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Synchronization and Chaos Propagation of Coupled Chaotic Circuits in Symmetric-Asymmetric Systems

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Abstract—In this study, we investigate synchronization and chaos propagation of 5 coupled chaotic circuits in various systems. We propose a ladder system model that the central circuit generates chaotic attractor and the other circuits generate three-periodic attractor. We observe that chaotic attractor of the central circuit propagates to all circuits. By measuring the phase difference between the circuits, we investigate synchronization in the entire system. Moreover, we compare the phase difference between symmetric and asymmetric systems in the cases of adding the coupling resistor from the ladder system.

1. Introduction

Synchronization of chaotic systems are good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Therefore, synchronization of coupled chaotic circuits has been interested by many researchers [1]-[4]. In particular, it is important to investigate synchronization phenomena of coupled circuits under some difficult situations for the circuits. In our research group, synchronization and chaos propagation have been reported in the ring of coupled chaotic circuits [5][6]. However, these research were considered about the only one ring system.

In this study, synchronization and chaos propagation of coupled chaotic circuits in various systems are researched. We propose a ladder system model of 5 chaotic circuits coupled by the resistors. In this model, the central circuit generates chaotic attractor and the other circuits generate the three-periodic attractors. First, we show synchronization and chaos propagation in the ladder system. By measuring the phase difference among all adjacent circuits, we investigate synchronization in the entire system. Moreover, the symmetric and asymmetric systems obtained from adding the coupling resistor from the ladder system, are studied.

2. System Model

Figure 1 shows the chaotic circuit. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. We propose the ladder system model as shown in Fig. 2. Each circuit is coupled by a resistor R

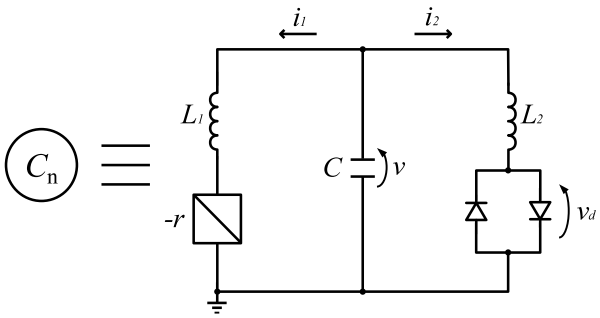

Figure 1: Chaotic circuit.

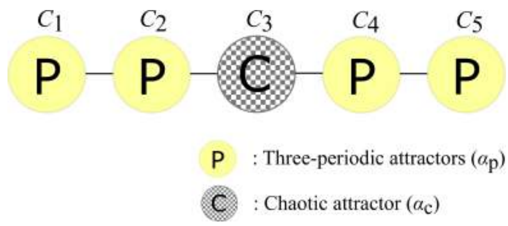

Figure 2: Proposed ladder system model.

which is corresponding the edge in this system. The number of the circuits in the system is set to 5. The central circuit (C_3) generates chaotic attractor and the other circuits generate three-periodic attractors.

The circuit equations of this circuit are described as follows:

$$\begin{cases} L_1 \dfrac{di_1}{dt} = v + ri \\ L_2 \dfrac{di_2}{dt} = v - v_d \\ C \dfrac{dv}{dt} = -i_1 - i_2, \end{cases} \tag{1}$$

where v_d is the characteristic of the nonlinear resistor consisting of the diodes, is described as follows:

$$v_d = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \tag{2}$$

By using the variables and parameters:

$$\begin{cases} i_1 = \sqrt{\frac{C}{L_1}} V x_n, \;\; i_2 = \frac{\sqrt{L_1 C}}{L_2} V y_n, \;\; v = V z_n, \\ \alpha = r\sqrt{\frac{C}{L_1}}, \;\; \beta = \frac{L_1}{L_2}, \;\; \delta = r_d \frac{\sqrt{L_1 C}}{L_2}, \\ g = \frac{1}{R}\sqrt{\frac{L_1}{C}}, \;\; t = \sqrt{L_1 C_2}\tau, \;\; \text{"}\cdot\text{"} = \frac{d}{d\tau}, \end{cases} \tag{3}$$

the normalized circuit equations are given as follows:

$$\begin{cases} \dot{x} = \alpha x + z \\ \dot{y} = z - f(y) \\ \dot{z} = -x - \beta y, \end{cases} \tag{4}$$

where α represents the chaos degree. $f(y)$ can be expressed as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \tag{5}$$

