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Synchronization in Complex Networks with Dispersion by Coupled Parametrically Excited Oscillators

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Abstract

In this study, we investigate synchronization in complex network with hubs and dispersion by using parametrically excited van der Pol oscillators. By means of computer simulation, we confirm various synchronous states and clustering phenomena and observe the effect of hubs in complex network.

1. Introduction

Synchronization is one of the fundamental phenomena in nature and it is observed over the various fields. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. The coupled van der Pol oscillator is one of typical coupled oscillators, and synchronization generated in the system can model certain synchronization of natural rhythm phenomena. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena of the parametric excitation circuits for future engineering applications. In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Refs. [4] and [5].

In our research group, we have investigated synchronization of parametrically excited van der Pol oscillators [6]. By carrying out computer calculations for two or three subcircuits cases, we have confirmed that various kinds of synchronization phenomena of chaos are observed. In the case of two subcircuits, the anti-phase synchronization is observed. In the case of three subcircuits, self-switching phenomenon of synchronization states is observed.

However, we have investigated the only simple network models. It is important to investigate more complex network for the broad-ranging future engineering applications. In our previous study, we have challenged to investigate the synchronization and clustering in more complex network modified from "Dolphin social network" [7] by using parametrically excited van der Pol oscillators with dispersion [8]. We have confirmed that the network with hubs can induce synchronization. Though, we have only investigated the case of densely-packed with hubs.

In this study, we focus on the location of the hubs, and investigate synchronization and clustering in the complex network scattering hubs. This network constructed by reference to the network modified from "Dolphin social network" [8]. In this network, the hubs are scattered about dual places. We focus the effect of the hubs in network, and investigate mechanism of synchronization clustering phenomena in the complex network scattering hubs.

2. System model

The circuit model of van der Pol oscillator under parametric excitation is shown in Fig. 1.

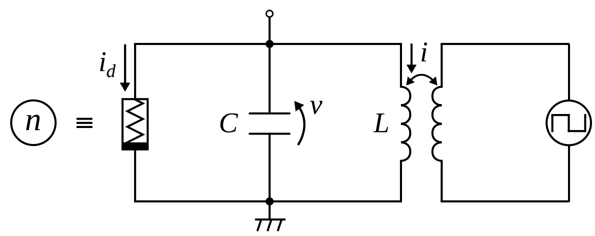


Figure 1: van der Pol oscillator under parametrically excitation

The circuit includes a time-varying inductor L whose characteristics are given as the following equation. The time-varying inductor is shown as Fig. 2.

$$L = L_0\gamma(\tau). \tag{1}$$

$\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 2, and its amplitude and angular frequency are termed α and ω, respectively. By changing the value of α, the amplitude of

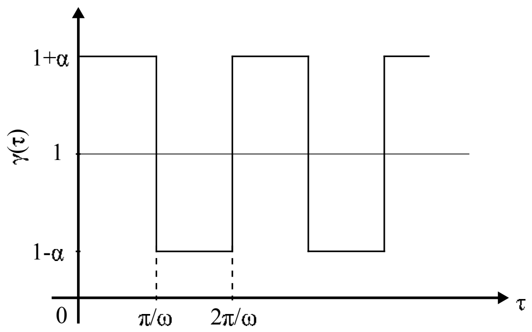


Figure 2: Function relating to parametric excitation

parametric excitation can be changed. The $v-i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v_k + g_3 v_k. \tag{2}$$

By changing the variables and the parameters,

$$\begin{cases} t = \sqrt{L_0 C}\tau,\ v_n = \sqrt{\dfrac{g_1}{g_3}} x_n \\ \omega = \omega_0 \sqrt{L_0 C} \\ i_n = \sqrt{\dfrac{g_1}{g_3}} \sqrt{\dfrac{C}{L_0}} y_n \\ \varepsilon = g_1 \sqrt{\dfrac{L_0}{C}},\ \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} \end{cases} \tag{3}$$

The normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon(x_n - x_n^3) - y_n - \delta \displaystyle\sum_{k \in S_n} (x_n - x_k) \\ \dfrac{dy_n}{d\tau} = \dfrac{1}{\gamma(\tau)} x_n \end{cases} \tag{4}$$

where n is the count of the nodes. S_n is the set of the nodes which are directly connected to the node n.

