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Synchronization of Small-World Coupled Chaotic Circuits with or without Parameter Mismatch

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Abstract

This paper considers the synchronization of 100 coupled chaotic circuits in five different topologies obtained from the small-world network model. By using the computer simulations, we calculate the synchronization probability of the entire network during a certain time interval when the coupling strength is changed. Moreover, the case under parameter mismatch is also investigated. From the simulation results, the small-world topology is shown to be effective for the synchronization compared with regular and random networks.

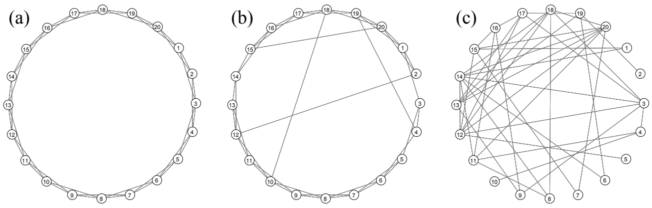

Figure 1: Illustration of the WS model for $N = 20$ and $k = 4$. (a) : Regular network, $p = 0$. (b) : Small-world network, $p = 0.1$. (c) : Random network, $p = 1$.

1. Introduction

Complex networks have attracted a great deal of attention from various fields since the discovery of "small-world" network [1] and "scale-free" network [2]. In particular, understanding the relation between topological structure and functional behavior is considered a significant hot topic for applying to practical applications in many disciplines [3]. As the dynamical behaviors on the networks, the synchronization is one of the typical phenomena. Therefore, many researchers have studied the synchronization on complex networks and reported that the small-world topology enhances the synchronization [4]-[6].

On the other hand, the synchronization of coupled chaotic systems [7]-[10] is very interesting and good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. However, there are not many studies for complex networks of continuous-time real physical systems such as electrical circuits. Wan and Chen reported the synchronization in the small-world coupled Chua's circuits [5]. In our previous work, we have studied the synchronization of 20 coupled chaotic circuits in the presence of parameter mismatch [11]. We considered three different network topologies, namely regular, small-world, and random networks, we found that the small-world network provides the best framework to realize synchronization. Synchronization under parameter mismatch is an important issue to be more clearly the structural function on the networks, but not frequently studied.

In this study, we investigate the synchronization of 100 coupled chaotic circuits as more larger-scale networks. We consider five different network topologies including regular, small-world, and random networks. We define the synchronization state in a quantitative way, and the synchronization probability of the entire network during a certain time interval is calculated in five networks. First, we investigate the relation between the synchronization probability and the coupling strength which corresponds to the value of the coupling resistors for chaotic circuits. Next, the maximum cluster sizes of small-world and random networks are investigated, it shows the difference of how to achieve the synchronization between two networks. Furthermore, the case under parameter mismatch is also considered. Thereby, the small-world topology is shown to be effective for the synchronization of the entire network compared with regular and random networks.

2. Small-World Network Model

In 1998, Watts and Strogatz introduced very interesting small-world network model, called the WS model [1]. The WS model can be generated as shown in Fig. 1. Starting from a ring lattice with N nodes and k edges per nodes in Fig. 1(a), each edge is rewired at randomly with probability p. By increasing probability p, the randomness of the network is also increased. The small-world network is known as the graph which is characterized by highly clustering coefficient like a regular graph and small path length like a random graph.

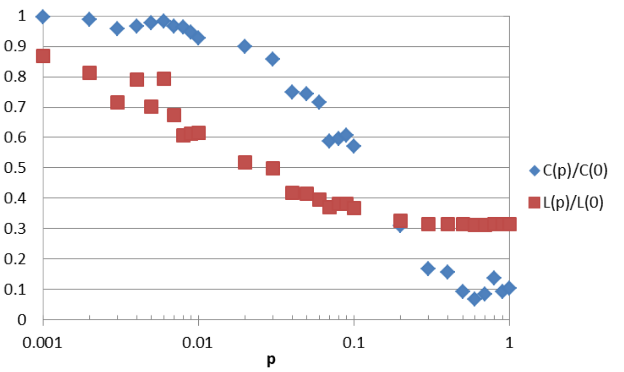

Figure 2: Characteristic clustering coefficient $C(p)$ and path length $L(p)$ for randomly rewired probability p. $N = 100$ and $k = 6$.

