Delay-Induced Synchronization States in Two Coupled Cubic Maps

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***Abstract*— Generally, complex dynamical phenomena can be observed in networks formed by many elements with nonlinearity. Coupled Map Lattice has proposed by Kaneko, to represent the complex high-dimensional dynamics, for example biological systems, networks in DNA, and neural networks. In this study, we investigate the influence of the delay in two coupled cubic maps with intermittency chaos. When we set a control parameter of two cubic maps to generate intermittency chaos near the six periodic window, various synchronization states are confirmed in laminar part. Moreover, the relation between average length of laminar part and the delay is investigated.**

I. Introduction

Generally, complex dynamical phenomena can be observed in networks formed by many elements with nonlinearity. Coupled Map Lattice (CML) has proposed by Kaneko [1]-[4], to represent the complex high-dimensional dynamics, for example biological systems, networks in DNA, economic activities and neural networks. Furthermore, we focus on intermittency chaos and delay. The delay naturally occurs from information transmission and processing speeds in the realistic networks[5]. In Ref.[5], the synchronization states of the coupled logistic maps are affected by the delay. Namely, the synchronization state of coupled chaotic maps are induced by the delay. Therefore, the studies considered the delay in coupled chaotic maps are investigated actively. In addition, intermittency chaos has stability and mobility and gains good result for information processing. We consider that intermittency chaos is related to various phenomena[6][7], e.g, information processing of the brain. In order to make clear the mechanism of such phenomena in various fields, unveiling the roles of intermittency chaos is very important. In this study, we focus on the influence of the delay in two coupled cubic maps with intermittency chaos. When we set a control parameter of two cubic maps to generate intermittency chaos near the six periodic window, various synchronization states are confirmed in laminar part. Moreover, the relation between average length of laminar part and the delay is investigated.

II. Two coupled cubic maps

A cubic map is expressed as follows:

$$f(x) = ax^3 + x(1 + a), \tag{1}$$

where a represents a control parameter.

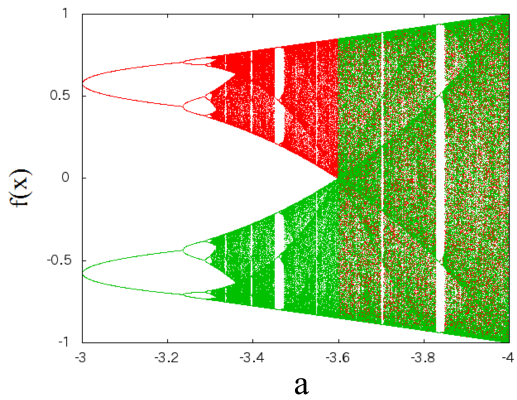


Fig. 1. The bifurcation diagram of cubic map.

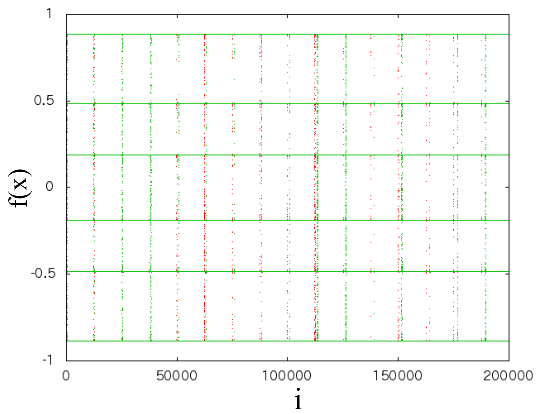


Fig. 2. Time series of cubic map with intermittency chaos (a = -3.69964153).

