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Synchronization in Two van der Pol Oscillators Coupled by Transmission Line Model

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Abstract—The internal wiring of high-speed VLSI is considered for transmission line, because its structure is very complex and high density.

In this study, we investigate synchronization phenomena in two van der Pol oscillators coupled by transmission line model. By using the computer simulations, we confirm several types of synchronization states such as coexistence in-phase and anti-phase by changing the circuit parameter.

1. Introduction

Recently, the synchronization observed from coupled oscillators and chaotic circuits systems have been studied actively [1], [2]. It is important to investigate basic synchronization observed in coupled oscillatory systems for future engineering applications such as chaotic communication and chaotic cryptography. In high-speed VLSI, the internal wiring is considered for transmission line because the structure of high-speed VLSI is complex and high density. However, there are not many discussions about effect caused by transmission line model for coupled oscillatory systems.

In our previous study, the synchronization of two chaotic circuits with transmission line coupled by the cross talk have been investigated [3]-[7]. We have observed in-phase and anti-phase synchronization phenomena from these circuits in computer simulations (see Figs. 1, 2) [6]. In this system model, the part of inductor and capacitor of chaotic circuit is modeled by transmission line. The synchronization of oscillators coupled by transmission line using the ladder circuit of inductor and capacitor is not really investigated.

This paper presents synchronization phenomena in two van der Pol oscillators coupled by transmission line model. We model transmission line using ladder circuits of inductor and capacitor as lossless transmission line. By using the computer simulations, several types of synchronization states such as coexistence in-phase and anti-phase are confirmed by changing the circuit parameter.

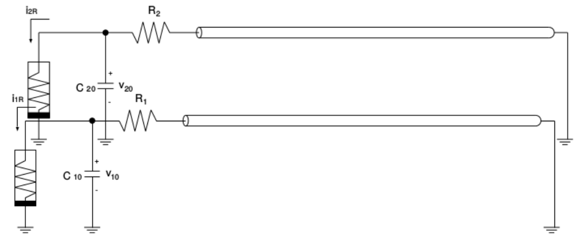
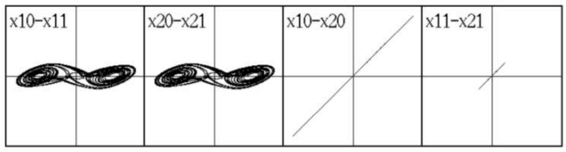
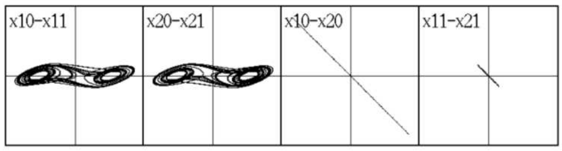


Figure 1: Two Chua's circuits with transmission lines.



(a) In-phase (coupling capacitors).



(b) Anti-phase (mutal inductors).

Figure 2: Synchronization states.

2. Two van der Pol Oscillators Coupled by Transmission Line Model

Figure 3 shows the conceptual circuit model of this study. Two van der Pol Oscillators are coupled by transmission line as shown in Fig. 4.

Next, we develop the expression for the circuit equations of the circuit model as shown in Fig. 4. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third order polynomial equation,

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), (k = 1, 2). \qquad (1)$$

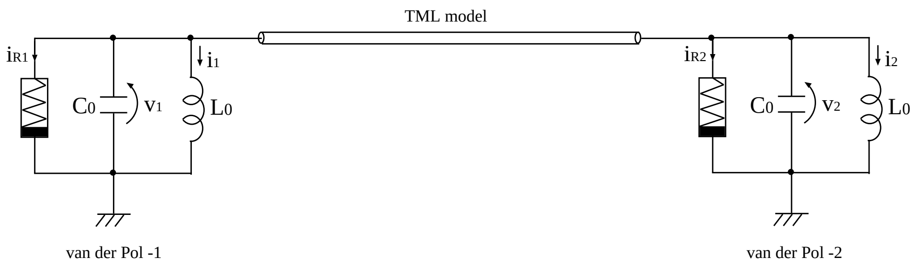


Figure 3: Conceptual circuit model.