Topological structures in complex networks of N nodes and E edges can be evaluated by the typical three structural metrics (degree, clustering coefficient and path length). First, degree (k) shows the number of edges on a node. Second, clustering coefficient (C) shows the number of actual links between neighbors of a node divided by the number of possible links between those neighbors. It is given as follows:

$$C = \frac{1}{N}\sum_{n=1}^{N} C_n = \frac{1}{N}\sum_{n=1}^{N}\frac{2E_n}{k_n(k_n-1)}. \quad (1)$$

Third, path length (L) shows the shortest path in the network between two nodes. It is given as follows:

$$L = \frac{2}{N(N-1)}\sum_{m=1}^{N-1}\sum_{n=m+1}^{N} l(m,n). \quad (2)$$

In this research, we consider 100 coupled chaotic circuits in five network topologies obtained from the WS model. The data shown in the Fig. 2 are the characteristic C and L for randomly rewired probability p of the rewiring process described in Fig. 1. $C(p)$ and $L(p)$ have been normalized by using the values $L(0)$ and $C(0)$ for a regular network ($p = 0$). All generated networks have $N = 100$ and the average degree of $k = 6$ edges per nodes. By increasing p, $L(p)$ drops rapidly, corresponding to the onset of the small-world phenomenon. On the other hand, during this drop of $L(p)$, $C(p)$ remains at highly value like the regular network. Generally, the networks at such regions of p are called small-world network. We choose five values as $p = 0, 0.06, 0.1, 0.2$, and 1, respectively. Each topologies is considered as regular ($p = 0$), random ($p = 1$), and small-world ($p = 0.06, 0.1$, and 1) networks. Namely, we obtain five network models for 100 coupled chaotic circuits and investigate the synchronization.

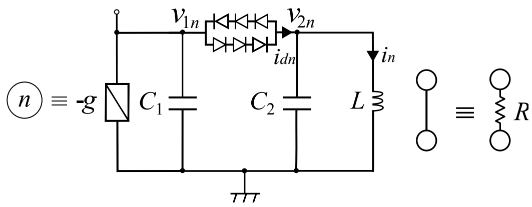

Figure 3: Chaotic circuit and coupling resistor.

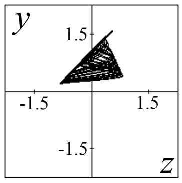

Figure 4: Chaotic attractor of the circuit as shown in Fig. 3. $\alpha = 0.5$, $\beta = 20$, and $\gamma = 0.5$.

3. Coupled Chaotic Circuits

Figure 3 shows the chaotic circuit which is three-dimensional autonomous circuit proposed by Shinriki *et al.* [12][13]. In this study, we propose 100 coupled chaotic circuits in five network topologies. In these network models, chaotic circuits are applied to each node of the network and each edge corresponds to a coupling resistor R (see Fig. 3).

The normalized circuit equations are given as follows:

$$\begin{cases} \dot{x_n} = z_n \\ \dot{y_n} = \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta \sum_{k\in S_n}(y_n - y_k) \\ \dot{z_n} = f(y_n - z_n) - x_n, \end{cases} \quad (3)$$

where $n = 1, 2, 3, ..., 100$ and S_n is the set of nodes which are directly connected to the node n. The parameter δ is corresponding to coupling resister R, it determines the coupling strength in this study. The nonlinear function $f(y_n - z_n)$ corresponds to the $i - v$ characteristics of the nonlinear resistor consisting of the diodes as follows:

$$f(y_n - z_n) = \begin{cases} \beta(y_n - z_n - 1) & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \le 1) \\ \beta(y_n - z_n + 1) & (y_n - z_n < -1). \end{cases} \quad (4)$$

This circuit generates asymmetric chaotic attractor as shown in Fig. 4. The values y and z in Fig. 4 correspond to v_1 and v_2 of the circuit in Fig. 3, respectively. In this study, we fix the circuit parameters as $\alpha = 0.5$, $\beta = 20$, and $\gamma = 0.5$ for all chaotic circuits. For the computer simulations, we calculate Eq. (3) using the fourth-order Runge-Kutta method with step size $h = 0.01$.

