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Investigation of Simulation Frequencies and Theoretical Frequencies of Synchronization States by Using Electric Power

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Abstract
In this study, theoretical synchronization frequencies are calculated by using electric powers on coupled oscillator system. This method is easily applied for complex and large systems when stable synchronization states are clarified. The calculated theoretical frequencies are compared from frequencies which are obtained by using simulations in changing a coupling parameter and a nonlinearity. We carry out that the theoretical frequencies can be applied for analyzing mechanisms of a phase-inversion waves or not.

1. Introduction

In this natural world, all kinds of oscillators are existing, their are moving mostly in cooperation. For example, a lot of fireflies which simultaneously flash are observed in southeastern Asia. In the sea, a lot of sardines simultaneously swim and make a motion like a big fish. Especially, synchronization phenomena can be easily observed on the electric circuit like coupled oscillators. Therefore, on the electric circuit, many researches of synchronization phenomena have been carried out up to now. There are a ladder system, a ring system and 2D lattice system which are easy shape of coupled oscillators. On the ring system, in-phase synchronizations, anti-phase synchronizations and in-and-anti-phase synchronizations can be observed when the number of oscillators are an even number. The in-and-anti phase synchronization is a synchronization phenomenon which the in-phase synchronizations and the anti-phase synchronizations are alternately existing.

Our previous study, the synchronization phenomena on the ladder system, the ring system and the 2D lattice system, which are constructed by using van der Pol oscillators and coupling inductors, were investigated. We discovered a special wave motion which is continuously existing. We call the wave motion phase-inversion waves[1]. The phase-inversion wave changes phase states between adjacent oscillators from an in-phase synchronization to an anti-phase synchronization or from the anti-synchronization to the in-phase synchronization and propagates. The phase-inversion waves have some characteristics. Frequencies of the in-phase synchronization, the anti-phase synchronization and the in-and-anti-phase synchronization are needed for analyzing mechanisms of the characteristics. There are a method of using averaging as one of method which the frequencies of the synchronization states are obtained[2]. However, the averaging method is hard to use for a large system, complex shape system and so on.

In this study, a method, which finds a theoretical frequency of each synchronization state, is developed by using reactive power of an instantaneous electric power on the ring system. It is already known that the ring system has the in-phase synchronization, the anti-phase synchronization and in-and-anti-phase synchronization. The theoretical values of frequencies are compared from the simulation results in changing circuit parameters. Finally, we check that the theoretical values can be used for analyzing mechanisms of the phase-inversion waves or can not.

2. Circuit Model

Our circuit model is shown in Fig. 1. Eight van der Pol oscillators are coupled by inductor L_C as a ring. An inductor and a capacitor of each van der Pol oscillator are assumed as "L" and "C" respectively. Characteristic of nonlinear negative resistor of k-th oscillator is shown as $f(v_k)$ in Eq. (1).

$$f(v_k) = -g_1 v_k + g_3 v_k^3 \quad (1 \le k \le 8) \tag{1}$$

Circuit equations of this circuit are normalized by using Eq. (2). The normalized equations are shown in Eq. (3).

$$\begin{aligned} i &= \sqrt{\frac{Cg_1}{3Lg_3}} x_k, \quad v = \sqrt{\frac{g_1}{3g_3}} y_k, \quad t = \sqrt{LC}\tau, \\ \alpha &= \frac{L}{L_c}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}}, \quad \delta = \frac{g_1^2}{3g_3}. \end{aligned} \tag{2}$$

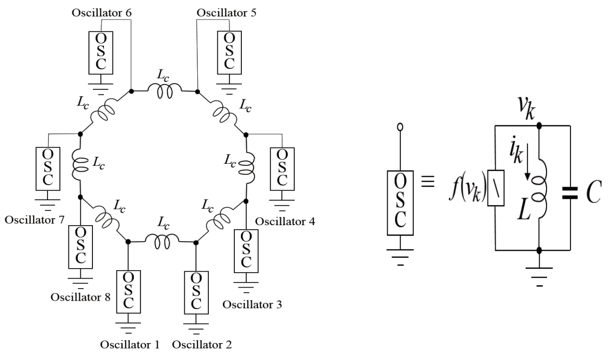

Figure 1: Circuit model.

$$\frac{dx_k}{d\tau} = y_k,$$

$$\frac{dy_k}{d\tau} = -x_k + \alpha(x_a - 2x_k + x_b) + \varepsilon(y_k - \frac{1}{3}y_k^3). \quad (3)$$

$$(If\ k = 1,\ a = N\ and\ b = 2.$$
$$If\ k = N,\ a = N - 1\ and\ b = 1.$$
$$If\ 2 \leq k \leq N - 1,\ a = k - 1\ and\ b = k + 1.)$$