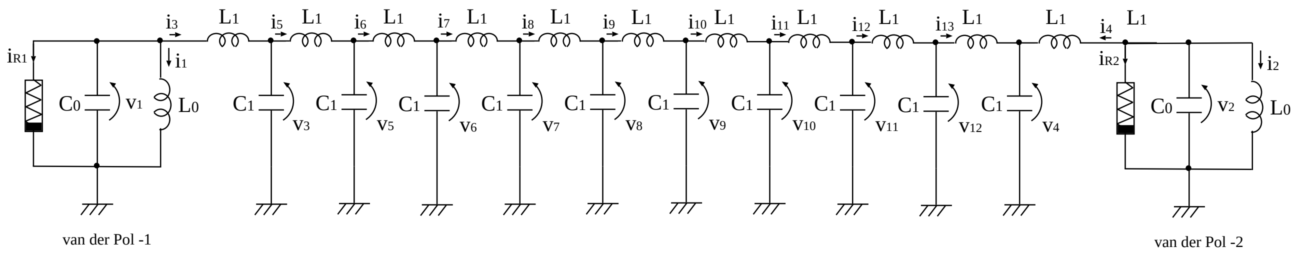


Figure 4: Two van der Pol Oscillators coupled by transmission line model.

The normalized circuit equations governing the circuit are expressed as [van der Pol Oscillator]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - y_k - \sqrt{\alpha}\sqrt{\dfrac{1}{\beta}}y_n \\ \dfrac{dy_k}{d\tau} = y_k \\ \qquad\qquad\qquad (k = 1, 2). \end{cases} \tag{2}$$

[Transmission Line Model]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \sqrt{\alpha}\sqrt{\beta}(y_k - y_n) \\ \dfrac{dy_k}{d\tau} = \sqrt{\alpha}\sqrt{\beta}(x_k - x_n) \\ \qquad\qquad\qquad (k = 3...13). \end{cases} \tag{3}$$

where

$$t = \sqrt{L_0 C_0}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}}x_k \qquad (k = 1...12),$$

$$i_k = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C_0}{L_0}}y_k \qquad (k = 1, 2),$$

$$i_k = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C_1}{L_1}}y_k \qquad (k = 3...13),$$

$$\alpha = \frac{L_0}{L_1}, \quad \beta = \frac{C_0}{C_1},$$

In this equations, ε denotes nonlinearity of the oscillators. α and β denote the ratio of inductor and capacitor of the oscillators and transmission line, respectively.

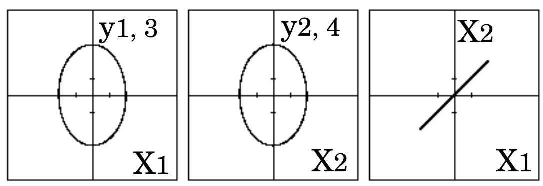
For the computer simulations, we calculate Eq. (3) using the fourth-order Runge-Kutta method with the step size h = 0.005. The parameter ε is fixed with 0.1.

3. Synchronization Phenomena

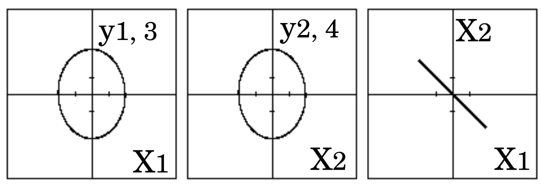
3.1. Attractor

Figures 5-7 show the attractors and the lissajous when the parameters α and β are changed.

In the case of α=β=1.0, coexistence of in-phase and anti-phase states of periodic solutions are observed as shown in Fig. 5. Figure 6 shows the simulation results when α and β are set to 1.9. In this case, we also confirm coexistence of in-phase and anti-phase state. We confirm that the torus attractor can be obtained when two oscillators are synchronized at in-phase, while the periodic attractor is generated when two oscillators are synchronized at anti-phase.



(a) In-phase.

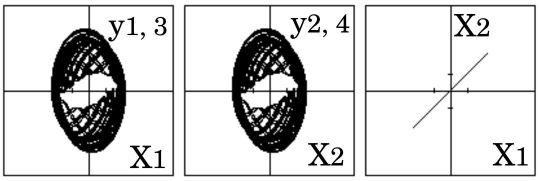


(b) Anti-phase.

