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Synchronization in Oscillators Coupled by Transmission Line Model with Different Length

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Abstract

This paper presents synchronization phenomena in two van der Pol oscillators coupled by transmission line model with different length. By using the computer simulations, several types of synchronization states such as coexistence in-phase and anti-phase are confirmed by changing the circuit parameters.

1. Introduction

Recently, the synchronization observed from coupled oscillators and chaotic circuits systems have been studied actively in various fields [1], [2]. It is important to investigate basic synchronization observed in coupled oscillatory systems for future engineering applications such as chaotic communication and chaotic cryptography. In high-speed VLSI, the internal wiring is considered for transmission line because the structure of high-speed VLSI is complex and high density. However, there are not many discussions about effect caused by transmission line model for coupled oscillatory systems.

In our research group, the synchronization of two chaotic circuits with transmission line coupled by the cross talk have been investigated [3]-[7]. We have observed in-phase and anti-phase synchronization phenomena from these circuits in computer simulations [6]. In this system model, the part of inductor and capacitor of chaotic circuit is modeled by transmission line. The synchronization of oscillators coupled by transmission line using the ladder circuit of inductor and capacitor has not really investigated.

In our previous study, we have investigated synchronization phenomena in two van der Pol oscillators coupled by transmission line model. We modeled transmission line using ladder circuits of inductor and capacitor as lossless transmission line. By using the computer simulations, several types of synchronization states such as coexistence in-phase and anti-phase have been confirmed by changing the circuit parameters.

However, we have investigated only basic synchronization state by using only one transmission line model. The transmission line length is determined by the number of inductors and capacitors. Previously, the number of inductors and capacitors of transmission line model is fixed to 11 and 10, respectively.

This study considers the effects of the transmission line length by changing the number of the inductors and capacitors. Furthermore, we investigate the parameter effects of the transmission line model.

2. Two van der Pol Oscillators Coupled by Transmission Line Model

Figure 1 shows the conceptual circuit model of this study. Two van der Pol Oscillators are coupled by transmission line as shown in Fig. 2. In this circuit model, the transmission line is modeled by the ladder circuit composed of inductors and capacitors.

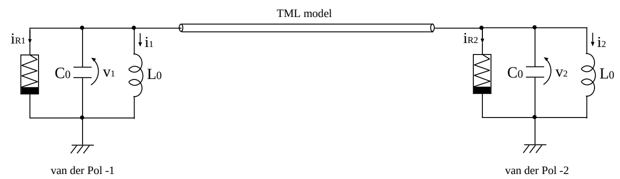


Figure 1: Conceptual circuit model.

The number of inductors and capacitors is fixed with 11 and 10, respectively. In this study, we change the number of inductors and capacitors of the transmission line model.

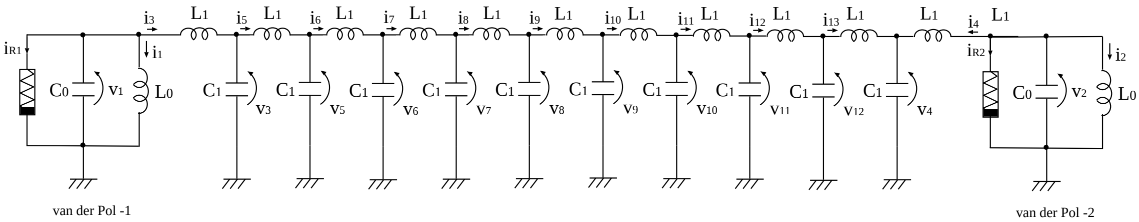


Figure 2: Two van der Pol Oscillators coupled by transmission line model (L_{tml}=10).

Three types of transmission line models are considered as follows.

- Inductors: 6, Capacitors: 5 (L_{tml}=5)
- Inductors: 11, Capacitors: 10 (L_{tml}=10)
- Inductors: 21, Capacitors: 20 (L_{tml}=20)

Next, we develop the expression for the circuit equations of the circuit model as shown in Fig. 2. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third order polynomial equation,

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), (k = 1, 2). \qquad (1)$$

The normalized circuit equations governing the circuit are expressed as [van der Pol Oscillator]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\Big(1 - \dfrac{1}{3}{x_k}^2\Big)x_k - y_k - y_n \\ \dfrac{dy_k}{d\tau} = y_k \\ \qquad\qquad\qquad\qquad (k = 1, 2). \end{cases} \qquad (2)$$

[Transmission Line Model]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \beta(y_k - y_n) \\ \dfrac{dy_k}{d\tau} = \alpha(x_k - x_n) \\ \qquad\qquad\qquad\qquad (k = 3...\mathrm{N}). \end{cases} \qquad (3)$$

where

$$t = \sqrt{L_0 C_0}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}}x_k \qquad (k = 1...N+2),$$

$$i_k = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C_0}{L_0}}y_k \qquad (k = 1...N+3),$$

$$\alpha = \frac{L_0}{L_1}, \quad \beta = \frac{C_0}{C_1},$$

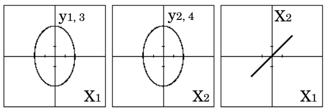
In this equations, ε denotes nonlinearity of the oscillators. α and β denote the ratio of inductor and capacitor of the oscillators and transmission line, respectively.

