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Synchronous States of Coupled Chaotic Circuits Measured by Oscillation Frequency

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Abstract—In this study, we investigate relationship between synchronous state and oscillation frequency in the coupled chaotic circuit. We consider that oscillation frequency moves closer to steady value when the chaotic circuits become synchronization. From the computer simulation, we confirm synchronous state in the coupled chaotic circuit is lowly dependent to oscillation frequency. Furthermore, we pay attention to timing of synchronization of among the coupled chaotic circuits.

1. Introduction

The synchronous phenomena are observed as not only fields of natural science but also various fields. For example, we can confirm the flashing of fireflies (a firefly is able to match frequency of other fireflies), metronome, heartbeat of the human, and so on. The synchronous phenomena have been researched extensively in physics [1] and biology [2]~[3]. In addition, applying synchronous phenomena to medical technology are developed. These synchronous phenomena are known as one of the nonlinear phenomenon. For the future engineering application, we consider it is important to investigate synchronous phenomena of coupled chaotic circuits.

Synchronous Discrimination of the chaotic circuit uses the phase difference generally. Not only the phase difference but also period and oscillation frequency exist in the coupled chaotic circuit. We pay attention to oscillation frequency in the coupled chaotic circuit. We can consider that entire circuit synchronizes if and when oscillation frequency included in a certain individual match. In this paper, we compare synchronous discrimination by using phase difference and oscillation frequency. We consider that oscillation frequency converges on a steady value when the coupled chaotic circuits synchronize.

As simulation result, relationship is uncommon between synchronization of the chaotic circuit and oscillation frequency, although oscillation frequency converges on a steady value conclusively. Next, we pay attention to timing of synchronization of among the coupled chaotic circuits. Chaotic circuits which we use in this paper do not synchronize at the same time when the coupling strength exceeds a certain threshold value. Chaotic circuits synchronize every partial circuit. From these, we focus on timing when chaotic circuits synchronize every partial circuit. We investigate whether there is relationship in the timing of synchronization as we determine synchronous state of circuit by using oscillation frequency or phase difference.

2. Circuit Model

Figure 1 shows the model of the chaotic circuit.

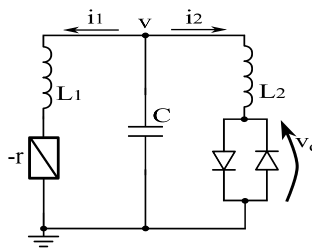

Figure 1: The model of the chaotic circuit.

This chaotic circuit is called Nishio-Inaba circuit [4]~[7]. This circuit consists of single linear negative resistance, single nonlinear resistance consisting of two diodes, two inductors and single capacitor. The linear negative resistance is realized by using the negative impedance converter made of an operational amplifier [4]. The circuit dynamics is described by the following piecewise-linear third-order ordinary differential equation,

$$\begin{cases} L_1 \frac{di_1}{dt} = v + ri_1, \\ L_2 \frac{di_2}{dt} = v - v_d(i_2), \\ C\frac{dv}{dt} = -i_1 - i_2. \end{cases} \quad (1)$$

We approximate the $I - V$ characteristic of the nonlinear resistance by the following function,

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \quad (2)$$

The parameter r_d is the slope of the nonlinear resistance. Equation (1) is normalized by changing the variables according to

$$i_1 = \sqrt{\frac{C}{L_1}} Vx \;;\; i_2 = \frac{\sqrt{L_1 C}}{L_2} Vy \;;\; v = Vz \;;\; \text{“}\cdot\text{”} = \frac{d}{d\tau},$$

$$r\sqrt{\frac{C}{L_1}} = \alpha \;;\; \frac{L_1}{L_2} = \beta \;;\; r_d \frac{\sqrt{L_1 C}}{L_2} = \gamma \;;\; t = \sqrt{L_1 C}\tau. \quad (3)$$