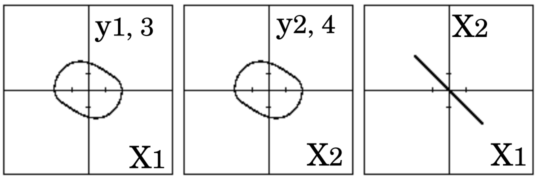
Figure 5: Attractor (α = 1.0).

Figure 7 shows the results when α and β are set to 2.4 and 3.0. In the case of α=β=2.4, torus attractor can be observed. In the case of α=β=3.0, two oscillators generate the periodic attractor. In the case of α=β=5.0, we observe coexistence of in-phase and anti-phase states again as shown in Fig. 8.

By increasing the value of α and β, only in-phase state can be observed (see. Fig. 9). Namely coexistence of in-phase and anti-phase disappears if the ratio of α and β is set to large values.

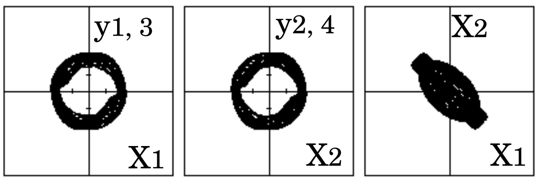


(a) In-phase.

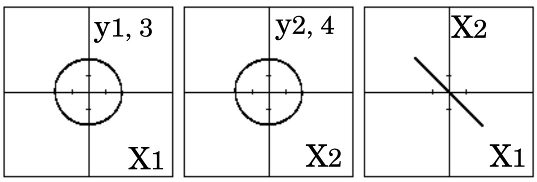


(b) Anti-phase.

Figure 6: Attractor (α = 1.9).



(a) α=2.4.

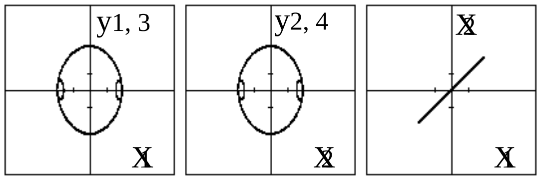


(b) α=3.0.

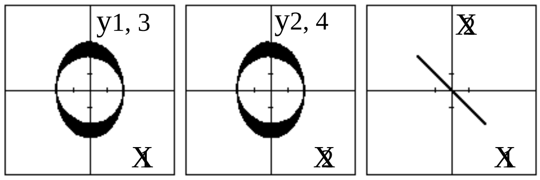
Figure 7: Attractor (α=2.4, 3.0).

3.2. Bifurcation diagram

Next, we consider one parameter bifurcation when the parameters α and β are changed. Figure 10 shows the simulation results when initial condition is set to in-phase and anti-phase mode, respectively. We can observe coexistence with different solutions when α, β is smaller than 2.3. By increasing the parameter α and β, only one solution exists. Furthermore, we observe periodic window around α=β=2.35 (see. Fig. 11).



(a) In-phase.



(b) Anti-phase.

Figure 8: Attractor (α=5.0).

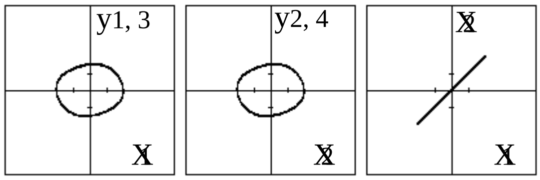
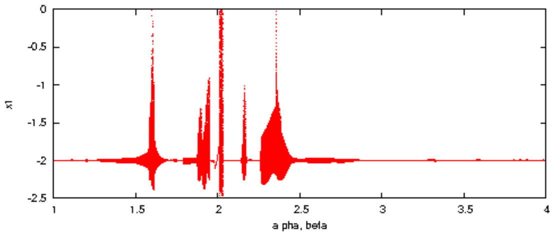


Figure 9: Attractor (α=8.0).

