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Synchronization in Bubble Structure using Coupled Oscillators

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***Abstract*—In this study, we investigate the characteristics of the bubble structure using coupled oscillators. By using computer simulations, we confirm that the stable type of the bubble structure has symmetrical characteristics for the amplitude, the phase difference and the energy.**

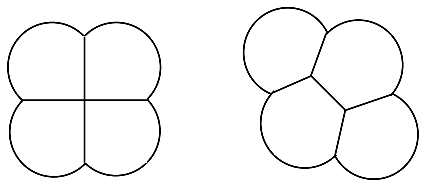
I. Introduction

The synchronization phenomena of coupled oscillators are suitable model to express essential behavior observed from the natural science. Therefore, many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena.

On the other hand, there are several types of polygonal network structures (e.g. honeycomb, soup bubbles and cracks of drying mud structure) in the natural science. Generally, for the studies of large-scale network using coupled oscillators, a ring, a ladder and a two dimensional array structure are often investigated. However, there are not many discussions about coupled polygonal oscillatory networks by using electrical oscillators.

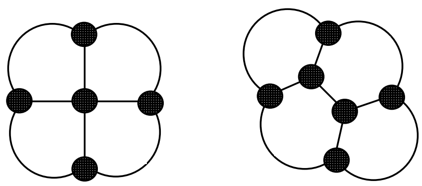
In our research group, we focus on synchronization phenomena of coupled oscillators under a difficult situation for the circuit. Setou et al. have reported the synchronization phenomena in N oscillators coupled by resistors as a ring. The oscillation stop in some range of the coupling resistors was confirmed [1]. We have investigated synchronization phenomena in coupled polygonal oscillatory networks sharing branches [2]. In this system, van der Pol oscillators are connected to every corner of polygonal network. By using computer simulations and theoretical analysis, we confirm that coupled oscillators tend to synchronize to minimize the power consumption of the whole system. Furthermore, the phase difference of the shared oscillators is solved by finding the minimum value of the power consumption function.

In this study, we consider the bubble structure using oscillators as one example of general nature patterns. Figure 1 shows the bubble structures with four bubbles. In the case of Fig. 1(a), the center (intersection point) of the four bubbles becomes unstable, then the four bubbles change the form as shown in Fig. 1(b). We apply the previous study of the polygonal oscillatory network for the two bubble structures. Figure 2 shows the conceptual circuit model of the unstable and the stable bubble structures using coupled oscillators. The circle corresponds to the van der Pol oscillators. By using computer simulations, we confirm that the stable type of the bubble structure has symmetrical characteristics for the amplitude, the phase difference and the energy.



(a) Unstable model. (b) Stable model.

Fig. 1. Bubble structure with four bubbles.



(a) N=5. (b) N=6.

Fig. 2. Conceptual circuit model.

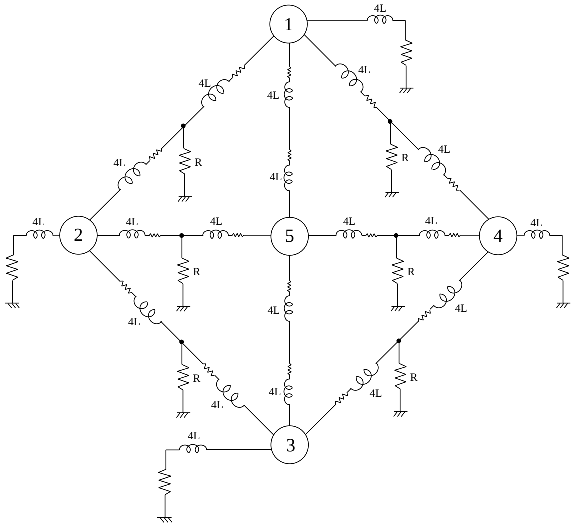
II. Bubble Structure using Coupled Oscillators

Figure 3 shows the circuit model of the unstable and stable bubble structures using the coupled oscillators. In this circuit model, we consider the coupling method which two adjacent oscillators are tend to synchronize at anti-phase state.