Where α expresses a coupling parameter and ε shows nonlinearity of each oscillator. An instantaneous electric power of each oscillators and an instantaneous electric power of each inductor between adjacent oscillators are calculated by which each oscillation wave shape is assumed as a sinusoidal wave. Each current x_k $(1 \leq k \leq 8)$ is assumed as Eq. (4)

$$\begin{aligned} x_k &= X sin(\omega\tau + \theta_k) \\ y_k &= \omega X sin(\omega\tau + \theta_k) \end{aligned} \quad (4)$$

<Instantaneous electric power of the indctor in each oscillator>

$$P_{Lk} = \frac{\delta}{\varepsilon} x_k y_k \quad (5)$$

<Instantaneous electric power of the capacitor in each oscillator>

$$P_{Ck} = \frac{\delta}{\varepsilon} y_k \frac{dy_k}{d\tau} \quad (6)$$

<Instantaneous electric power of the coupling indoctor>

$$P_{Lc(k+1,k)} = \frac{\alpha\delta}{\varepsilon}(y_{k+1} - y_k)(x_{k+1} - x_k) \quad (7)$$

A normalized equation of a total instantaneous reactive power by using Eq. (2) are assumed as Eq. (8).

$$\begin{aligned} P_{rall} = & \sum_{k=1}^{8}\left(\frac{\delta}{\varepsilon}x_k y_k + \frac{\delta}{\varepsilon}y_k\frac{dy_k}{d\tau}\right) \\ & + \sum_{k=1}^{7}\frac{\alpha\delta}{\varepsilon}(y_{k+1} - y_k)(x_{k+1} - x_k) \\ & + \frac{\alpha\delta}{\varepsilon}(y_1 - y_8)(x_1 - x_8) \end{aligned} \quad (8)$$

In this system, we can consider that a power effect is best when a reactive power, which can be assumed as including instantaneous electric powers of L, C and L_C, is zero. In other words, we can guess that this system is a steady state when the reactive power is zero. In this study, angular frequencies of the in-phase synchronizations, the anti-phase synchronizations and the in-and-anti-phase synchronizations are obtained when the reactive powers are zero.

$$P_{rall} = 0 \quad (9)$$

Each phase θ_k is set as follows.

<Phase angles of the in-phase synchronization>

$$(\theta_k = 0) \quad (10)$$

<Phase angles of the in-and-anti-phase synchronization>

$$\begin{pmatrix} \theta_1 & = \theta_2 & = \theta_5 & = \theta_6 & = 0 \\ \theta_3 & = \theta_4 & = \theta_7 & = \theta_8 & = \pi \end{pmatrix} \quad (11)$$

<Phase angles of the anti-phase synchronization>

$$\begin{pmatrix} \theta_1 & = \theta_3 & = \theta_5 & = \theta_7 & = 0 \\ \theta_2 & = \theta_4 & = \theta_6 & = \theta_8 & = \pi \end{pmatrix} \quad (12)$$

These angle frequencies are applied for Eq. (9), and we achieve Eqs. (13)–(15).

<Angle freqency of the in-phase synchronization>

$$\omega_{in} = 1 \quad (13)$$

<Angle freqency of the in-and-anti-phase synchronization>

$$\omega_{in-and-anti} = \sqrt{1 + 2\alpha} \quad (14)$$

<Angle freqency of the anti-phase synchronization>

$$\omega_{anti} = \sqrt{1 + 4\alpha} \quad (15)$$

Each frequency of each phase state is shown in Table 1. An arbitrary value of parameters α is applied for each equation and a frequency of each phase state are obtained.

Table 1: Frequency of each synchronization states.

	in-phase	in-and-anti-phase	anti-phase
f	$\frac{1}{2\pi}$	$\frac{\sqrt{1+2\alpha}}{2\pi}$	$\frac{\sqrt{1+4\alpha}}{2\pi}$

3. Comparison between theoretical frequencies and simulation frequencies

In this circuit, each oscillation frequency is calculated when the synchronization state is the in-phase synchronization, the in-and-anti-phase synchronization or the anti-phase synchronization. Equations (13)-(15) depend on only coupling parameter α. However, the frequencies of using numerical simulation depend on the coupling parameter α and the nonlinearity ε. The frequencies of the simulations are investigated while the coupling parameter α and the nonlinearity ε are changed. Calculation conditions are set as follows.