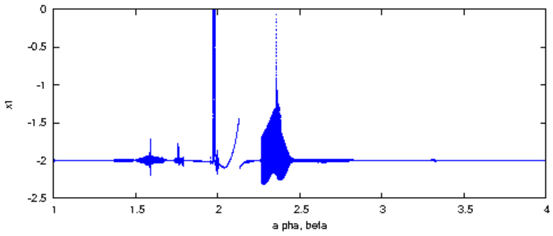
3.3. Phase Difference

In this subsection, we calculate the phase difference between two oscillators. The simulation result of the phase difference is shown in Fig. 12. We can observe coexistence with different solutions when α, β is smaller than 2.3 as known from the bifurcation diagram. When the parameters α and β is larger than 2.3, in-phase synchronization disappear and only anti-phase synchronization is observed. After that, coexistence of in-phase and anti-phase states can be observed around α=β=5.0. Finally, by increasing the value of α and β, only in-phase state can be observed.

In order to confirm the switching several synchronization states, Fig. 13 shows the expanded graph of Fig. 12. From this figure, we can see that synchronization state is classified in in-phase, anti-phase, coexistence and asynchronous states depending on the parameter. Investigation of synchronization state with small step size of the parameter is one of our future work.



(a) Initial condition: In-phase mode.



(b) Initial condition: Anti-phase mode.

Figure 10: Bifurcation diagram.

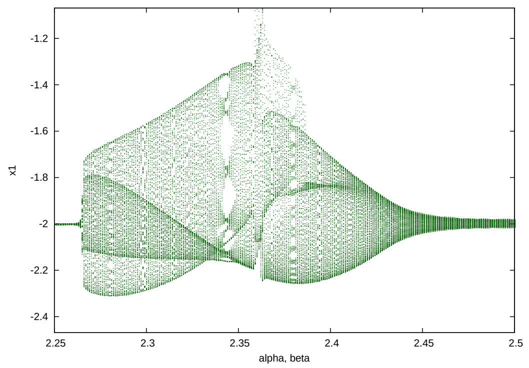


Figure 11: Periodic solution in bifurcation.

4. Conclusions

We have investigated synchronization phenomena in two van der Pol oscillators coupled by transmission line model. By using the computer simulations, several types of synchronization states such as coexistence in-phase and anti-phase were confirmed by changing the circuit parameter.

In our future works, we would like to investigate effect of the length of the transmission line and apply this model to more complex networks such as smart grid network and social network.

Acknowledgment

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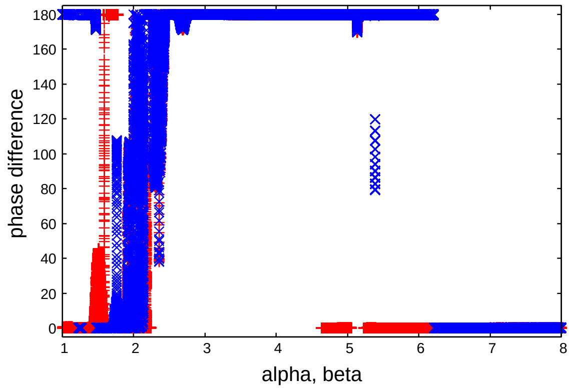

Figure 12: Phase difference.

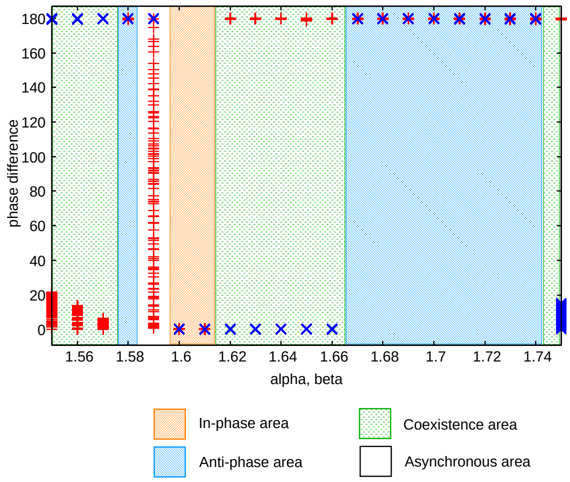

Figure 13: Phase difference.

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