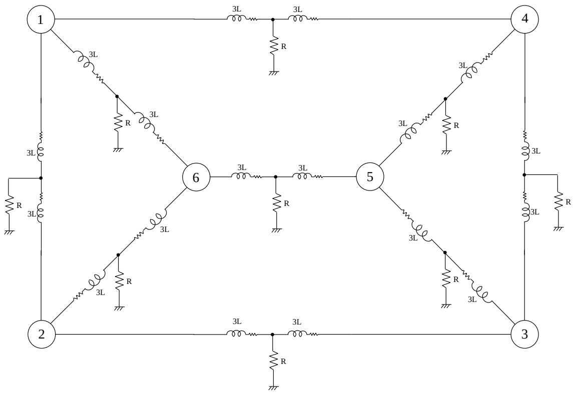
Next, we develop the expression for the circuit equations of the unstable and the stable bubble models as shown in Fig. 3(a) and (b). The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third order polynomial equation,

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), (k = 1, 2, 3, 4, 5). \quad (1)$$

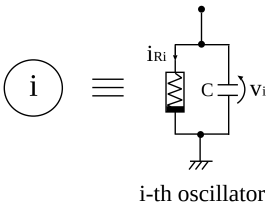
The normalized circuit equations governing the circuit are expressed as



(a) Unstable bubble model (N=5).



(b) Stable bubble model (N=6).



(c) van der Pol oscillator.

Fig. 3. Bubble structure models using coupled oscillators.

[kth oscillator of the unstable bubble model]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\Big(1 - \dfrac{1}{3}{x_k}^2\Big)x_k - (y_{ak} + y_{bk} + y_{ck} + y_{dk}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{4}\Big\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\Big\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{4}\Big\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\Big\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{4}\Big\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\Big\} \\ \dfrac{dy_{dk}}{d\tau} = \dfrac{1}{4}\Big\{x_k - \eta y_{ck} - \gamma(y_{dk} + y_n)\Big\} \\ \qquad\qquad\qquad\qquad (k = 1, 2, 3, 4, 5). \end{cases} \tag{2}$$

[kth oscillator of the stable bubble model]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\Big(1 - \dfrac{1}{3}{x_k}^2\Big)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\Big\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\Big\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\Big\} \\ \qquad\qquad\qquad\qquad (k = 1, 2, 3, 4, 5, 6). \end{cases} \tag{3}$$

where

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}}x_k, \quad i_{ak} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{ak},$$

$$i_{bk} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{bk}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}},$$

$$\gamma = R\sqrt{\frac{C}{L}}, \quad \eta = r_m\sqrt{\frac{C}{L}},$$

In this equations, γ is the coupling strength, ε denotes the nonlinearity of the oscillators and y_n denotes the current of neighbor oscillator on coupling resistor. We obtain the similar circuit equations for the stable bubble structure model.

For the computer simulations, we calculate Eqs. (2),(3) using a fourth-order Runge-Kutta method with the step size $h = 0.005$. The parameters of this circuit model are fixed as $\varepsilon = 0.1$, $\gamma = 0.1$, $\eta = 0.0001$.

III. Synchronization Phenomena

A. Amplitude and Phase Difference

Figures 4, 5 show the observed attractors of the two types of the bubble structure models. The value of the amplitude, the phase difference obtained from each bubble model are summarized in Tables I, II and III. From Table I, the amplitude of 5th oscillator has smaller value the other oscillators in the unstable bubble model. While, all amplitude have same value in the stable bubble model.

Next, we focus on the phase difference in two types of the bubble models. In the case of the unstable bubble model, there are three phase state such as ≈ 99, ≈ 106 and ≈ 153. In the case of the stable bubble model, there are two phase state such as ≈ 180 (anti-phase state) and ≈ 120 (3-phase state). From these results, we assume that the synchronization state of the stable bubble model is also stable.

TABLE I

Amplitude of the unstable and the stable bubble models.

Oscillator No.	Unstable model	Stable model
1st oscillator	1.94	1.89
2nd oscillator	1.93	1.89
3rd oscillator	1.94	1.89
4th oscillator	1.93	1.89
5th oscillator	1.80	1.89
5th oscillator	-	1.89

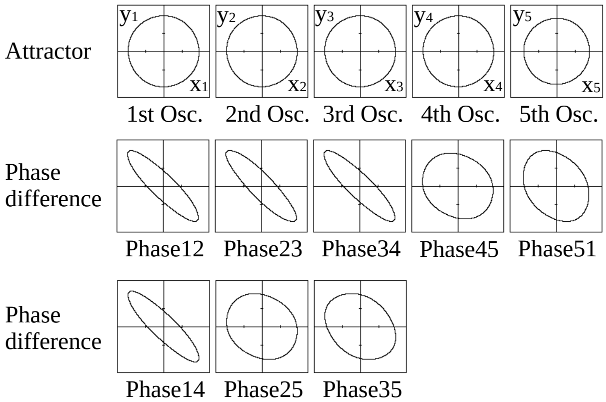


Fig. 4. Attractor and phase diffrence for the unstable bubble model (N=5).