Equation (1) is normalized as

$$\begin{cases} \dot{x} = \alpha x + z, \\ \dot{y} = z - f(y), \\ \dot{z} = -x - \beta y. \end{cases} \quad (4)$$

The nonlinear function $f(y)$ corresponds to the $I - V$ characteristics of the nonlinear resistors consisting of the diodes is assumed to be described as follow,

$$f(y) = \frac{\gamma}{2}\left(\left|y + \frac{1}{\gamma}\right| - \left|y - \frac{1}{\gamma}\right|\right). \quad (5)$$

3. Simulation Method

3.1. Circuit Model

We consider a ladder network using chaotic circuits as shown in Fig. 2. Ladder network is the simple network. Namely, ladder network is easier to understand the flow of electric signal. Each circuit is coupled via resistor R.

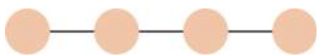

Figure 2: Ladder network.

Normalized equation of coupled chaotic circuits is described as follow,

$$\begin{cases} \dot{x_n} = \alpha x_n + z_n. \\ \dot{y_n} = z_n - f(y_n). \\ \dot{z_n} = -x_n - \beta y_n + \sigma(z_(n+1) - z_n), \\ \qquad\qquad (n = 1) \\ \dot{z_n} = -x_n - \beta y_n + \sigma(z_(n-1) - z_n), \\ \qquad\qquad (n = \textit{the maximum value}) \\ \dot{z_n} = -x_n - \beta y_n + \sigma(z_(n+1) + z_(n-1) - 2z_n), \\ \qquad\qquad (\textit{otherwise}) \end{cases} \quad (6)$$

For this simulation, the parameters are set as follows, $\beta_n = 3.0$ and $\gamma_n = 470.0$. Own oscillation frequency and phase difference of the chaotic circuits depend on the parameter α. Each circuit is assigned α to investigate transition of oscillation frequency when the chaotic circuits are synchronized. Parameter α is fixed value $0.40 \leq \alpha \leq 0.48$. In addition, the parameter α_1 and α_n located in both ends of coupled chaotic circuit are fixed $\alpha_1 = 0.40$ and $\alpha_n = 0.48$ to make it easier to compare transition of oscillation frequency as shown in Fig. 3. Five chaotic circuits is used in this paper.

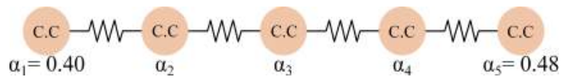

Figure 3: Setting condition of parameter α_n (n = 5, α_2=α_3=α_4=the variable).

The coupling strength σ_n is chosen as a control parameter. All σ_n are set same value. Equation of coupling strength is shown as

$$\sigma_n = \frac{1}{R}\sqrt{\frac{L_{n1}}{C_n}}. \quad (7)$$

There is inverse relationship between the coupling strength σ and resistor R.

3.2. Oscillation Frequency

Equation of resonance frequency is described as

$$f = \frac{1}{2\pi\sqrt{L_{n1}C_n}}. \quad (8)$$

We can not obtain exact value of oscillation frequency by using Eq. (8) because the proposed chaotic circuit includes a linear negative resistance and a nonlinear resistance consisting of two diodes. New measuring procedure is proposed to obtain exact value of oscillation frequency.

Equation of new measuring procedure of oscillation frequency is described as,

$$F = \frac{1}{T}. \quad (9)$$

The cycle T is obtained from the chaotic attractor. The chaotic attractor of proposed model is shown in Fig. 4.

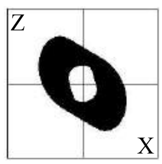

Figure 4: The chaotic attractor.

Poincare section of attractor is set between the first and the fourth quadrant. The number of dots to describe attractor is counted from Poincare section. First 10,000 laps of attractor are ignored because attractor is in an unstable state at first. Total dots from 10,001 laps to 30,000 laps are measured for obtaining average dots per one lap. In addition, average dots are replaced the cycle of the chaotic attractor. We can obtain exact value of oscillation frequency by assigning the cycle T to Eq. (9).