In the proposed ladder system, the circuits are connected to only adjacent circuits by the resistors. The normalized circuit equations of the system are given as follows:

$$\begin{cases} \dot{x_n} = \alpha x_n + z_n \\ \dot{y_n} = z_n - f(y_n) \\ \dot{z_n} = -x_n - \beta y_n - \sum_{m \in s_n} g(z_n - z_m), \end{cases} \tag{6}$$

where n represents the circuit number up to 5 in this study. S_n is the set of circuits which are directly connected to C_n. g represents the coupling strength corresponding the coupling resistor R. For the computer simulations, we calculate Eq. (6) using the fourth-order Runge-Kutta method with the step size $h = 0.01$.

3. Simulation Result

In this study, we fix the circuit parameters of the system as $\alpha_c = 0.460$, $\alpha_p = 0.412$, $\beta = 3.0$ and $\delta = 470.0$. First, we investigate synchronization and chaos propagation in the ladder system. Moreover, we consider synchronization of the symmetric and asymmetric systems in the cases of adding the edge from the ladder system.

3.1. Ladder System

Figure 3 shows some examples of the computer simulation results. Figure 3(a) shows the state when all circuits are not connected. We can observe the state that the chaos are propagated to the only adjacent circuits from the central chaotic circuit in a range of the coupling strength g (see Fig. 3(b)). By increasing the coupling strength g, all

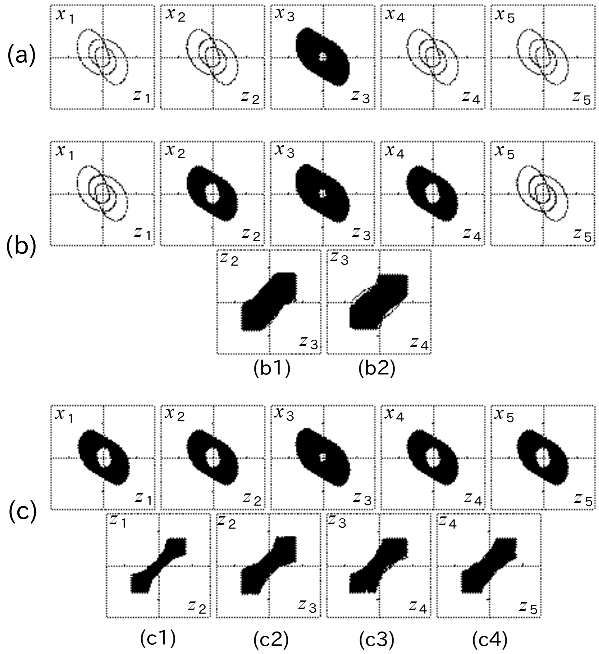

Figure 3: Attractors and phase differences in the ladder system, $\alpha_c = 0.460$, $\alpha_p = 0.412$, $\beta = 3.0$ and $\delta = 470.0$. (a) $g = 0$, (b) $g = 0.005$, (b1) 30.00°, (b2) 41.93° and (c) $g = 0.010$, (c1) 12.88°, (c2) 18.12°, (c3) 22.68°, (c4) 11.02°.

three-periodic attractors are affected from chaotic attractor and all circuits close to the synchronous state. In order to investigate the synchronous state, we measure the phase difference between the adjacent circuits.

Figure 4 shows the relation between the phase difference and the coupling strength. In Fig. 4, the phase difference shows the average among all adjacent circuits. If all circuits are not synchronized, the phase difference shows 90°. We confirm that the phase difference is smaller and close to 0° by increasing the coupling strength. Namely, all circuits reach the synchronous state by chaos propagation.