4. Synchronization

In this section, we show the computer simulation results about the synchronization of 100 coupled chaotic circuits. In particular, two cases of interest are considered: global synchronization of coupled chaotic circuits with or without parameter mismatch in five networks ($p = 0, 0.06, 0.1, 0.2$, and 1) described in Sec. 2.

4.1 Definition of Synchronization

Now, we define the synchronization state in a quantitative way by the following equation:

$$|y_i - y_j| < 0.01 \quad (i \neq j), \tag{5}$$

where y corresponds to v_1 of the circuit in Fig. 3, it denotes the voltage in the coupling terminal. Namely, if the two nodes (i and j) are synchronized, the value of $|y_i - y_j|$ should be almost zero. However, this chaotic circuit model is difficult to achieve complete synchronization. Therefore, we determine the synchronization threshold as 0.01, and temporal phase synchronization states in the networks are considered.

By means of the above definition of the synchronization, we propose and investigate the "synchronization probability" denoted the synchronization rate during a certain time interval. In this research, we fix a certain time interval as ($\tau = 1,00,000$ and $step = 0.01\tau$) and statistically investigate the synchronization probability in the entire network of 100 coupled chaotic circuits.

4.2 Synchronization Probability

Figure 5 shows the investigation results of the synchronization probability in five network topologies when the coupling strength is changed by $\delta = 0.01$. The vertical axis denotes the synchronization probability in the entire network during the certain time interval. We check every step whether all nodes pares are satisfied Eq. (5). If the synchronization probability is 1, we can consider that the entire network is synchronized perfectly. On the other hand, if the synchronization probability is 0, we can consider that all nodes have no time for the synchronization during the time interval. We compare the relation between the synchronization probability and the coupling strength δ among five network topologies. By increasing δ, the synchronization probabilities in four networks excluding $p = 0$ are also increased. Namely, all nodes become to be easily synchronized. However, the synchronization probability remains constant value at 0 in the network only $p = 0$ (regular network) even though the coupling strength is increased. This result could not be obtained in the case of $N = 20$ of our previous work. Therefore, we consider its reason the influence of the path length among nodes according to $N = 100$. From Fig. 5, we can evaluate effective topologies for the synchronization among five networks as follows; $p = 0.2 > 1 > 0.1 > 0.06 > 0$.

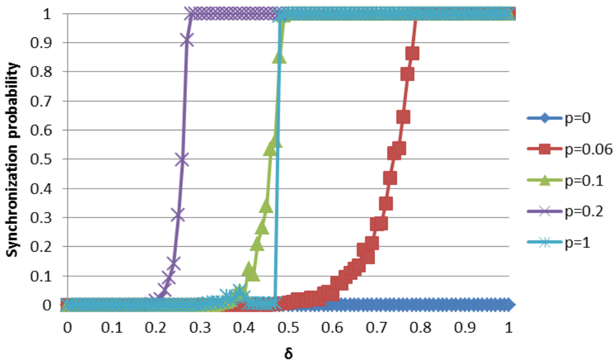

Figure 5: Relation between the synchronization probability and the coupling strength δ in five network topologies.

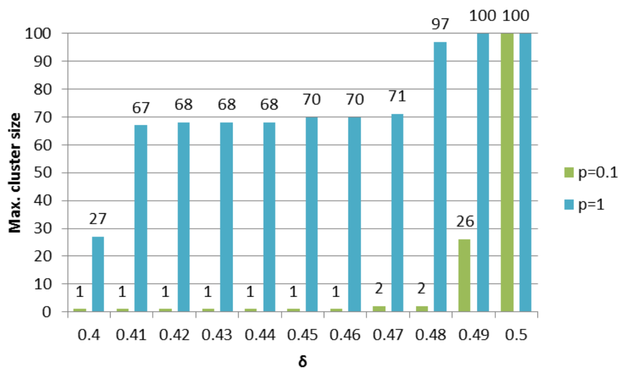

Figure 6: Difference of the maximum cluster sizes in small-world ($p = 0.1$) and random ($p = 1$) networks.

