Synchronization Phenomena of Complex Networks by Using Parametrically Excited Oscillators

Kosuke Oi
Dept. Electrical and Electronic Eng.
Tokushima University
Email: ooi@ee.tokushima-u.ac.jp

Yoko Uwate
Dept. Electrical and Electronic Eng.
Tokushima University
Email: uwate@ee.tokushima-u.ac.jp

Yoshifumi Nishio
Dept. Electrical and Electronic Eng.
Tokushima University
Email: nishio@ee.tokushima-u.ac.jp

***Abstract*— Recently, many studies of synchronization of chaotic circuits have been investigated. Synchronization phenomenon is one of the typical phenomena observed in nature. And, it is applicable to the fields of medical science and biology and so on. In this study, we investigate synchronization phenomena of complex networks by using parametrically excited oscillators.**

I. Introduction

Synchronization is one of the fundamental phenomena in nature and it is observed over the various fields. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. We consider that it is important to investigate the synchronization phenomena of coupled oscillators for the future engineering application. The coupled van der Pol oscillator is one of coupled oscillators, and synchronization generated in the system can model certain synchronization of natural rhythm phenomena. The van der Pol oscillator is studied well because it is expressed in simple circuit. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena of the parametric excitation circuits for future engineering applications. In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Refs. [4] and [5].

In our research group, we have investigated synchronization of parametrically excited van der Pol oscillators [7]. By carrying out computer calculations for two or three subcircuits case, we have confirmed that various kinds of synchronization phenomena of chaos are observed. In the case of two subcircuits, the anti-phase synchronization is observed. In the case of three subcircuits, self-switching phenomenon of synchronization states is observed.

However, we have investigated the only simple network models. It is important to investigate more complex network for the broad-ranging future engineering applications. In our previous study, we have challenged to investigate more complex network using parametrically excited van der Pol oscillators with small mismatch. First, two oscillators are combined by resisters in one-dimensional coordinate system. We have investigated synchronization between two oscillator by changing the value of coupling strength. We also have investigated synchronization phenomena of complex network by applying this circuit model to ten coupled oscillators as random network model with hub.

In this study, we compare synchronization phenomena of parametrically excited van der Pol oscillators with small mismatch in complex network with hub and without hub in order to investigate effect of hub for the synchronization and clustering.

II. Circuit Model

Figure 1 shows the parametrically excited van der Pol oscillator.

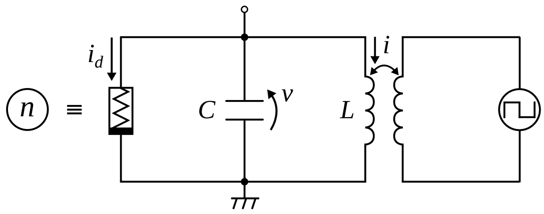


Fig. 1. Parametrically excited van der Pol oscillator.

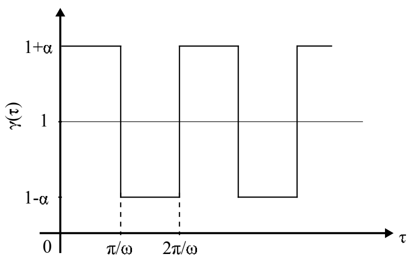


Fig. 2. Function relating to parametrically excitation.

The circuit includes a time-varying inductor L whose characteristic is given as the following equation. The operation of the time-varying inductor is shown as Fig. 2.

$$L = L_0\gamma(\tau). \tag{1}$$

$\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 2, and its amplitude and angular frequency are termed α and ω, respectively. The $v - i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v_k + g_3 v_k. \tag{2}$$

The normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon(x_n - x_n^3) - y_n + \delta \sum_{k \in S_n} (x_k - x_n) \\ \dfrac{dy_n}{d\tau} = \dfrac{1}{\gamma(\tau)} x_n \quad (n = 1, 2, 3, \cdots, 10)) \end{cases} \tag{3}$$

S_n is the set of nodes which are directly connected to the node n.

III. Random Network With Hub

We apply the circuit model to ten coupled oscillators as random network model with hub. The ten coupled oscillator model is shown in Fig. 3. In this circuit system, there is a hub (1st oscillator) which is connected to many oscillators. Here, the small mismatch is added to α which is corresponding to the amplitude of the function relating to parametrically excitation. The mismatch is generated by random and the range of the mismatch is set to [-0.01:0.01]. Table I shows the pattern of five types of the small mismatch used in this computer simulations.

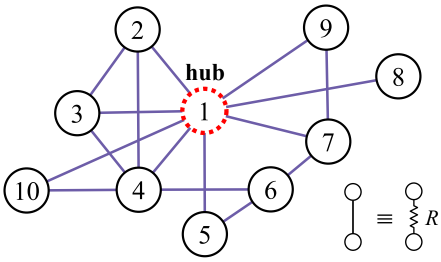


Fig. 3. A random network model with a hub.

