2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Partial Decline of Synchronization Rate in Chaotic Circuits Coupled by a Resistor

Tatsuki Sobue†, Yasuteru Hosokawa‡, Yoko Uwate† and Yoshifumi Nishio†

†Dept. Electrical and Electronic Eng., Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email:{sobue, uwate, nishio}@ee.tokushima-u.ac.jp,
‡Dept. Media and Information Syst., Shikoku University
Furukawa, Ohjin, Tokushima, Japan
Email: hosokawa@keiei.shikoku-u.ac.jp

Abstract—In our previous study, a modified Shinriki-Mori circuit was proposed. The circuit includes two LC resonators which are coupled by resistors. By changing a parameter, switching phenomena of synchronization and asynchronization between two resonators were observed.

In this study, we propose new coupled system which two previous study's systems are coupled by a resistor. As a result, interesting phenomena about relationship between LC resonators and coupling strength are observed.

1. Introduction

In coupled chaotic circuits, many interesting phenomena, for instance, chaotic synchronization phenomena, intermittency chaos, spatio-temporal chaos and so on are observed. Therefore, there are many studies about coupled chaotic circuits. In almost these studies, some of famous chaotic circuits have been applied. One of famous chaotic circuits is a Shinriki-Mori circuit [1] [2]. There are many investigations of coupled chaotic circuits using a Shinriki-Mori circuit [4] [5].

In our previous study, a modified Shinriki-Mori circuit [6] has been proposed. The Shinriki-Mori circuit consists of a LC resonator, by-directional diodes and the others. In the proposed circuit, a LC resonator and by-directional diodes included in the original circuit are copied and added to the original circuit. By applying different parameters of LC resonators, two similar chaotic waveforms which are observed as voltages of LC resonators are obtained. Additionally, these waveforms are influenced by changing circuit parameters.

In this study, two modified Shinriki-Mori circuits coupled by a resistor are proposed. This system includes parameter mismatches. Therefore, LC resonators could not synchronized at all. In this system, similar chaotic waveforms are also observed and these are not synchronized at all. Additionally, these waveforms are influenced by changing circuit parameters. Namely, it means that in spite of asynchronous state, the relationship among LC resonators exist. The aim of this study is to reveal the relationship.

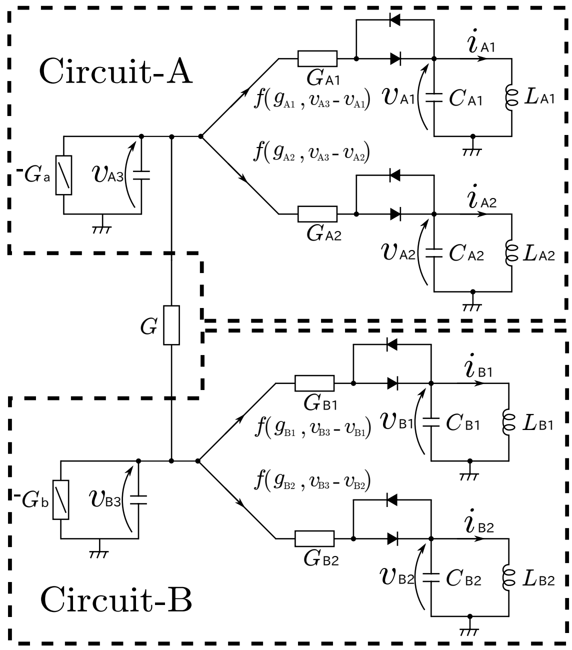

Figure 1: Proposed System.

2. Proposed System

Figure 1 shows the proposed system. Two modified Shinriki-Mori circuits are coupled by a resistor. In this study, two modified Shinriki-Mori circuits are called as Circuit A and Circuit B, respectively. Additionally, LC resonators included in two circuits are called as subcircuit A1, A2, B1 and B2, respectively. A difference between Circuit A and Circuit B is only parameters of subcircuits. In order to change parameters of nonlinearity, resistors G_{A1}, G_{A2}, G_{B1} and G_{B2} are added. Bidirectionally coupled diodes are modeled as a piecewise linear function shown in Figure 2. The others are modeled as linear elements.

Using this model, the circuit equation is described as follows.