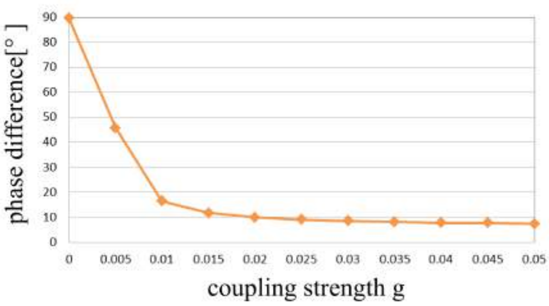

Figure 4: Relation between the phase difference and the coupling strength in the ladder system.

Table 1: Phase differences in all symmetric systems.

Edge	Phase difference [°]
5-A	17.89
5-B	16.15
6-A	15.02
6-B	9.57
6-C	15.04
7-A	9.35
7-B	15.32
7-C	10.05
7-D	15.32
8-A	9.39
8-B	15.64
8-C	9.41
9-A	9.27
9-B	9.67

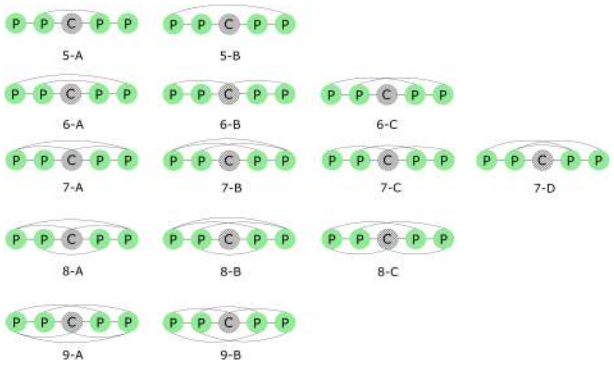

Figure 5: Symmetric system patterns.

3.2. Symmetric System

From this section, we consider the symmetric and asymmetric systems in the cases of adding the edge from the ladder system. We focus on the number of the edge and the symmetric and asymmetric systems. In the case of the ladder system, the number of edge is 4. On the other hand, when the number of edge is 10, the system shows full-coupled system. We consider the system in the intermediate number of the edge between 4 and 10.

In this section, the symmetric systems are considered. Figure 5 shows the illustration of the conceivable all symmetric systems. Table 1 shows the phase differences of all symmetric systems in the case of $g = 0.01$ and each system pattern corresponds to Fig. 5. The phase difference shows the average among all adjacent circuits in Table 1. From Table 1, the phase difference is decreased by increasing the number of edges. Figure 6 shows the relation between the average phase difference of the symmetric systems and the number of edges. The average phase difference close to 0° by adding the edges. Namely, the entire system easy to reach the synchronous state by increasing the number of

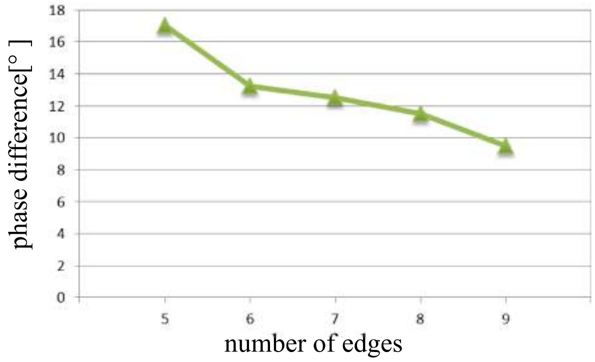

Figure 6: Relation between the average phase difference and the number of edges in the symmetric systems.

the edges. However, the phase difference in 6-B is smaller than 7-B, 7-C, 7-D and 8-B in Table 1. For this reason, we consider that the phase difference is affected from the coupling way of each circuit.

3.3. Asymmetric System

In this section, the asymmetric systems are considered. Figure 7 shows the illustration of the all conceivable asymmetric systems. Table 2 shows the phase differences of all symmetric systems in the case of $g = 0.01$ and each system pattern corresponds to Fig. 7. The phase difference shows the average among all adjacent circuits in Table 2. Figure 8 shows the relation between the average phase difference of the asymmetric systems and the number of edges. From Table 2 and Fig. 8, in the asymmetric systems, the average phase difference is not decreased by increasing the number of the edges like the symmetric systems. We consider that this result is affected from the asymmetry of the system.

