2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Relationship between Synchronization Rates and Parameter Mismatches in Coupled Chaotic Circuits Using Memristors

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Abstract—In our previous studies, the relationship between synchronization rates and small parameter mismatches on an asymmetrical coupled chaotic systems has been investigated. As a result of investigating these systems, interesting phenomenon can be observed. We consider that the phenomenon can be observed in the case of any kinds of chaotic oscillators.

In this study, a memristor based chaotic circuit are applied to our proposed system. By this investigation, our hypothesis for the phenomenon observed in our system is reinforced.

1. Introduction

Many kinds of coupled chaotic systems have been proposed and investigated. In these system, most important point is that various kinds of interesting phenomena such as spatio-temporal chaotic phenomena, clustering phenomena, and so on can be observed.

In our previous studies [1]-[2], the relationship between synchronization rates and small parameter mismatches on an asymmetrical coupled chaotic systems has been investigated. The system is coupled globally and coupling elements are resistors. Sub-circuits are divided into two groups. The difference of two groups is circuit parameters, coupled nodes or circuits as coupled elements. The small parameter mismatches were given to the sub-circuits as an mismatch of the oscillation frequency. As a result of investigating these systems, interesting phenomenon can be observed. This phenomenon is that the synchronization rates of the one sub-circuit increases in spite of increasing parameter mismatches of the other group. We consider that the phenomenon can be observed in the case of any kinds of chaotic oscillators.

On the other hand, a memristor proposed by L. O. Chua [3] is called as the fourth fundamental circuit element. A physical implementation was realized by a group of researchers from Hewlett-Packard laboratories in 2007 [4]. Many researchers consider that the memristor has a potential of application for computer science, neural networks and so on. However, the physical implementation is nano scale size and is not available on the market. Some researchers who are interested in a memristor developed the emulator circuits [5]-[8]. By using the emulator, some chaotic circuits are proposed and some applications are proposed.

In this study, a memristor based chaotic circuit [5] are applied to our proposed system. By this investigation, our hypothesis for the phenomenon observed in our system is reinforced.

2. System Model

2.1. Memristor Based Chaotic Circuit

Figure 1 shows a memristor based chaotic circuit (MBCC) proposed by B. Muthuswamy et. al. [5]. This circuit consists of two capacitors, one inductor and one memristor. Note that the memristor used in the circuit is an active element. By connecting a well-known passive memristor with a negative resistor in parallel, the active memristor can be realized [9].

Figure 2 shows an implementation of the memristor. This memristor is a flux-controlled memristor. Voltage v_m is corresponding to flux ϕ of the memristor. The characteristic of the memristor is defined as a cubic nonlinearity for the q-ϕ function:

$$q(\phi) = m_1\phi + m_2\phi^3. \tag{1}$$

Therefore, the memductance function $W(\phi)$ is given by :

$$W(\phi) = \frac{dq}{d\phi} = m_1 + 3m_2\phi^2. \tag{2}$$

A circuit experimental result and a computer simulation result are shown in Fig. 3. Chaotic attractors are observed.

2.2. System Model

Figure 4 is a proposed system. This system consists of resistors R and MBCCs. Two kinds of MBCCs (which are called as Group A and Group B) are included in the system. A difference between Group A and Group B is a value of circuit parameters. The number of circuit is defined as $k = m + n$. Namely, the numbers of Group A and

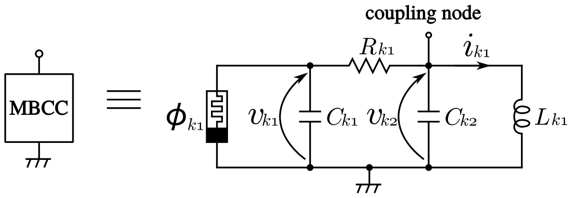


Figure 1: Memristor Based Chaotic Circuit [5].

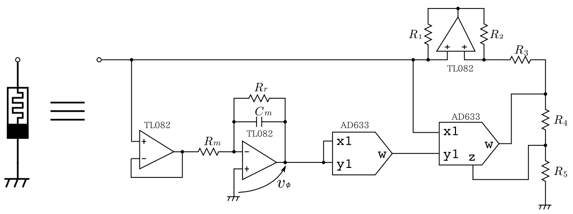
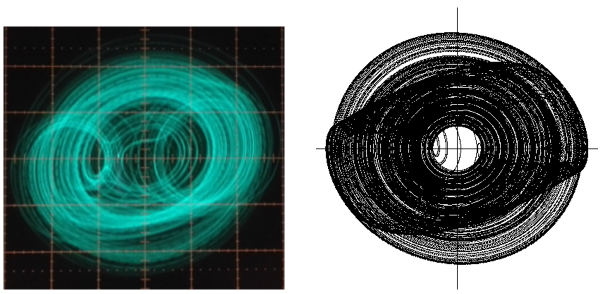


Figure 2: Implementation of a Memristor.