3.3. Definition of Synchronous State

Synchronous condition is necessary to investigate relationship between synchronous state and transition of oscillation frequency. In order to derive synchronous state numerically, the phase difference θ is used. Equation of the phase difference is shown as

$$\theta = \arctan \frac{z}{x}. \qquad (10)$$

The phase difference is obtained from chaotic attractor as with oscillation frequency. For example, we measure the phase difference between circuits 1 and 2. The phase difference is obtained by assigning coordinate of dot drawn the chaotic attractor of circuit 2 to Eq. (10) when the chaotic attractor of circuit 1 runs through the poincare section. We define that two circuits 1 and 2 synchronize when the phase difference θ of each circuit is below 30 degrees.

4. Simulation Results

We set parameter $\alpha_1 = 0.40$ and $\alpha_5 = 0.48$. Figures 5~8 show transition of oscillation frequency when the coupling strength is changed.

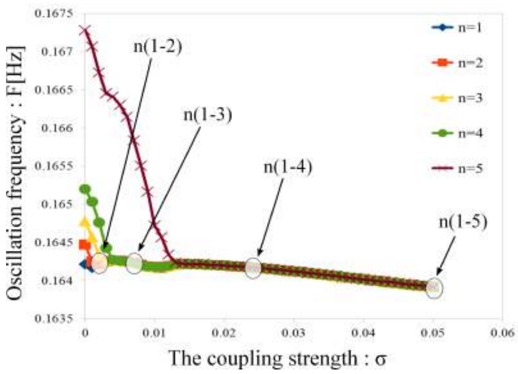

Figure 5: Transition of oscillation frequency when the value of α are set equability($\alpha_2 = 0.41$, $\alpha_3 = 0.435$, $\alpha_4 = 0.46$).

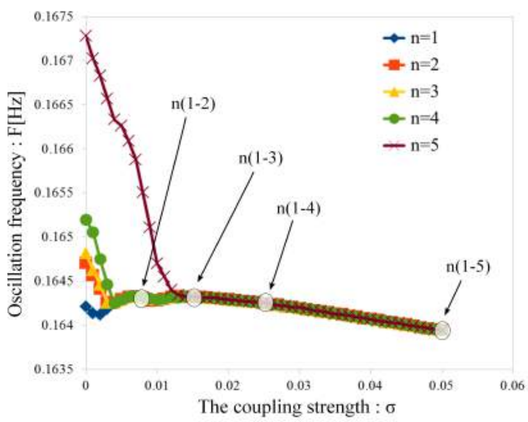

Figure 6: Transition of oscillation frequency when the value of α are set nearer α_1($\alpha_2 = 0.41$, $\alpha_3 = 0.43$, $\alpha_4 = 0.44$).

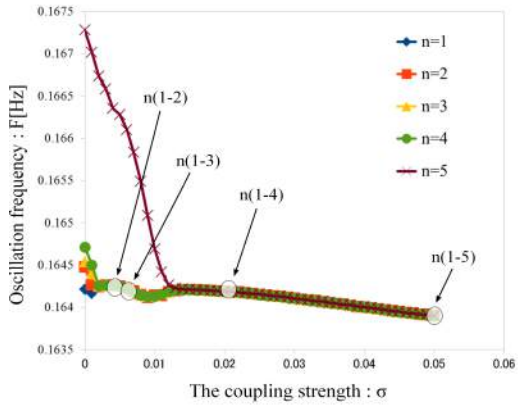

Figure 7: Transition of oscillation frequency when the value of α are set nearer α_5($\alpha_2 = 0.45$, $\alpha_3 = 0.46$, $\alpha_4 = 0.47$).