Figure 9 shows the comparison of the phase difference among the ladder system, the symmetric systems, the asymmetric systems and the full-coupled system. From Fig. 9, the entire system easy to reach the synchronous state by increasing the number of the edges. Additionally, we consider that synchronization and chaos propagation can

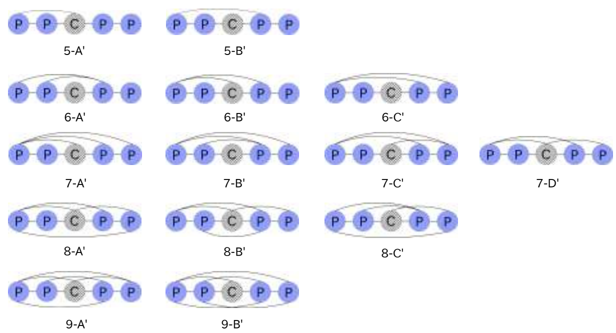

Figure 7: Asymmetric system patterns.

Table 2: Phase differences in all asymmetric systems.

Edge	Phase difference [°]
5-A'	12.25
5-B'	16.28
6-A'	17.39
6-B'	13.26
6-C'	15.83
7-A'	11.43
7-B'	15.46
7-C'	11.43
7-D'	9.71
8-A'	9.52
8-B'	10.02
8-C'	11.65
9-A'	9.51
9-B'	11.62

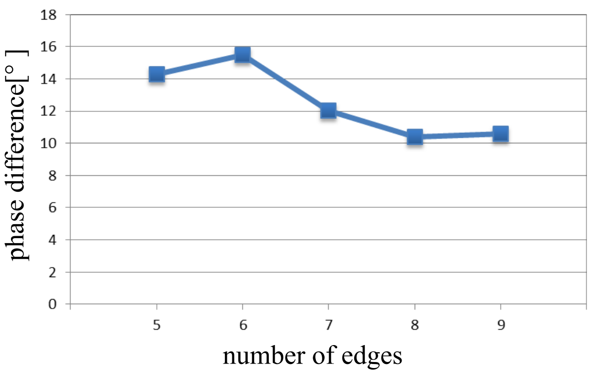

Figure 8: Relation between the average phase difference and the number of the edges in the asymmetric systems.

be achieved when the circuit generating chaotic attractor is the hub in the system.

4. Conclusion

In this study, we have researched about synchronization and chaos propagation of 5 coupled chaotic circuits in the symmetric and asymmetric systems. First, we proposed the ladder system model that the central circuit generates chaotic attractor and the other circuits generate three-periodic attractor. We have observed that the three-periodic attractors are affected from the chaos of the central circuit, and all circuits are synchronized by increasing the coupling strength in the ladder system. Moreover, we compared the phase difference between the symmetric and asymmetric systems in the cases of adding the coupling resistor from the ladder system. As a result, the entire system is easy to reach a state of synchronization when the central chaotic circuit is connected to many circuits.

For the future works, we would like to investigate "chaos propagation speed" in the proposed system. Considering the other types of chaotic circuits and the other coupling systems are also important subjects for us.

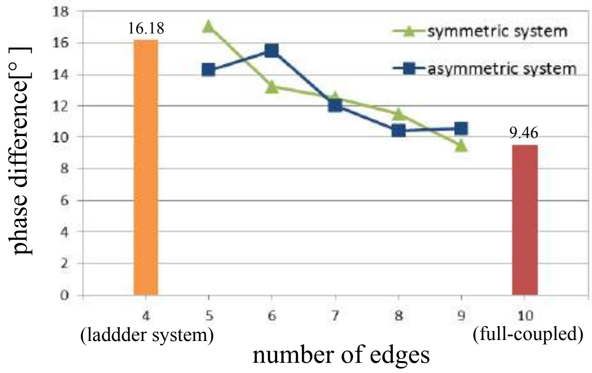

Figure 9: Comparison of the phase difference among the ladder system, the symmetric systems, the asymmetric systems and the full-coupled system.

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