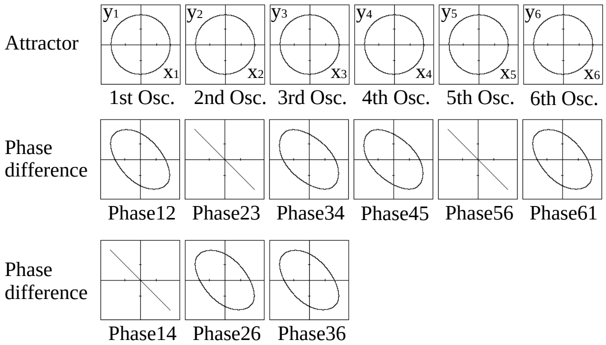


Fig. 5. Attractor and phase diffrence for the stable bubble model (N=6).

TABLE II

PHASE DIFFERENCE (UNSTABLE BUBBLE MODEL

Bridge No.	Phase difference [°]	State
bridge: 1-2	153.21	state 3
bridge: 2-3	153.05	state 3
bridge: 3-4	153.21	state 3
bridge: 4-5	99.40	state 1
bridge: 5-1	104.94	state 2
bridge: 1-4	153.21	state 3
bridge: 2-5	99.40	state 1
bridge: 3-5	107.25	state 2

TABLE III

PHASE DIFFERENCE (STABLE BUBBLE MODEL)

Bridge No.	Phase difference [°]	State
bridge: 1-2	120.69	state 1 (3-phase)
bridge: 2-3	179.78	state 2 (anti-phase)
bridge: 3-4	119.45	state 1 (3-phase)
bridge: 4-5	119.23	state 1 (3-phase)
bridge: 5-6	179.81	state 2 (anti-phase)
bridge: 6-1	120.62	state 1 (3-phase)
bridge: 1-4	179.91	state 2 (anti-phase)
bridge: 2-6	120.56	state 1 (3-phase)
bridge: 3-5	120.52	state 1 (3-phase)

B. Energy

Finally, we investigate the energy of the coupling resistor in the bubble circuit models. The simulation results of the energy are summarized in Table IV and V. As the results of the phase difference, in the case of the unstable bubble model, there are three energy states. While, in the case of the stable bubble model, there are two energy states.

However, the total energy of the unstable bubble model is smaller than the stable bubble model.

TABLE IV

ENERGY (UNSTABLE BUBBLE MODEL)

Bridge No.	Energy
bridge: 1-2	0.025
bridge: 2-3	0.025
bridge: 3-4	0.025
bridge: 4-5	0.179
bridge: 5-1	0.157
bridge: 1-4	0.025
bridge: 2-5	0.179
bridge: 3-5	0.157
Total energy	**0.773**

TABLE V

ENERGY (STABLE BUBBLE MODEL)

Bridge No.	Energy
bridge: 1-2	0.197
bridge: 2-3	0.000
bridge: 3-4	0.197
bridge: 4-5	0.197
bridge: 5-6	0.000
bridge: 6-1	0.197
bridge: 1-4	0.000
bridge: 2-6	0.197
bridge: 3-5	0.197
Total energy	**1.182**

IV. CONCLUSIONS

In this study, we have investigated the characteristics of the two types of the bubble structures using coupled oscillators. By using computer simulations, we confirmed that the stable type of the bubble structure has symmetrical characteristics for the amplitude, the phase difference and the energy.

For the future work, we would like to investigate the stability of the solution obtained the bubble networks.

ACKNOWLEDGMENT

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[1] Y. Setou, Y. Nishio and A. Ushida, "Synchronization Phenomena in Many Oscillators Coupled by Resistors as a Ring," *Proc. of APCCAS'94*, pp. 570-575, Dec. 1994.

[2] Y. Uwate and Y. Nishio, "Synchronization in Several Types of Coupled Polygonal Oscillatory Networks," *IEEE Trans. Circuits Syst. I*, vol. 59, no. 5, pp. 1042-1050, May 2012.