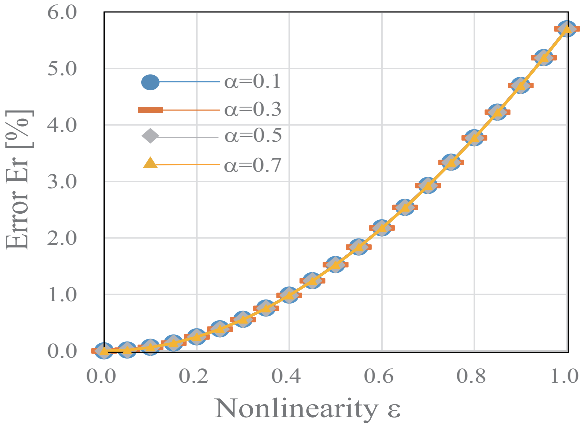

Figure 2: Errors between f_{th} and f_{sim} of the in-phase synchronization.

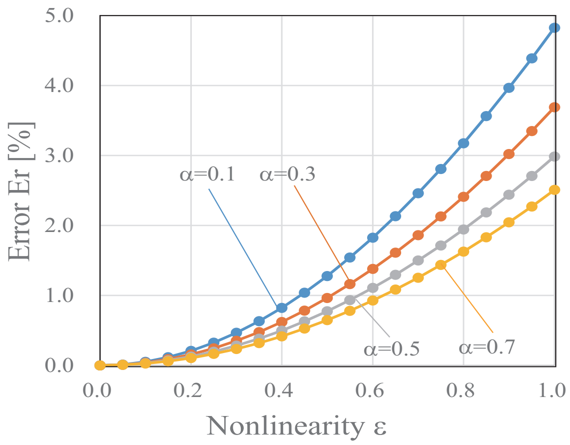

Figure 3: Errors between f_{th} and f_{sim} of the in-and-anti-phase synchronization.

1. The coupling parameter α is changed from 0.00 to 1.00 every 0.05.
2. The nonlinearity ε is changed from 0.1 to 0.7 every 0.2.

The errors Er between the theoretical frequencies and the simulation frequencies are obtained by using Eq. (16). The frequency by using simulation is called f_{sim}. The theoretical frequency is named f_{th}.

$$Er[\%] = \left| \frac{f_{sim} - f_{th}}{f_{th}} \right| \times 100 \qquad (16)$$

When the synchronization state of the ring is the in-phase synchronization, relations between the errors Er and the parameters are shown in Fig. 2. The errors Er are shown in Fig. 3 when the synchronization state is the in-and-anti-phase synchronization. The errors Er are shown in Fig, 4 when the synchronization state is the anti-phase synchronization.
Values of f_{th} and f_{sim} of which the errors Er are under 1% are shown in Table 2 when the synchronization state of the system is the in-phase synchronization. When the synchronization state is the in-and-anti-phase synchronization, values of f_{th} and f_{sim} are shown in Table 3. When the synchronization state is the anti-phase synchronization, values of f_{th} and f_{sim} are shown in Table 4.

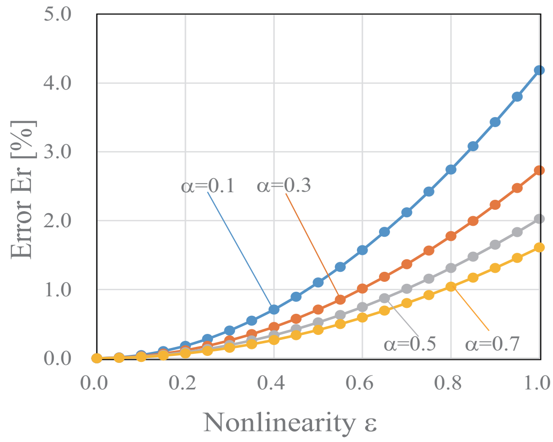

Figure 4: Errors between f_{th} and f_{sim} of the anti-phase synchronization.

Table 2: Frequencies of the in-phase synchronization (Er<1%).

	α=0.1	α=0.3	α=0.5	α=0.7
ε=0.40	0.99014	0.99016	0.99016	0.99017
theory	1.00000	1.00000	1.00000	1.00000
error[%]	0.98%	0.98%	0.98%	0.98%

Table 3: Frequencies of the in-and-anti-phase synchronization (Er<1%).

	α=0.1	α=0.3	α=0.5	α=0.7
ε=0.40	1.08641	1.25705	1.40715	1.54272
theory	1.09544	1.26491	1.41421	1.54919
error[%]	0.82%	0.62%	0.49%	0.41%

Table 4: Frequancies of the anti-phase synchronization (Er<1%).

	α=0.1	α=0.3	α=0.5	α=0.7
ε=0.45	1.17263	1.47473	1.72472	1.94281
theory	1.18321	1.48323	1.73205	1.94935
error[%]	0.89%	0.71%	0.42%	0.33%

4. Instantaneous Freqencies of Phase-Inversion waves

We check that the frequency equations and value can be used or can not be used in analysis of mechanism of phase-inversion waves. An phase-inversion wave, which changes phase states between adjacent oscillators from the in-phase synchronization to the anti-phase synchronization, propagates with another phase-inversion wave, which changes the phase states from the anti-phase synchronization to the in-phase synchronization.
Observation conditions of the phase-inversion waves are configured as follows.