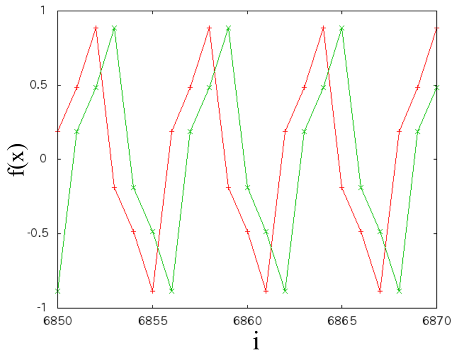
Figure 1 shows the bifurcation diagram of cubic map. This figure shows period-doubling bifurcations and periodic windows (near a = -3.69,-3.83). At the boundary of periodic windows intermittency chaos is observed. Intermittency chaos is switching between laminar part and burst part. Laminar represents the periodic state and burst represents the chaotic state. Figure 2 shows the time series of cubic map with intermittency chaos. In this study, we consider two coupled cubic maps with the delay. The coupling system is expressed as follows:

$$\begin{cases} x_{(1,i+1)} = (1-g)f(x_{(1,i)}) + gf(x_{(2,i-\tau)}) \\ x_{(2,i+1)} = (1-g)f(x_{(2,i)}) + gf(x_{(1,i-\tau)}), \end{cases} \quad (2)$$

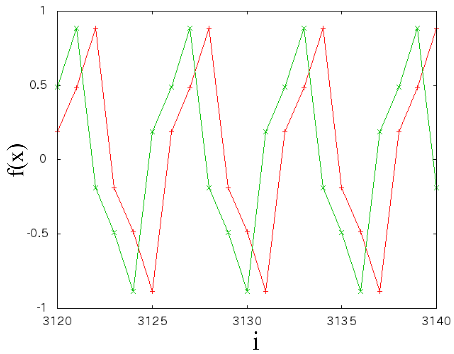
where g represents the coupling strength, τ represents the delay between the maps.

III. Synchronization States

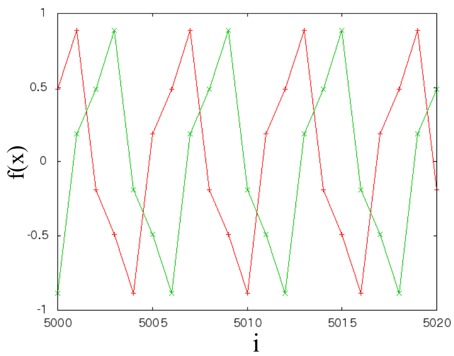
In this study, we focus on synchronization states of laminar part in two coupled maps. Additionally, laminar part is composed of some periodic patterns. In the case of a = -3.69964153, g = 0.0000001, intermittency chaos including six periodic laminars are observed in Fig. 3. Here, the results are considered that intermittency chaos including six periodic laminar generates six patterns of synchronization states.



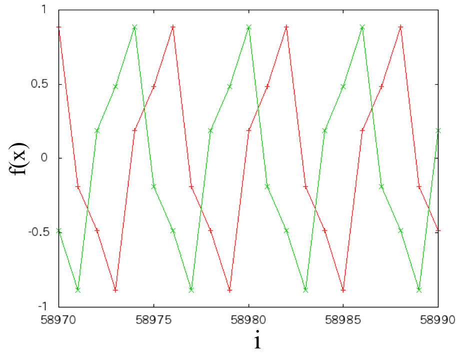
(a) i = 6850 ∼ 6870.



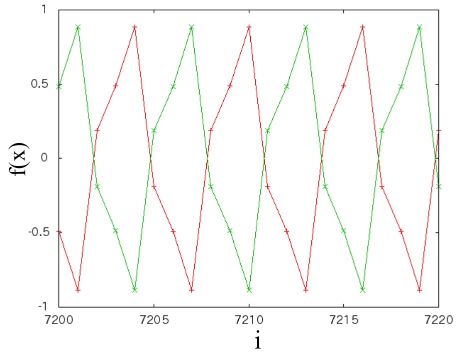
(b) i = 3120 ∼ 3140.



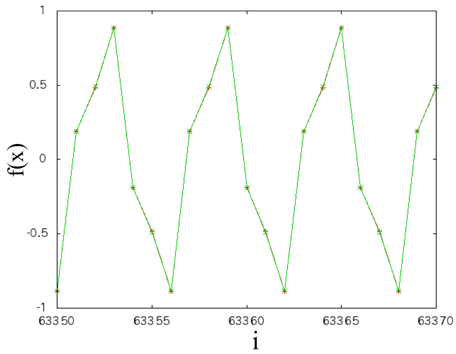
(c) i = 5000 ∼ 5020.



(d) i = 58970 ∼ 58990.



(e) i = 7200 ∼ 7220.