For the computer simulations, we calculate Eqs. (2), (3) using the fourth-order Runge-Kutta method with the step size $h = 0.005$. The parameter ε is fixed with 0.1.

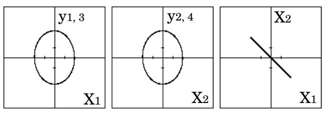
3. Basic Synchronization Phenomena (L_{tml}=10)

Figures 3-5 show the attractors and the Lissajous when the parameters α and β are changed.

In the case of α=β=1.0, coexistence of in-phase and anti-phase states of periodic solutions are observed as shown in Fig. 3. Figure 4 shows the simulation results when α and β are set to 1.9. In this case, we also confirm coexistence of in-phase and anti-phase state. We confirm that the torus attractor can be obtained when two oscillators are synchronized at in-phase, while the periodic attractor is generated when two oscillators are synchronized at anti-phase. In the case of α=β=5.0, we observe coexistence of in-phase and anti-phase states as shown in Fig. 5.

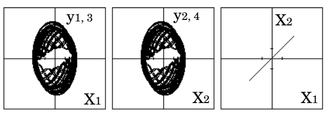


(a) In-phase.

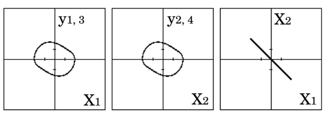


(b) Anti-phase.

Figure 3: Attractor ($\alpha = \beta = 1.0$).

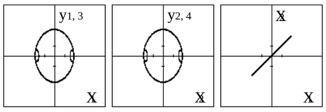


(a) In-phase.

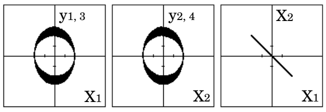


(b) Anti-phase.

Figure 4: Attractor ($\alpha = \beta = 1.9$).



(a) In-phase.



(b) Anti-phase.

Figure 5: Attractor ($\alpha = \beta 5.0$).

4. Synchronization Phenomena Depending on Parameters

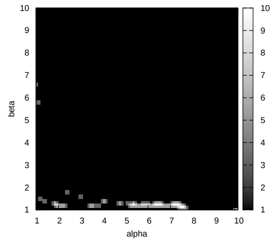
In this section, we investigate the effects of the transmission line length and the parameter effects of the transmission line model.

4.1 Solution

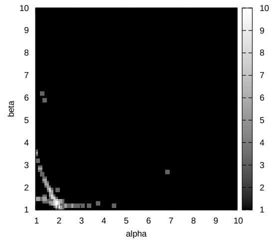
We investigate the solution type by changing the length of the transmission line and the circuit parameters α and β. α corresponds to the ratio of inductors between van der Pol and the transmission line. β corresponds to the ratio of capacitors between van der Pol and the transmission line. The obtained solutions are distinguished to one periodic solution and the others. The simulation results of the solutions when the α and β are changed from [1:10] are shown in Figs. 6-8. In these figures, the black and white region denote one periodic and quasi-periodic solutions, respectively. The horizontal axis denotes α and the vertical axis denotes β.

In the case of $L_{tml} = 5$, one periodic solution can be observed almost every area in the both cases as shown in Fig. 6(a) and (b). The extended graphs of quasi-periodic solution area are shown in Fig. 6(c) and (d). The structure of the quasi-periodic solution is very simple. Figure 7 shows the results of $L_{tml} = 10$. The area of one periodic solution becomes large when α and β show small value. The extended graphs of quasi-periodic solution area are shown in Fig. 7(c) and (d). The area of the quasi-periodic solution seems like shrimp structure. By increasing the length of the transmission line ($L_{tml} = 20$), the area of one periodic solution becomes large for whole area. The form of the quasi-periodic solution also seems like shrimp structure as shown in Fig. 8.

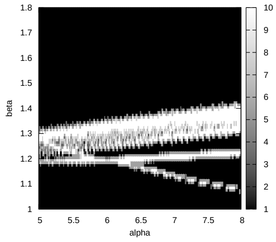
From these results, we can see that the quasi-periodic solution area becomes large when the length of transmission line becomes long. Furthermore, we observe the shrimp structure of quasi-periodic solutions when the length of transmission line is set to $L_{tml} = 10$ and 20.



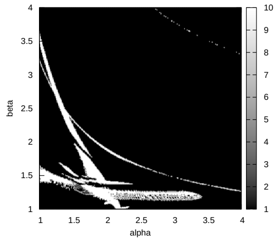
(a) Initial: in-phase.



(b) Initial: anti-phase.