Table 1: Feature quantities of proposed network

Average degree	3.194
Average clustering coefficient	0.145
Average path length	4.384

In our system, parametrically excited van der Pol oscillators are coupled by one resistor R. Figure 3 shows proposed complex network and Fig. 4 shows degree distribution of this network. In this figure, vertical axis denotes the number of nodes, and horizontal axis denotes the value of degree. In this network, the number of nodes which is parametrically excited van der Pol oscillator is 62 and the number of edges which is one resister is 99. Other feature quantities of proposed network is expressed in Table 1. This network can be divided two clusters as shown in Fig. 3. The 58th node is the hub in cluster 1, and the 15th node is the hub in cluster 2. The bond number of 58th node is 11, and the bond number of 15th node is 13.

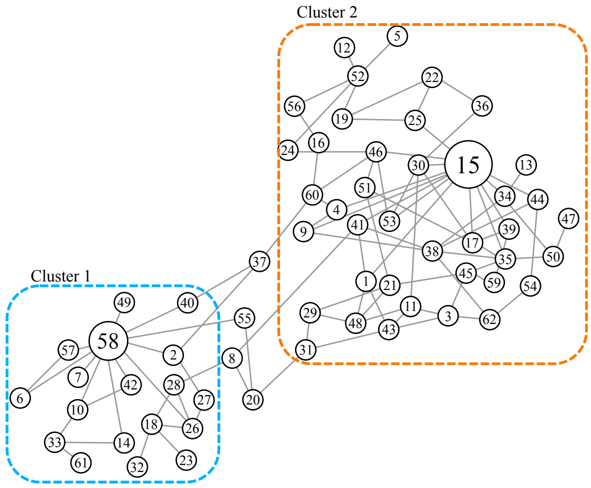


Figure 3: Proposed network

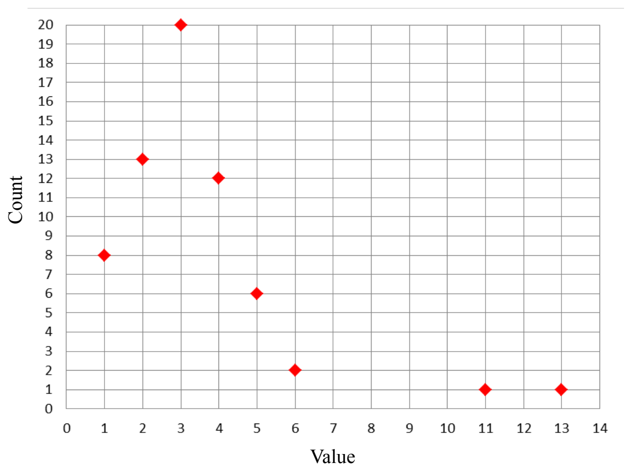
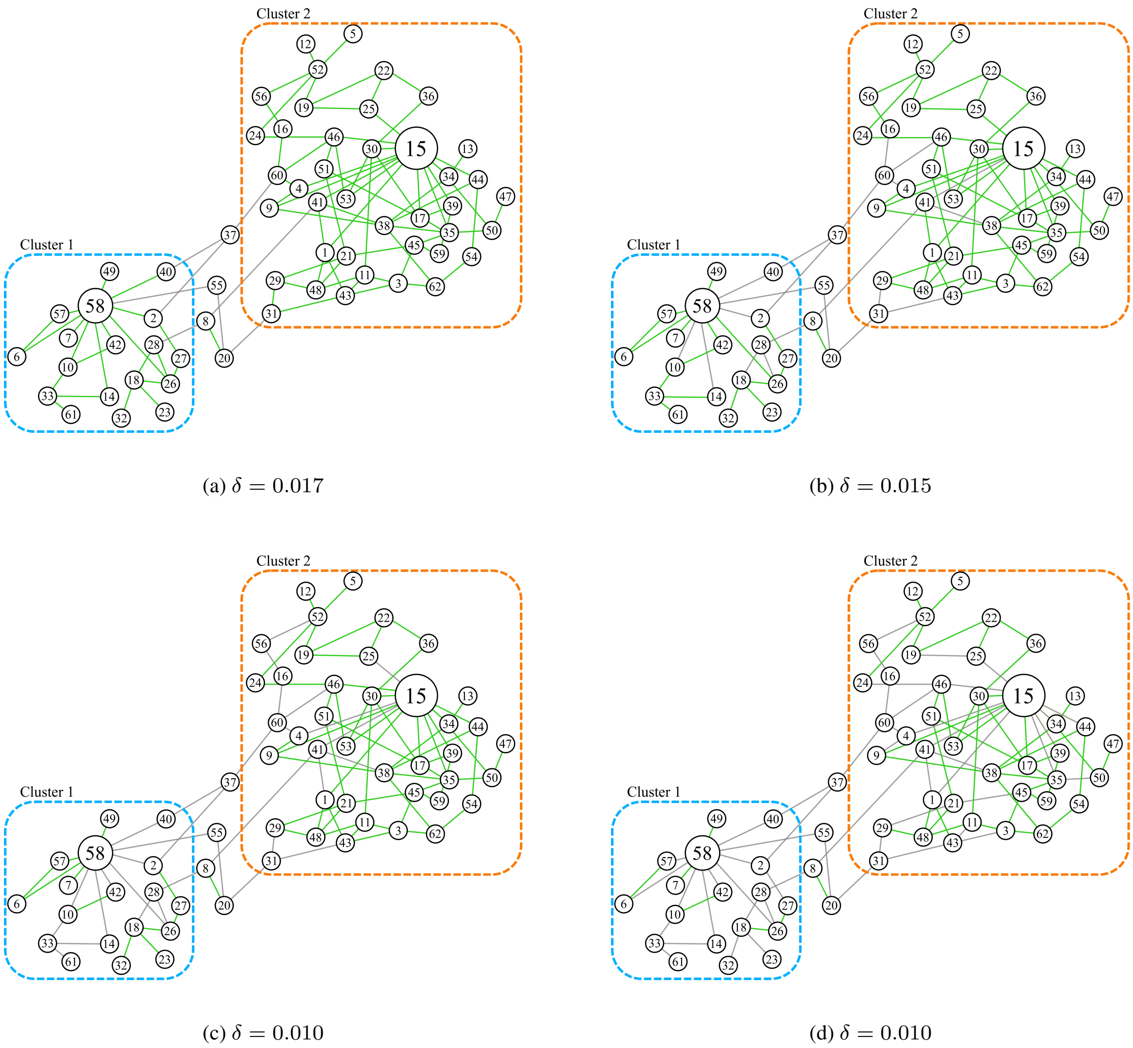