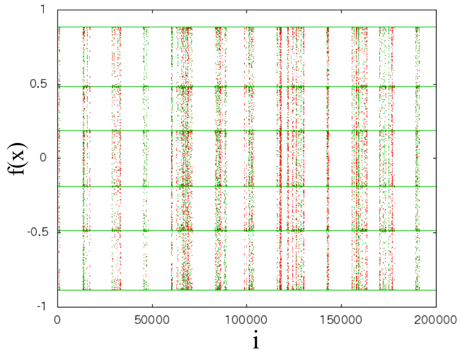


(f) i = 63350 ∼ 63370.

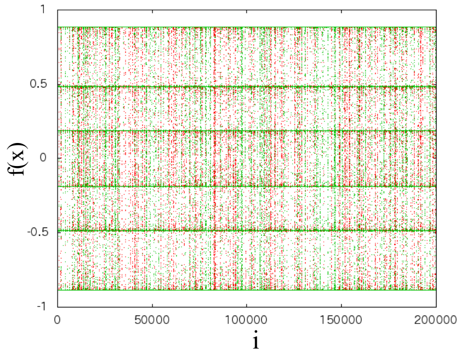
Fig. 3. The pattern of synchronization state (a = -3.69964153, g = 0.0000001).

IV. Simulation Results

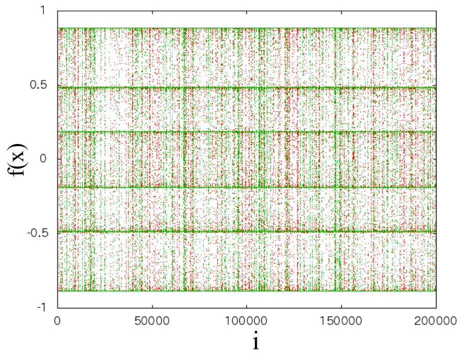
In this study, the initial conditions and the parameters of two cubic maps are fixed with $x_{(1,0)}$ = 0.1, $x_{(2,0)}$ = -0.2, g = 0.0000001, 0.00001 and a = -3.69964153, respectively. The iteration time is fixed with i = 200000. In the case of a = -3.69964153, intermittency chaos including six periodic laminars are observed in Figs. 4 and 5. Figures 4 and 5 shows the time series of coupled cubic maps with the delay. From Figs. 4 and 5, laminar part is longer than the others, when we set to τ = 3, 6.



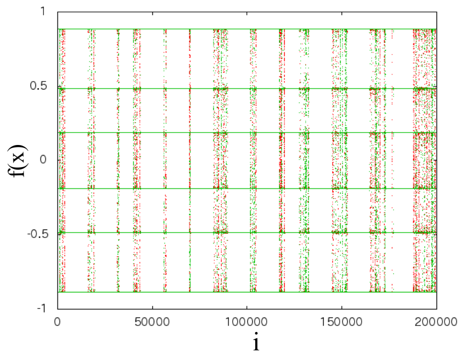
(a) τ= 0.



(b) τ= 1.

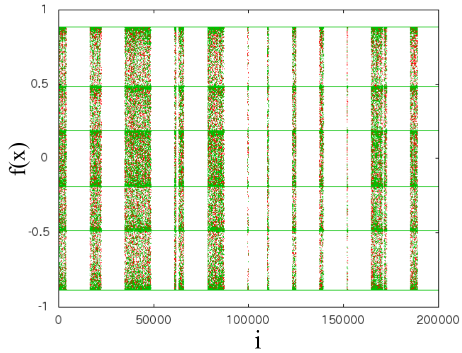


(c) τ= 2.

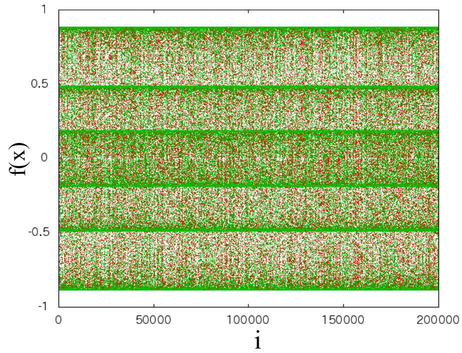


(d) τ= 3.