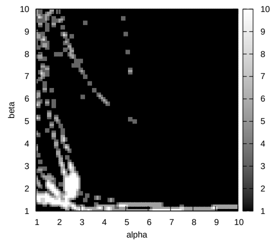


(c) Initial: in-phase.

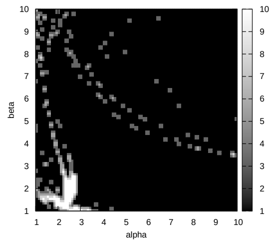


(d) Initial: anti-phase.

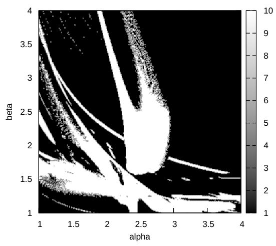
Figure 6: Solution ($L_{tml} = 5$).



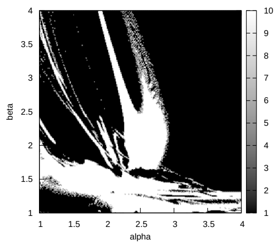
(a) Initial: in-phase.



(b) Initial: anti-phase.

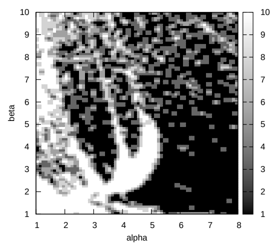


(c) Initial: in-phase.

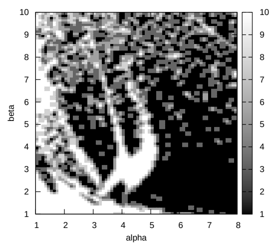


(d) Initial: anti-phase.

Figure 7: Solution ($L_{tml} = 10$).



(a) Initial: in-phase.



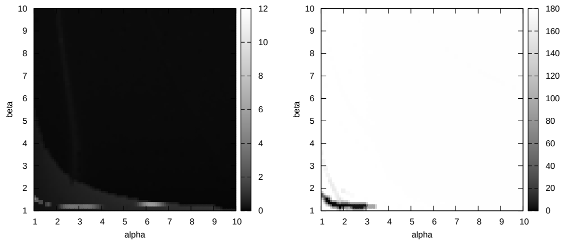
(b) Initial: anti-phase.

Figure 8: Solution ($L_{tml} = 20$).

4.2 Phase Difference

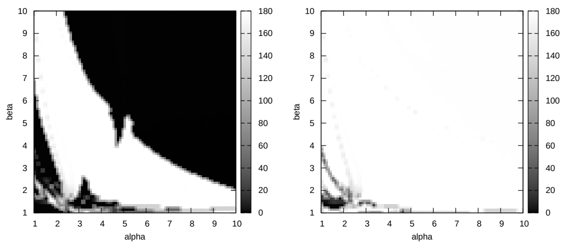
The simulation results of the phase difference are shown in Figs. 9-11. In these figures, the black and white region denotes in-phase and anti-phase states, respectively. The horizontal axis denotes α and the vertical axis denotes β.

In the case of $L_{tml} = 5$, the coexistence of in-phase and anti-phase states can be observed almost every area as shown in Fig. 9. In the case of $L_{tml} = 10$, the anti-phase area increases when the initial condition is set to in-phase mode as shown in Fig. 10(a). Figure 11 shows the phase difference of $L_{tml} = 20$. We confirm that anti-phase area can be obtained large area in Fig. 11(a). We can see that the coexistence area decreases by increasing the length of the transmission line.



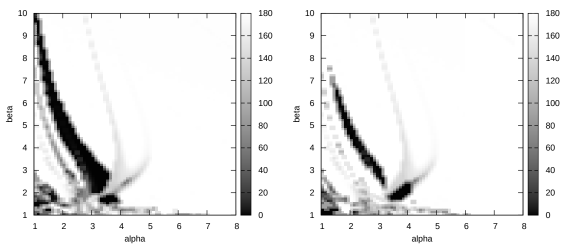
(a) Initial: in-phase. (b) Initial: anti-phase.

Figure 9: Phase difference ($L_{tml} = 5$).



(a) Initial: in-phase. (b) Initial: anti-phase.

Figure 10: Phase difference ($L_{tml} = 10$).



(a) Initial: in-phase. (b) Initial: anti-phase.

Figure 11: Phase difference ($L_{tml} = 20$).

5. Conclusions

We have investigated synchronization phenomena in two van der Pol oscillators coupled by transmission line model with different length. By using the computer simulations, we observe the shrimp structure of quasi-periodic solution in the circuit parameter space when the length of the transmission line is longer than $L_{tml} = 10$. Furthermore, several types of synchronization states such as coexistence in-phase and anti-phase were confirmed by changing the circuit parameter.

In our future works, we would like to investigate mechanism of the obtained synchronization phenomena in detail and apply this model to more complex networks such as smart grid network and social network.

Acknowledgment

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