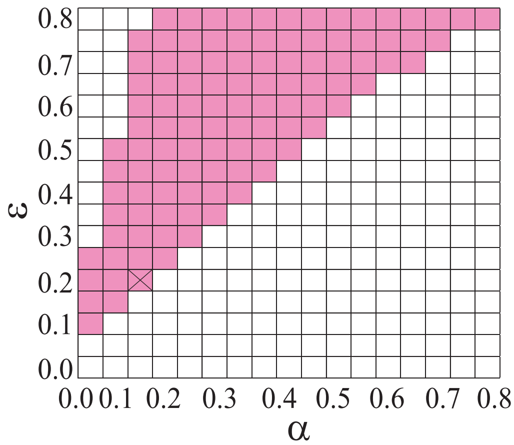

Figure 5: Domains which the phase-inversion waves are observed.

1. α is changed from 0.00 to 0.80 every 0.05.

2. ε is changed from 0.00 to 0.80 every 0.05.

A domains, which the phase-inversion waves are observed, are shown in Fig. 5. The phase-inversion waves can be observed in the colored domain. We investigate the phase-inversion waves when α and ε are set 0.1 and 0.2 respectively(see X-mark in Fig. 5). Instantaneous frequencies and phase differences of when the phase-inversion waves are propagating are shown in Figs. 6 and 7. The phase-inversion wave, which change from the anti-phase synchronization to the in-phase synchronization propagate immediately after the phase-inversion wave, which change from the in-phase synchronization to the anti-phase synchronization, propagates(see around 700τ in Fig. 7). Therefore, the instantaneous frequencies start to decrease before arriving at the anti-phase synchronization frequency(see around 700τ in Figs. 6 and 7). The instantaneous frequencies arrive at the in-phase synchronization frequency and become to a stable state(see around 770τ in Figs. 6 and 7). In this Fig. 6, we can think that our theoretical value can be used for analysing mechanism of the phase-inversion wave because the instantaneous frequencies arrive at near the in-phase synchronization frequency and does not arrive at the anti-phase synchronization frequency.

5. Conclusions

In this study, we developed the method which frequencies are obtained by using reactive powers. The errors between theoretical frequencies and simulation frequencies were investigated in changing the coupling parameters and the nonlinearities. Furthermore, we considered whether the theoretical frequencies could be used for analyzing the mechanisms of the phase-inversion waves.

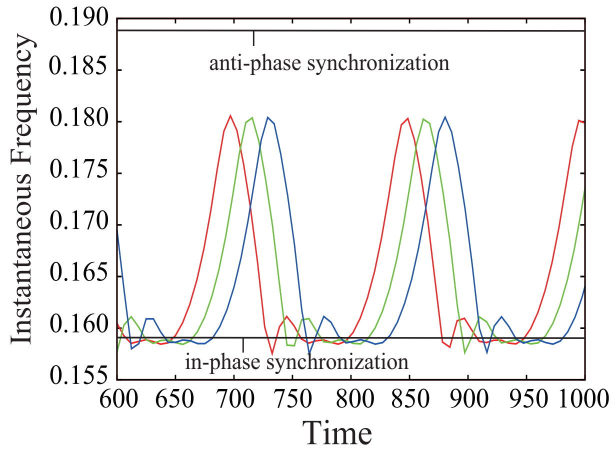

Figure 6: Itinerancies of the instantaneous freqencies by two phase-inversion waves.

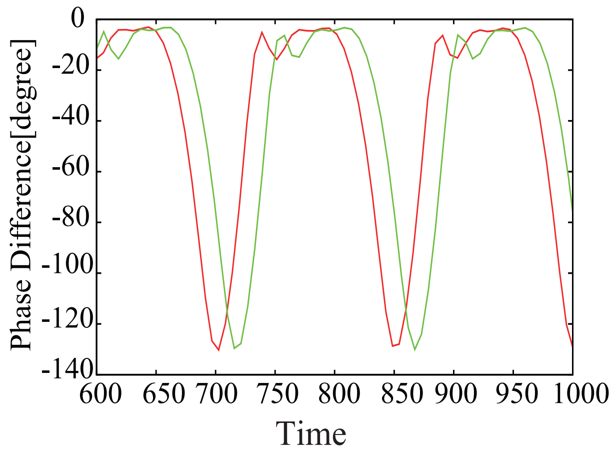

Figure 7: Itinerancies of the phase differences by two phase-inversion waves.

Acknowledgment

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