TABLE I
SMALL MISMATCH

small mismatch	w=1	w=2	w=3	w=4	w=5
α_1	0	0	0	0	0
α_2	-0.009	0.008	0.008	-0.01	-0.009
α_3	-0.006	-0.006	0.009	0.009	0.01
α_4	-0.001	-0.005	-0.01	-0.003	-0.007
α_5	0.009	0.007	-0.01	0.001	-0.001
α_6	-0.002	-0.01	-0.004	-0.002	0.004
α_7	0	-0.001	-0.003	0.007	0.009
α_8	0	-0.004	0.005	-0.006	0.007
α_9	-0.001	-0.01	0.009	-0.01	0.005
α_{10}	0.005	0.009	-0.009	-0.009	-0.001

Figure 4 shows the simulation results. The horizontal axis denotes the coupling strength δ, and the vertical axis denotes α of the second oscillator. In this graph, the lower area of each line denotes synchronous area and the upper area of each line denotes unsynchronous area. By increasing the value of the coupling strength, the synchronous area becomes large. We can see that the synchronous area is determined by the small mismatch pattern. Namely, several types of synchronization states can be observed from ten coupled oscillators with the small mismatch.

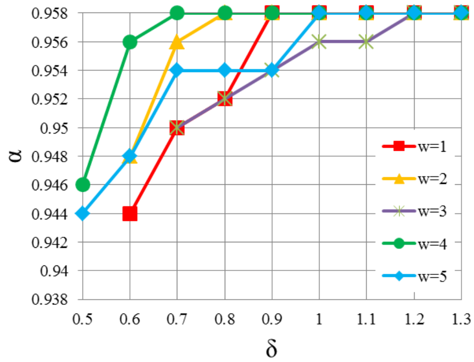


Fig. 4. Synchronous state with δ.

IV. Random Network Without Hub

Next, we focus on comparison with synchronous state of the random network without hub. Figure 5 shows a random network model without hub. We investigate the synchronous state of random network model without hub in order to compare with synchronous state of random network model with hub. Table II shows the pattern of five types of the small mismatch without hub used in this computer simulations. Figure 6 shows synchronous state of random network model without hub. In this graph, the lower area of each line denotes synchronous area and the upper area of each line denotes unsynchronous area in common with Fig. 4. In Fig. 7, we draw a comparison between Fig 4 and Fig 6. The synchronous state of random network model without hub is smaller than the synchronous state of random network model with hub. In these results, we have confirmed that the synchronous state depends on the network structure.

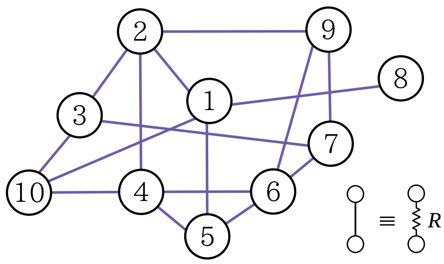


Fig. 5. A random network model with a hub.

V. Clustering Phenomena

We investigate clustering phenomena of ten coupled oscillators as random network with hub and without hub. Figure 8 shows the clustering phenomena (how do the oscillators change the fully synchronization state) in random network with hub and without hub when the coupling strength is changed. The horizontal axis is the coupling strength, and the vertical axis denotes the node number between the oscillators. By comparison Fig. 8 (a). and Fig. 8 (b), we confirm the difference of clustering phenomena. In Fig. 8 (a), the network completely synchronize in $\delta = 0.60$. However, in Fig. 8 (b),

TABLE II

SMALL MISMATCH (WITHOUT HUB)

small mismatch	w=1	w=2	w=3	w=4	w=5
α_1	0	-0.001	0.009	0.003	-0.006
α_2	-0.009	0.008	0.008	-0.01	-0.009
α_3	-0.006	-0.006	0.009	0.009	0.01
α_4	-0.001	-0.005	-0.01	-0.003	-0.007
α_5	0.009	0.007	-0.01	0.001	-0.001
α_6	-0.002	-0.01	-0.004	-0.002	0.004
α_7	0	-0.001	-0.003	0.007	0.009
α_8	0	-0.004	0.005	-0.006	0.007
α_9	-0.001	-0.01	0.009	-0.01	0.005
α_{10}	0.005	0.009	-0.009	-0.009	-0.001

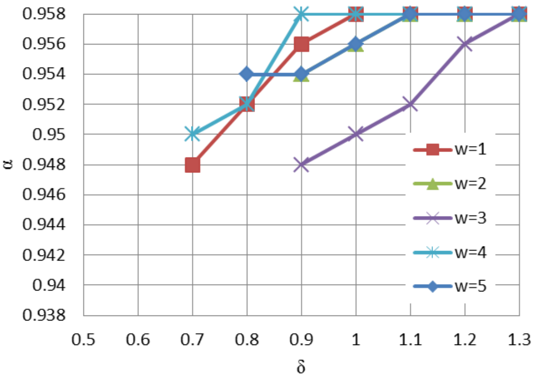


Fig. 6. Synchronous state with δ (without hub).