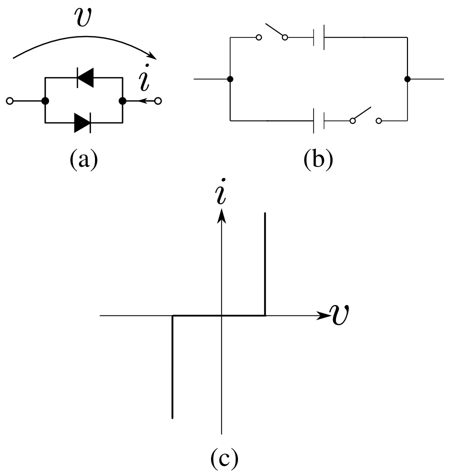

Figure 2: Bidirectionally coupled diodes model. (a) Circuit schematic. (b) Circuit model. (c) $v - i$ characteristic.

$$
\begin{cases}
C_{a1}\dfrac{dv_{a1}}{dt} = g_{a1} f(v_{a3} - v_{a1}) - i_{a1}, \\
C_{a2}\dfrac{dv_{a2}}{dt} = g_{a2} f(v_{a3} - v_{a2}) - i_{a2}, \\
C_{a3}\dfrac{dv_{a3}}{dt} = G_a v_{a3} - g_{a1} f(v_{a3} - v_{a1}) \\
\qquad - g_{a2} f(v_{a3} - v_{a2}) - G(v_{a3} - v_{b3}), \\
L_{a1}\dfrac{di_{a1}}{dt} = v_{a1}, \\
L_{a2}\dfrac{di_{a2}}{dt} = v_{a2}, \\
C_{b1}\dfrac{dv_{b1}}{dt} = g_{b1} f(v_{b3} - v_{b1}) - i_{b1}, \\
C_{b2}\dfrac{dv_{b2}}{dt} = g_{b2} f(v_{b3} - v_{b2}) - i_{b2}, \\
C_{b3}\dfrac{dv_{b3}}{dt} = G_b v_{b3} - g_{b1} f(v_{b3} - v_{b1}) \\
\qquad - g_{b2} f(v_{b3} - v_{b2}) + G(v_{a3} - v_{b3}), \\
L_{b1}\dfrac{di_{b1}}{dt} = v_{b1}, \\
L_{b2}\dfrac{di_{b2}}{dt} = v_{b2},
\end{cases}
\tag{1}
$$

where

$$
f(v) = v + (|v - v_{th}| - |v + v_{th}|)/2. \tag{2}
$$

By substituting the variables and the parameters,

$$
\begin{aligned}
& x_{\{A1,A2,A3\}} = \frac{v_{\{a1,a2,a3\}}}{V_{th}}, \quad x_{\{A4,A5\}} = \sqrt{\frac{L_{a1}}{C_{a1}}}\frac{i_{\{a1,a2\}}}{V_{th}}, \\
& x_{\{B1,B2,B3\}} = \frac{v_{\{b1,b2,b3\}}}{V_{th}}, \quad x_{\{B4,B5\}} = \sqrt{\frac{L_{a1}}{C_{a1}}}\frac{i_{\{b1,b2\}}}{V_{th}}, \\
& \frac{d}{dt} = \text{“ . ”}, \quad \alpha_{\{a1,a2,a3,b1,b2,b3\}} = g_{\{a1,a2,a3,b1,b2,b3\}}\sqrt{\frac{L_{a1}}{C_{a1}}}, \\
& \beta_{\{a2,a3,b1,b2,b3\}} = \frac{C_{a1}}{C_{\{a2,a3,b1,b2,b3\}}}, \quad \gamma_{\{a2,b1,b2\}} = \frac{L_{a1}}{L_{\{a2,b1,b2\}}}, \\
& \delta_{\{a,b\}} = G_{\{a,b\}}\sqrt{\frac{L_{a1}}{C_{a1}}}, \quad \varepsilon = G\sqrt{\frac{L_{a1}}{C_{a1}}},
\end{aligned}
\tag{3}
$$

equations (1) and (2) are described as follows.