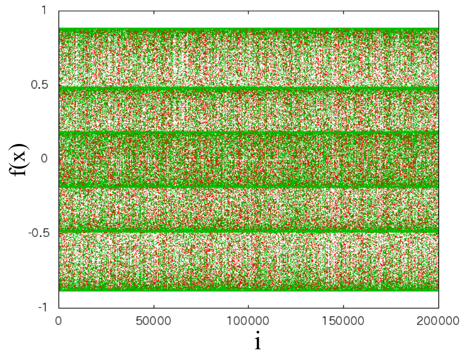
Fig. 4. Time series of coupled cubic maps (g = 0.0000001).



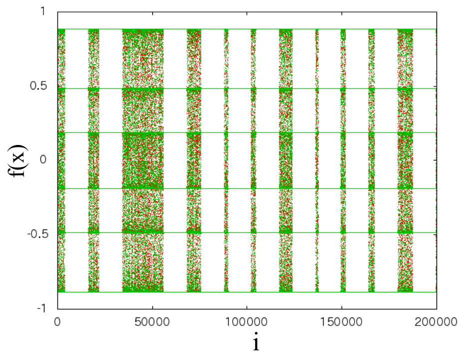
(a) τ = 0.



(b) τ = 1.



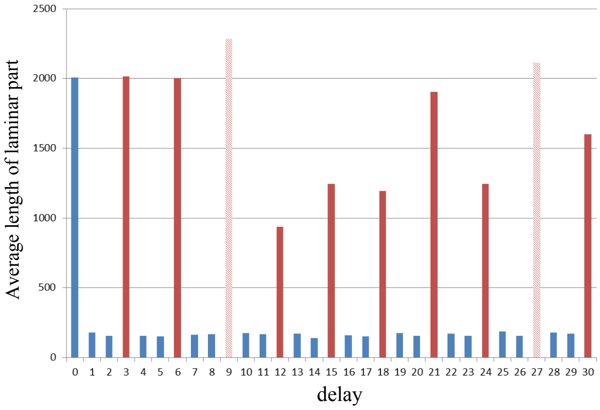
(c) τ = 2.



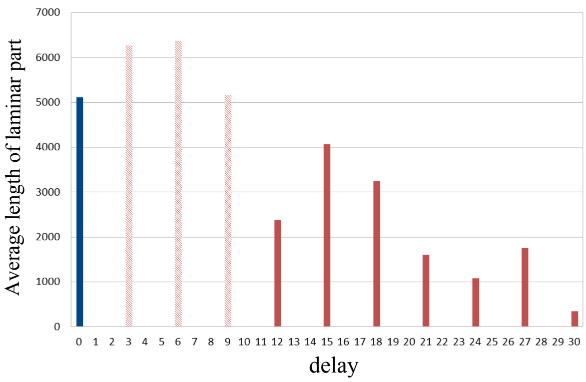
(d) τ = 3.

Fig. 5. Time series of coupled cubic maps (g = 0.00001).

Next, we investigate the length of laminar part in coupled cubic maps with the delay. In order to investigate quantitative average length of laminar part, we define laminar part by $|x - \pm 0.1895$ or ± 0.4865 or $\pm 0.8873| < 0.0001$. Figure 6 shows average length of laminar part during the iteration time. From Fig.6(a), when we set to τ = 9 and 27, the average length of laminar part is longer than the case of non-delay (τ = 0). In addition, when we set to τ = 3, 6 and 21, the average length of laminar part is similar length as much as the case of non-delay (τ = 0). From Fig.6(b), when the time delay is set to τ = 3, 6 and 9, the average length of laminar part is longer than the case of non-delay (τ = 0). Namely, the average length of laminar part is longer than the others when the delay is set to a multiple of three.



(a) g = 0.0000001.



(b) g = 0.00001.

Fig. 6. Average length of laminar part (a = -3.69964153).

V. Conclusions

In this study, we have investigated the influence of the delay in two coupled cubic maps with intermittency chaos. First, we observe the time series of coupled cubic maps with the delay. Next, we investigate the length of laminar part in coupled cubic maps with the delay. The average length of laminar part is longer than others when the delay is set to a multiple of three. Thereby, we could consider two maps easily to become to the synchronous states when delay's number is the least common multiple of periodic number.

Acknowledgment

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