of coupled cubic maps with the delay. Next, we investigated the length of laminar part in coupled cubic maps with the delay. The average length of laminar part is longer than the other when the delay is set to $\tau_1 + \tau_2 = 6n$ ($n = 1, 2, 3, 4$). Thereby, we could consider two maps with six periodic solution easily to become to the synchronization states when $\tau_1 + \tau_2 = 6n$. In addition to that we searched the synchronization state consists of in-phase synchronous to anti-phase synchronous, it returns to the in-phase synchronous regularity in the combination of $\tau_1 + \tau_2 = 6n$.



(a) Pattern 1. (b) Pattern 2. (c) Pattern 3.

(d) Pattern 4. (e) Pattern 5. (f) Pattern 6.

Figure 8: Synchronization Patterns.

	$\tau_1+\tau_2=6$						
$\tau_{1,}\tau_2$	0,6	1,5	2,4	3,5	4,2	5,1	6,0
Synchronization Pattern	a	c	e	b	f	d	a
	$\tau_1+\tau_2=12$						
	0,12	1,11	2,10	3,9	4,8	5,7	
	a	c	e	b	f	d	
	6,6	7,5	8,4	9,3	10,2	11,1	12,0
	a	c	e	b	f	d	a
	$\tau_1+\tau_2=18$						
	6,12	7,11	8,10	9,9	10,8	11,7	12,6
	a	c	e	b	f	d	a
	$\tau_1+\tau_2=24$						
	12,12						
	a						

Figure 9: Regularity of the pattern.

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