$$
\begin{cases}
\dot{x}_{A1} = \alpha_{a1} f'(x_{A3} - x_{A1}) - x_{A4}, \\
\dot{x}_{A2} = \beta_{a2}\{\alpha_{a2} f'(x_{A3} - x_{A2}) - x_{A5}\}, \\
\dot{x}_{A3} = \beta_{a3}\{\delta_a x_{A3} - \alpha_{a1} f'(x_{A3} - x_{A1}) \\
\qquad - \alpha_{a2} f'(x_{A3} - x_{A2}) - \varepsilon(x_{A3} - x_{B3})\}, \\
\dot{x}_{A4} = x_{A1}, \\
\dot{x}_{A5} = \gamma_{a2} x_{A2}, \\
\dot{x}_{B1} = \beta_{b1}\{\alpha_{b1} f'(x_{B3} - x_{B1}) - x_{B4}\}, \\
\dot{x}_{B2} = \beta_{a2}\{\alpha_{b2} f'(x_{B3} - x_{B2}) - x_{B5}\}, \\
\dot{x}_{B3} = \beta_{b3}\{\delta_b x_{B3} - \alpha_{b1} f'(x_{B3} - x_{B1}) \\
\qquad - \alpha_{b2} f'(x_{B3} - x_{B1}) + \varepsilon(x_{A3} - x_{B3})\}, \\
\dot{x}_{B4} = \gamma_{b1} x_{B1}, \\
\dot{x}_{B5} = \gamma_{b2} x_{B2},
\end{cases}
\tag{4}
$$

where

$$
f'(x) = x + (|x - 1| - |x + 1|)/2. \tag{5}
$$

3. Computer Simulations

In our previous study, a modified Shinriki-Mori circuit was investigated [6]. Following results were obtained. In the case of applying same parameters to subcircuits, subcircuits are synchronized. Additionally, switching phenomena between synchronization and asynchronization are observed in the case of applying different circuit parameters of subcircuits. As a result of investigations of the relationship between synchronization rates and circuit parameters, it was confirmed that some similar chaotic waveforms could be obtained at the same time.

In this study, two modified Shinriki-Mori circuits coupled by a resistor are investigated. This system includes four subcircuits. Additionally, four subcircuits are not equivalent relations. Therefore, the relationship among four subcircuits is very interesting.

Figure 3 shows one of computer simulation results. Double scroll type attractors can be observed. Double

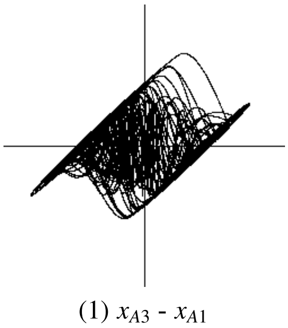

Figure 3: Attractors of Circuit A and Circuit B. $\alpha_{a1} = \alpha_{a2} = \alpha_{b1} = \alpha_{b2} = 9, \beta_{a2} = 0.8, \beta_{b1} = 0.4, \beta_{b2} = 0.7, \beta_{a3} = \beta_{b3} = 0.32, \gamma_{a2} = \gamma_{b1} = \gamma_{b2} = 1.0$ and $\delta_a = \delta_b = 1.4$. (1) Circuit A. Horizontal axis is x_{A3} and vertical axis is x_{A1}. (2) Circuit B. Horizontal axis is x_{B3} and vertical axis is x_{B1}.

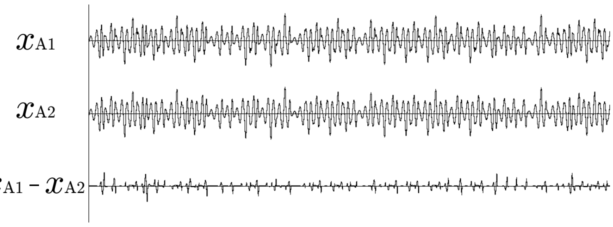

Figure 4: Waveforms of subcircuit A1 and A2. $\alpha_{a1} = \alpha_{a2} = \alpha_{b1} = \alpha_{b2} = 9, \beta_{a2} = 1.0, \beta_{b1} = 0.4, \beta_{b2} = 0.7, \beta_{a3} = \beta_{b3} = 0.32, \gamma_{a2} = \gamma_{b1} = \gamma_{b2} = 1.0$ and $\delta_a = \delta_b = 1.4$.