In order to understand the process of the synchronization, we investigate the maximum cluster sizes which show the maximum number of the synchronized nodes at all times during the time interval. Figure 6 shows the characteristics of the maximum cluster sizes in small-world ($p = 0.1$) and random ($p = 1$) networks. Here, the synchronization probability corresponds to 1 when the maximum cluster size is 100. In the case of the small-world network, the maximum cluster size remains constant value at 1 even though the synchronization probability is gradually increased (see Fig. 5). From this result, we find that the small-world network have no pairs of nodes that are synchronized at all time. Thereby, we consider that the small-world network behave like chaotic itinerancy and approach to achieve the synchronization. On the other hand, in the case of the random network, the maximum cluster size shows the large number (around 70) even when the

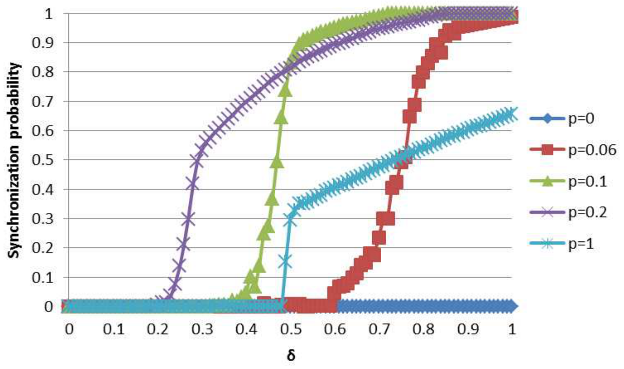

Figure 7: Relation between the synchronization probability and the coupling strength δ in five network topologies under parameter mismatch.

synchronization probability is almost 0 (see Fig. 5). Whereas the small-world network is gradually approached to achieve the synchronization, the random network becomes to be synchronized rapidly by exceeding a certain value δ. This reason can be considered that a few nodes tend to be isolated in the random network by spreading the degree distribution. Thus, we find the difference of how to achieve the synchronization in two networks.

4.3 Parameter Mismatch

Moreover, we consider the synchronization under parameter mismatch. Now, the parameter mismatches $\Delta\alpha$ are added for all circuits parameter α which is related to the range of chaotic trajectory. We choose $\alpha = 0.5$ as a standard parameter which is same parameter in the non-mismatch case. The parameter mismatches $\Delta\alpha$ are generated at randomly and the range of $\Delta\alpha$ is set to [-0.01:0.01]. Namely, the parameter α of all circuits are shown as $\alpha = 0.5 + \Delta\alpha$, respectively.

Figure 7 shows the synchronization probability of 100 coupled chaotic circuits with the parameter mismatches. The network condition excluding parameter mismatch is the same condition in the case of Fig. 5. By comparing between Fig. 5 and Fig. 7, we can find the influence of parameter mismatch and more clearly the structural function of the networks. From Fig. 7, we can evaluate effective topologies for the synchronization under parameter mismatch among five networks as follows; $p = 0.1 > 0.2 > 0.06 > 1 > 0$. It is striking that the random network is greatly influenced by parameter mismatch and difficult to achieve synchronization. In addition, the network of $p = 0.2$ is also difficult to achieve synchronization perfectly although rising the fastest among five networks. We consider that the network of $p = 0.2$ has high randomness along many rewiring edges. Therefore, the graph of $p = 0.2$ in Fig. 7 is very similar to random network ($p = 1$). Correctively, we conclude that the small-world topology provides the best framework to be effective for the synchronization.

5. Conclusion

This paper considered the synchronization of 100 coupled chaotic circuits in five different topologies obtained from the WS model. In particular, we focused on the global synchronization of the networks when the coupling strength was changed. By means of the computer calculations, we proposed and investigated the synchronization probability during a certain time interval. From the simulation results, we found that the regular network was difficult to achieve synchronization according to the influence of large-scale network. Furthermore, the case under parameter mismatch was also investigated. By comparing between two cases with or without parameter mismatch, we found that the networks with high randomness are greatly influenced by parameter mismatch. Hence, it can be said that small-world topology provides the best framework to realize synchronization even if there is variability among the nodes on the networks. More detailed investigation considering more various networks should be carried out in our future works.

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