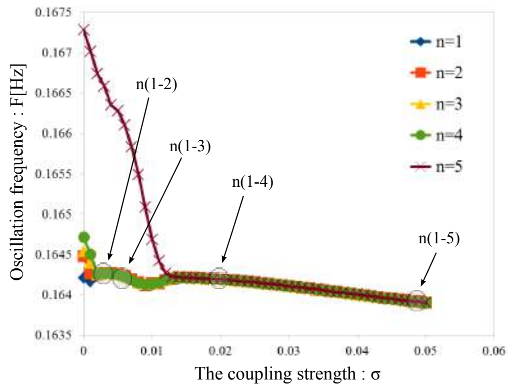

Figure 8: Transition of oscillation frequency when the value of α are set nearer medium value($\alpha_2 = 0.43$, $\alpha_3 = 0.44$, $\alpha_4 = 0.45$).

There are four patterns to set parameter α_2, α_3 and α_4. We set these parameters : approximately at equal interval while avoiding the values that affect periodic solution, nearer α_1, nearer α_5 and nearer medium value.

From the Figs. 5~8, $n(a-b)$ is indicated circuit state between circuit number a and b. In addition, we surround the coupling strength with circle when the phase difference is below 30 degrees. It is clear that relationship is uncommon between synchronous state and oscillation frequency in any patterns of $\alpha_2 \sim \alpha_4$. Each circuit does not synchronize when oscillation frequency converges on a steady value. And, oscillation frequencies are obtained a stable value before entire circuit becomes synchronization.

Next, we pay attention to timing of synchronization of among the coupled chaotic circuits. Timing of synchronization forced on oscillation frequency and the phase difference when the values of $\alpha_2 \sim \alpha_4$ are set equability is shown in Tables 1 and 2. Table 1 is visualized Fig. 9 to be easily understandable timing of synchronization. There are four symbols ○, ◎, △, ◇. These symbols show that circuits have some condition. For example, when the coupling strength is set 0.002, chaotic circuit of circuit number n=1,2 becomes synchronization as shown in Table 1. With respect to Tab. 2, when coupling strength is set 0.003, oscillation frequency is obtained a stable value in circuit number n=1,3 and n=2,3. It will be easy to understand this method by compare Table 1 and 2 with Fig. 5. By the expectation, we consider that it has similarities for two investigations. As compared to two table, there is no similarity in each timing.

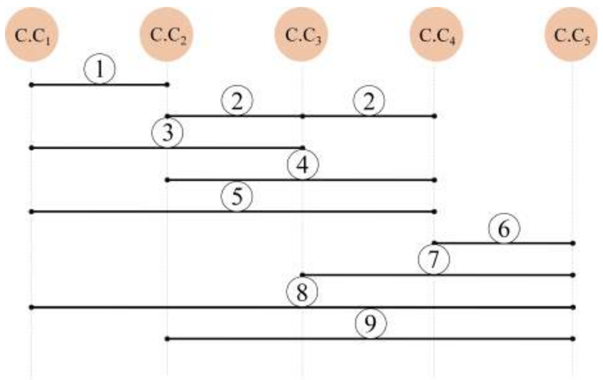

Figure 9: Timing of synchronization based on phase difference.

Table 1: Timing of synchronization based on phase difference.

		Circuit number : n				
		1	2	3	4	5
σ	0.002	○	○			
	0.006		○	○, ◎	◎	
	0.007	○		○		
	0.021		○		○	
	0.024	○			○	
	0.033				○	○
	0.045			○		○
	0.050	○				○
	0.051		○			○

Table 2: Timing of synchronization based on oscillation frequency.

		Circuit number : n				
		1	2	3	4	5
σ	0.002	○	○			
	0.003	○	◎	○, ◎		
	0.005	○	◎	△	○, ◎, △	
	0.021	○	◎	△	◇	○, ◎, △, ◇

5. Conclusions

In this study, we have investigated relationship between synchronous state and oscillation frequency in the coupled chaotic circuit. From simulation results, we could obtain a steady value of oscillation frequency before entire circuit becomes synchronization. These results are not determined by the value of α. That is, synchronous state in the coupled chaotic circuit is lowly dependent to oscillation frequency.

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