scroll type attractor of Figure 3(2) is larger attractor of Figure 3(1). In this case, waveforms of subcircuit A1 and A2 are shown in Figure 4. Quasi-synchronization and asynchronization states are appear at randomly. In order to investigate this phenomenon, the relationship between voltage differences of subcircuits and a coupling strength is paid attention. Figure 5 shows the relationship between the voltage differences and the coupling strength. The voltage difference between v_{A1} - v_{A2} is corresponding to x_{A1} - x_{A2}. In the same way, subcircuit A2 and B1, B1 and B2 are corresponding to x_{A2} - x_{B1} and x_{B1} - x_{B2}, respectively. In among these subcircuits, synchronization and asynchronization states are observed. By increasing the coupling strength ε, synchronization rate of x_{A1} - x_{A2} is decreased in Figure 5(1)-(3). On the other hand, synchronization rate of x_{B1} - x_{B2} is increased.

In order to make this phenomenon clear, a definition of the synchronization is defined as follows.

$$\begin{aligned} |x_{A1} - x_{A2}| &< 0.1 \\ |x_{A2} - x_{B1}| &< 0.1 \\ |x_{B1} - x_{B2}| &< 0.1 \end{aligned} \quad (6)$$

Equation 6 shows a definition of the synchronization as shown in Figure 5. Namely, a range of the synchronous state is from 0.1 to -0.1. Using this definition, the relationship between synchronous states and coupling strength is investigated.

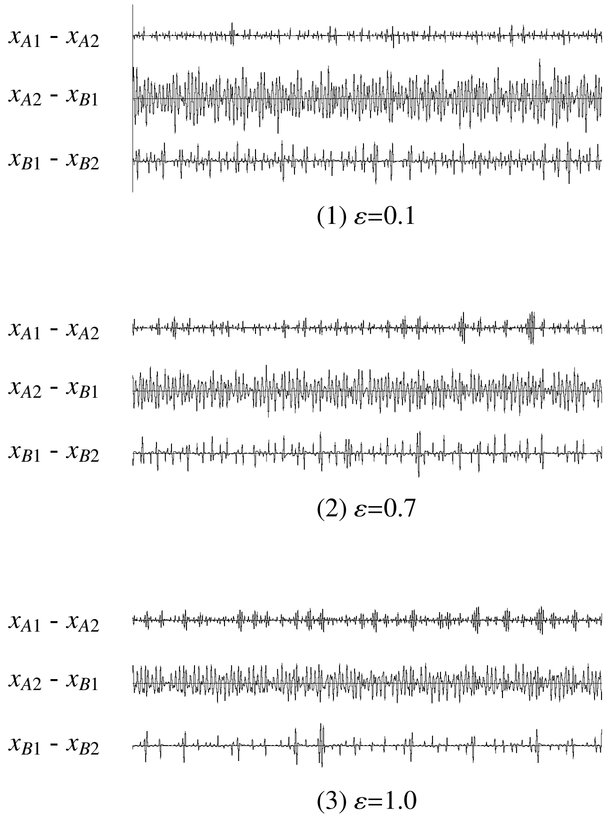

Figure 5: Differential waveforms of Circuit A and Circuit B. $\alpha_{a1} = \alpha_{a2} = \alpha_{b1} = \alpha_{b2} = 9$, $\beta_{a2} = 0.8$, $\beta_{b1} = 0.4$, $\beta_{b2} = 0.7$, $\beta_{a3} = \beta_{b3} = 0.32$, $\gamma_{a2} = \gamma_{b1} = \gamma_{b2} = 1.0$ and $\delta_a = \delta_b = 1.4$.

Figure 7 shows the statistical data of synchronous rates and the coupling strength ε. A step size of the coupling strength ε is 0.01. Calculation time of each parameter is 500. Time step is 0.005. The synchronous rate between subcircuits A1 and A2 is decreasing by increasing coupling strength ε from 0 to 1. After that the rate is increasing and becomes 100%. On the other hand, the synchronous rate between subcircuits B1 and B2 is increasing by increasing coupling strength ε. Additionally, the synchronous rate between subcircuits A2 and B1 remains very low rate. In $\varepsilon > 1.3$, the rate becomes 100%. In $0.0 < \varepsilon < 1.0$, double scroll type attractors can be observed in two circuits. However, In $\varepsilon > 1.0$, oscillations becomes periodic orbits. Therefore, rates become 100%.