Figure 4: Degree distribution of proposed network

3. Simulation method

In this research, we investigate the synchronization by using computer simulation. We fix the the circuit parameters as $\varepsilon = 1.00$ and $\omega = 1.00$ for all circuits. Each circuit is given different initial values for computer simulations. In



(a) $\delta = 0.017$ (b) $\delta = 0.015$

(c) $\delta = 0.010$ (d) $\delta = 0.010$

Figure 5: Clustering phenomena.

this simulation, all of the nodes involve dispersion m_n in α (in Fig. 2) which is corresponding to the amplitude of the function relating parametric excitation within the compass of $-0.01 \leq m_n \leq 0.01$.

In order to analyze synchronous state, we define the synchronization by the following equation:

$$|x_n - x_k| < 0.10 \quad (k \in S_n). \tag{5}$$

We fix the count of calculation as 100,000.

4. The results of simulation

In this simulation, we investigate change of synchronous state and observe clustering phenomena by changing coupling strength with uniformity. Figure 5 shows clustering phenomena in the proposed complex network. In this figure, the green edges express synchronous state and the gray edges express unsynchronous state. First of all, this network becomes full synchronous state when the value of δ is 0.022. In this research, we decrease the coupling strength δ from 0.022 in order to observe synchronization and clustering.

Next, in the case of $\delta = 0.017$, this network is divided two cluster as shown in Fig. 5 (a). In addition, we investigate clustering phenomena by decreasing the coupling strength δ. When the case of Fig. 5. (b), the unsynchronous state comes into existence centering around 58th node. When coupling strength decrease to $\delta = 0.010$, the synchronized nodes in the cluster 1 decrease in a marked fashion. There are more synchronized nodes in the cluster 2 compared to cluster 1, and 58th node has more unsynchronized edges than

15th node whichever coupling strength δ. In each case of Fig. 5 (b), (c) and (d), we can observe the synchronous state of cluster 1 crumples up faster than cluster 2. From the above, we can confirm that the clustering phenomena of complex network draw influence from location of the hubs.

5. Conclusions

In this study, we have investigated synchronization and clustering phenomena in proposed complex network with hubs and dispersion by using parametrically excited van der Pol oscillators.

We could observed various synchronous state and clustering phenomena, and confirmed that the clustering phenomena of complex network draw influence from location of the hubs.

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