(a) (b).

Figure 3: Circuit experimental result and computer simulation result of a MBCC. (a) Horizontal axis is i_{k1}. Vertical axis is v_{k2}. R_{k1} = 1800 [Ω], C_{k1} = 6.8 [nF], C_{k2} =68 [μF], L_{k1} = 20 [mF], R_m = 10 [kΩ], C_m = 10 [nF], R_r = 1000 [kΩ], R_1 = 2 [kΩ], R_2 = 2 [kΩ], R_3 = 1.48 [Ω], R_4 = 30 [kΩ] and R_5 = 30 [kΩ]. (b) Horizontal axis is x_{k4}. Vertical axis is x_{k3}. $\alpha = 0.037$, $\beta_a = 11$, $\beta_b = 10$, $\gamma_a = 0.28$, $\gamma_b = 0.29$, $M_1 = -0.379$, $M_2 = 0.065$, $\delta = 0.09$, $\varepsilon = 1.1$ and $\zeta = 1.0$.

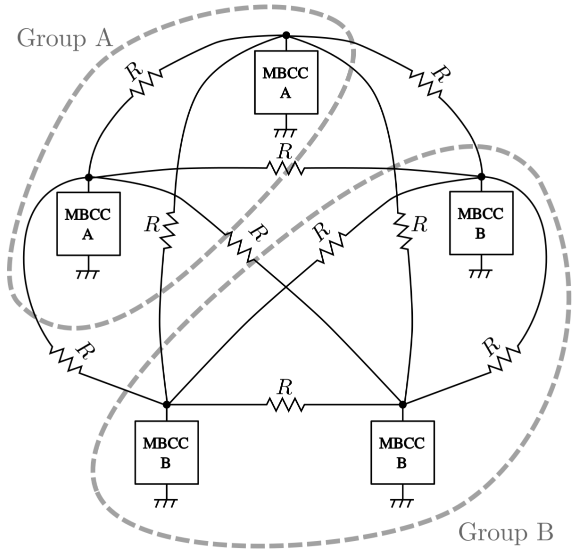


Figure 4: System Model.

Group B circuits are m and n, respectively. Additionally, small parameter mismatches of Group A and Group B are included in the system. Circuits are coupled globally by resistors. This system is asymmetrically coupled chaotic system. Now, in order to carry out computer simulations, Circuit equations are derived.

Group A ($1 \le k \le m$)**:**

$$\begin{cases} \dfrac{d\phi_k}{dt} = -\dfrac{v_{k1}}{R_m C_m} \\ C_{1a}\dfrac{dv_{k1}}{dt} = \dfrac{1}{R_a}(v_{k2}-v_{k1}) - (m_1 + 3m_2\phi_k^2)v_{k1} \\ C_{2a}\dfrac{dv_{k2}}{dt} = \dfrac{1}{R_a}(v_{k1}-v_{k2}) - i_k \\ \qquad\qquad + \dfrac{1}{R}\left\{\displaystyle\sum_{i=1}^{m+n} v_{i2} - (m+n)v_{k2}\right\} \\ L_a\dfrac{di_k}{dt} = (1+p_k)v_{k2} \end{cases} \tag{3}$$

Group B ($m+1 \le k \le m+n$)**:**

$$\begin{cases} \dfrac{d\phi_k}{dt} = -\dfrac{v_{k1}}{R_m C_m} \\ C_{1b}\dfrac{dv_{k1}}{dt} = \dfrac{1}{R_b}(v_{k2}-v_{k1}) - (m_1 + 3m_2\phi_k^2)v_{k1} \\ C_{2b}\dfrac{dv_{k2}}{dt} = \dfrac{1}{R_b}(v_{k1}-v_{k2}) - i_k \\ \qquad\qquad + \dfrac{1}{R}\left\{\displaystyle\sum_{i=1}^{m+n} v_{i2} - (m+n)v_{k2}\right\} \\ L_b\dfrac{di_k}{dt} = (1+q_k)v_{k2} \end{cases} \tag{4}$$

By using the following variables and parameters,

$$
\begin{aligned}
&x_{k1} = \phi_k, \quad x_{k2} = v_{k1}, \quad x_{k3} = v_{k2}, \quad x_{k4} = \sqrt{\frac{L_a}{C_{2a}}} i_k, \\
&\tau = \frac{1}{\sqrt{C_{2a}L_a}} t, \quad \text{“ . ”} = \frac{d}{d\tau}, \quad \alpha = \frac{\sqrt{C_{2a}L_a}}{R_m C_m}, \quad \beta_a = \frac{C_{2a}}{C_{1a}}, \\
&\beta_b = \frac{C_{2a}}{C_{1b}}, \quad \gamma_a = \frac{1}{R_a}\sqrt{\frac{L_a}{C_{2a}}}, \quad \gamma_b = \frac{1}{R_b}\sqrt{\frac{L_a}{C_{2a}}}, \\
&\delta = \frac{1}{R}\sqrt{\frac{L_a}{C_{2a}}}, \quad \varepsilon = \frac{C_{2a}}{C_{2b}}, \quad \zeta = \frac{L_a}{L_b}, \\
&M_1 = m_1\sqrt{\frac{L_a}{C_{2a}}} \quad \text{and} \quad M_2 = 3m_2\sqrt{\frac{L_a}{C_{2a}}}.
\end{aligned}
\tag{5}
$$

the normalized circuit equations of Group A and Group B circuits are described as follows:

Group A ($1 \leq k \leq m$)**:**

$$
\begin{cases}
\dot{x}_{k1} = -\alpha x_{k2}, \\
\dot{x}_{k2} = \beta_a \left\{ \gamma_a (x_{k3} - x_{k2}) - (M_1 + M_2 x_{k1}^2) x_{k2} \right\}, \\
\dot{x}_{k3} = \gamma_a (x_{k2} - x_{k3}) - x_{k4} + \delta \left\{ \sum_{i=1}^{m+n} x_{i3} - (m+n) x_{k3} \right\}, \\
\dot{x}_{k4} = (1 + p_k) x_{k3}.
\end{cases}
\tag{6}
$$

Group B ($m+1 \leq k \leq m+n$)**:**

$$
\begin{cases}
\dot{x}_{k1} = -\alpha x_{k2}, \\
\dot{x}_{k2} = \beta_b \left\{ \gamma_b (x_{k3} - x_{k2}) - (M_1 + M_2 x_{k1}^2) x_{k2} \right\}, \\
\dot{x}_{k3} = \varepsilon \left[\gamma_b (x_{k2} - x_{k3}) - x_{k4} + \delta \left\{ \sum_{i=1}^{m+n} x_{i3} - (m+n) x_{k3} \right\} \right], \\
\dot{x}_{k4} = \zeta (1 + q_k) x_{k3}.
\end{cases}
\tag{7}
$$

where $k = 1, 2, 3, \cdots, m+n$, i.e. the number of Group A circuits is m, the number of Group B circuits is n, and p_k and q_k are introduced to give parameter mismatches of the oscillation frequencies. By using these equations, computer simulations are carried out.

3. Computer Simulations

Figure 5 shows the voltage differences between two MBCCs in the case of using following initial values and parameters.

$$
\begin{cases}
x_{k1}(0) = 0.0, \ x_{k2}(0) = 0.1 + 0.1 \cdot k, \ x_{k3}(0) = 0.0, \\
x_{k4}(0) = 0.0, \ m = 2, \ n = 3, \ \alpha = 0.037, \ \beta_a = 11, \\
\beta_b = 10, \ \gamma_a = 0.28, \ \gamma_b = 0.29, \ \delta = 0.09, \ \varepsilon = 1.1, \\
\zeta = 1.0, \ p_k = 0.01(k-1) \text{ and } q_k = Q_b(k-1).
\end{cases}
\tag{8}
$$

Vertical axes show voltage differences and horizontal axes show time. Namely, in the case of synchronizing two MBCCs, the amplitude becomes zero. First graph $x_1 - x_2$ shows a voltage difference between the two MBCCs of

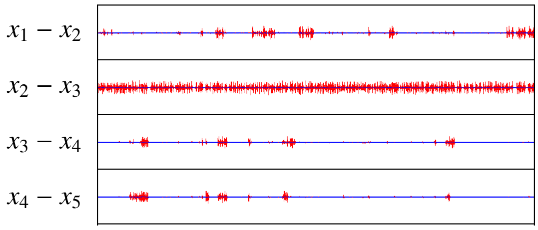
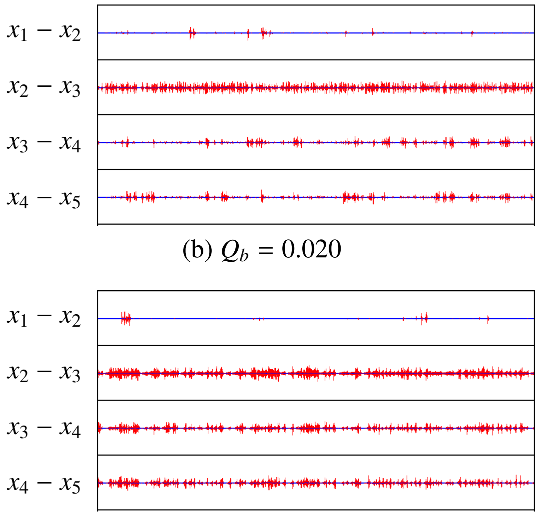


Figure 5: Voltage differences between two circuits. $\alpha = 0.037$, $\beta_a = 11$, $\beta_b = 10$, $\gamma_a = 0.28$, $\gamma_b = 0.29$, $\delta = 0.09$, $\varepsilon = 1.1$, $\zeta = 1.0$ and $Q_b = 0.005$.