This result shows a following phenomenon observed in this system. Two chaotic circuits coupled by a resistor have same circuit structure. By changing a value of the coupling resistor, synchronization rate of a part of one circuit is increased and the other is decreased. Namely, in spite of

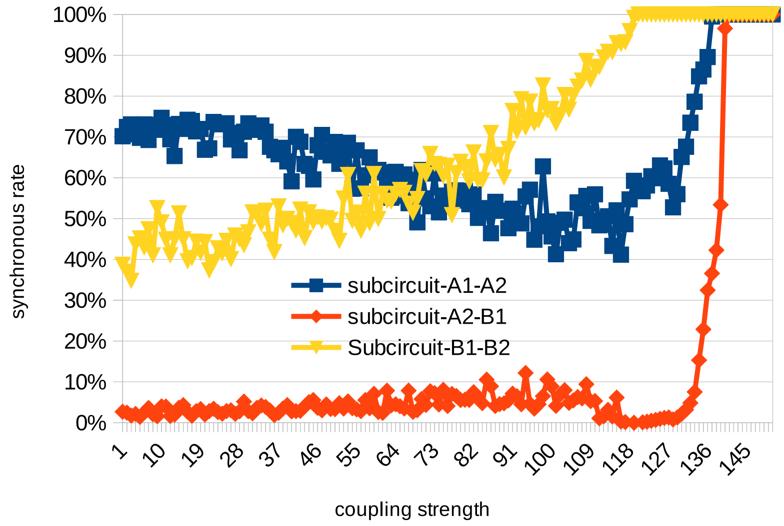

Figure 7: Synchronous rate of Circuit A and Circuit B. $\alpha_{a1} = \alpha_{a2} = \alpha_{b1} = \alpha_{b2} = 9$, $\beta_{a2} = 0.8$, $\beta_{b1} = 0.4$, $\beta_{b2} = 0.7$, $\beta_{a3} = \beta_{b3} = 0.32$, $\gamma_{a2} = \gamma_{b1} = \gamma_{b2} = 1.0$ and $\delta_a = \delta_b = 1.4$.

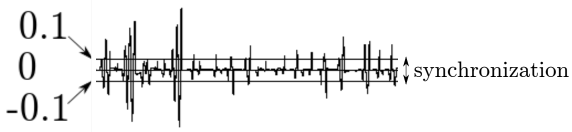

Figure 6: Definition of synchronization.

being same circuit structure, an inverse result is obtained. Additionally, one important point of this result is that two circuits are not synchronized at all. In this condition, it is interesting that these two circuits influence each other.

4. Conclusion

In this study, two modified Shinriki-Mori circuits coupled by a resistor are proposed. By investigating the relationship between subcircuits and a coupling strength, it is confirmed that synchronization rate of a part of one circuit is increased and the other is decreased by changing a coupling strength. We consider that the most important point of this result is that two circuits are not synchronized at all and two circuits influence each other.

In the future works, we will investigate this phenomenon in detail.

References

[1] M. Shinriki, M. Yamamoto and S. Mori, "Multimode Oscillations in a Modified van der Pol Oscillator containing a positive nonlinear conductance," *Proc. IEEE*, vol. 69, no. 3, pp. 394-395, 1981.

[2] N. Inaba, T. Saito and S. Mori, "Chaotic Phenomena in a Circuit with a Negative Resistance and an Ideal Switch of Diodes," *IEICE Trans.*, vol. E-70, pp. 744–754, 1987.

[3] H. Sekiya, S. Moro, S. Mori, and I. Sasase, "Synchronization Phenomena on Four Simple Chaotic Oscillators Full-Coupled by Capacitors," *IEICE Trans. Fundamentals*, vol. J82-A, No. 3, pp.375–385, 1999.

[4] M. Miyamura, Y. Nishio and A. Ushida, "Clustering in Globally Coupled System of Chaotic Circuits," *Proc. ISCAS'02*, vol. 3, pp. 57–60, 2002.

[5] L. M. Pecora and T. L. Carrol, "Synchronization in Chaotic Systems," *Physical Review Letters*, vol. 64, pp. 821–824, 1990.

[6] T. Sobue, Y. Hosokawa and Y. Nishio, "Coupled Chaotic System Based on Shinriki-Mori Circuit," *Proc. NCSP'14*, pp. 529–532, 2014.