(b) $Q_b = 0.020$

(c) $Q_b = 0.060$

Figure 6: Voltage differences between two circuits. $\alpha = 0.037$, $\beta_a = 11$, $\beta_b = 10$, $\gamma_a = 0.28$, $\gamma_b = 0.29$, $\delta = 0.09$, $\varepsilon = 1.1$ and $\zeta = 1.0$.

Group A. Synchronizations and un-synchronized burst appear alternately in a random way. Note that this synchronization is called as a quasi-synchronization. The second graph $x_2 - x_3$ shows a voltage difference between a MBCC of Group A and a MBCC of Group B. These are not synchronized at all. The third and fourth graphs $x_3 - x_4$ and $x_4 - x_5$ show voltage differences between two MBCCs of Group B.

By increasing Q_b which is corresponding to small parameter mismatches of Group B as shown in Fig. 6, unsynchronized bursts of Group A and B are decreased and increased, respectively.

In order to investigate this relationship, the synchronization is defined as following inequality.

$$|x_{k2} - x_{(k+1)2}| < 0.3 \tag{9}$$

Figure 7 shows a relationship between synchronization rates and Q_b. The synchronization rate is defined as a ratio of synchronization time and total time of the calculation. In $Q_b = 0$, it means parameter mismatches free, a synchro-

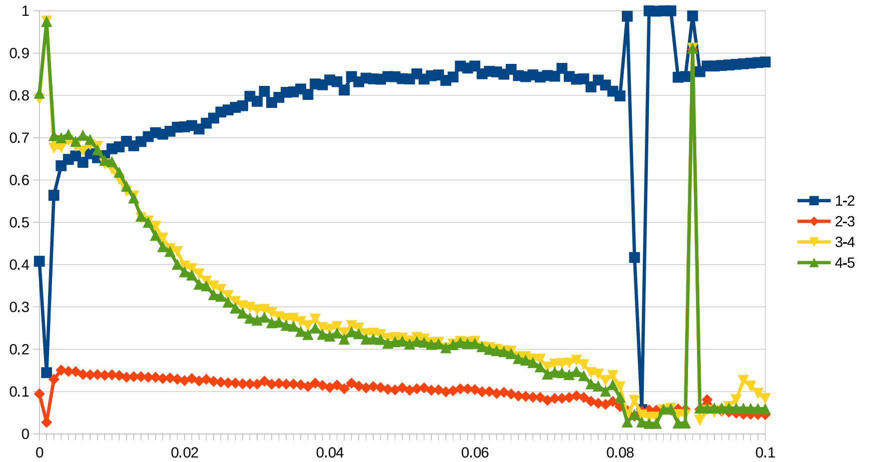


Figure 7: Relationship between synchronization rates and small parameter mismatches Q_b. Horizontal axis is Q_b, Vertical axis is a synchronization rate. $\alpha = 0.037$, $\beta_a = 11$, $\beta_b = 10$, $\gamma_a = 0.28$, $\gamma_b = 0.29$, $\delta = 0.09$, $\varepsilon = 1.1$ and $\zeta = 1.0$. Initial values are set as $x_{k1} = 0.0$, $x_{k2} = 0.1 + 0.1 * k$ $x_{k1} = 0.0$ and $x_{k1} = 0.0$. Simulation time of each parameter is 100000.

nization rate of Group A is about 0.4, a synchronization rate between Group A and B is about 0.1 and a synchronization rate of Group B is about 0.8. In $Q_b = 0.001$, MBCCs become periodic oscillation states. Therefore, very high or low rates are observed. Namely, the case of synchronizing two MBCCs perfectly becomes very high rate, and the case of phase shifted synchronization like an anti-phase synchronization becomes very low rate. By increasing Q_b from $Q_b = 0.001$ to $Q_b = 0.083$, a synchronization rate of Group A is increased and synchronization rates of Group B are decreased. In a area between $Q_b = 0.084$ and $Q_b = 0.090$, MBCCs become periodic oscillation states. In the case of $Q_b = 0.090$ and more, MBCCs do not oscillate.

In previous studies, similar results are obtained in other systems. We have hypothesized that this phenomenon can be observed all kinds of chaotic coupled element of the same couple system. This result reinforces this hypothesis.

4. Conclusions

In this study, in order to verifier the phenomenon of previous studies, relationship between synchronization rates and parameter mismatches in coupled chaotic circuits using memristors. In the case of five circuits, we confirmed a phenomenon similar to our previous studies in computer simulations. By this result, our hypothesis for the phenomenon observed in our system is reinforced.

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