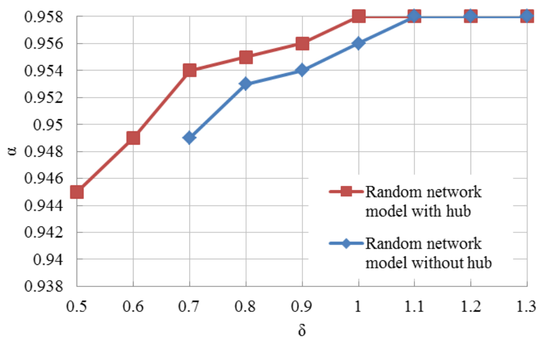
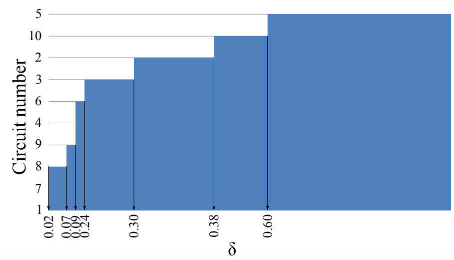
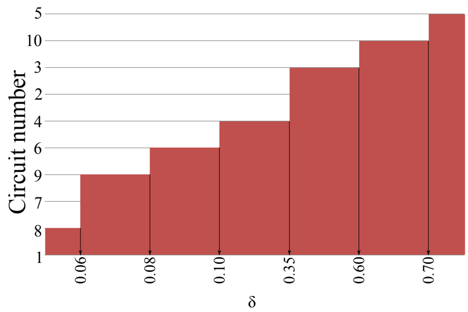


Fig. 7. Comparison of synchronous state.



(a) Clustering phenomena with hub



(b) Clustering phenomena without hub

Fig. 8. Clustering phenomena with δ (w=1).

network completely synchronize in $\delta = 0.70$. Basically, the presence or absence of hub effect of process of complete synchronization. In addition, we characterize Fig. 9 and Fig. 10 in order to investigate more further process of complete synchronization. In Fig, 9 (c), the value of bond number for synchronization is 12. However in Fig. 10 (c), the value of bond number for synchronization is only 5. In these results, clustering phenomena depend on the presence or absence of hub.

VI. Conclusions

In this study, we have investigate synchronization phenomena of parametrically excited van der Pol oscillators with small mismatch. In the case of two subcircuits, we have confirmed that the two subcircuits are synchronized at in-phase state when the adding mismatch is small. By increasing the small mismatch, we observe unsynchronous phenomena. Furthermore, we have applied this circuit model to ten coupled oscillators as random network model with hub and random network model without hub. In this result, we have confirmed that the synchronous state depend on the net work structure. For the future work, we would like to consider the influence of the small mismatch for synchronization state of other type network or more large scale networks.

Acknowledgment

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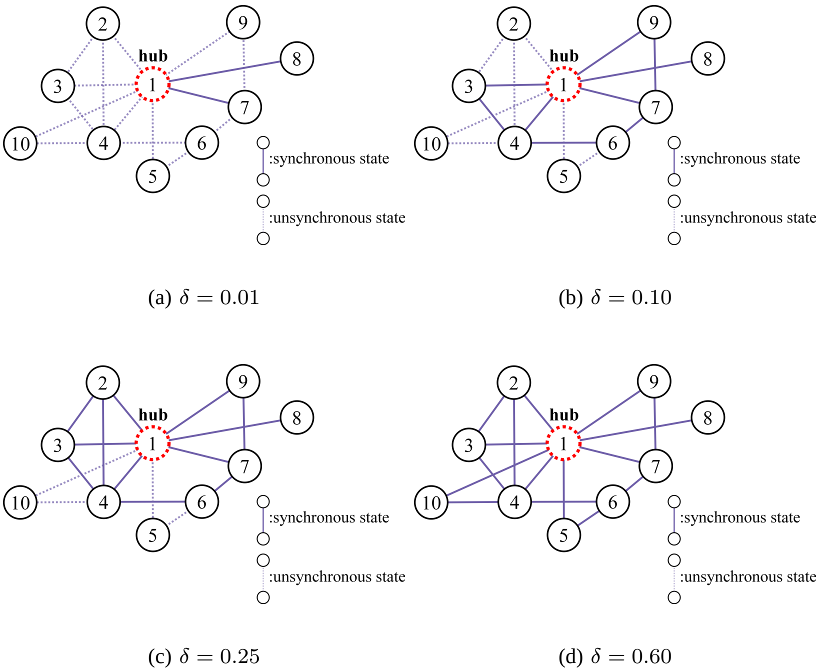
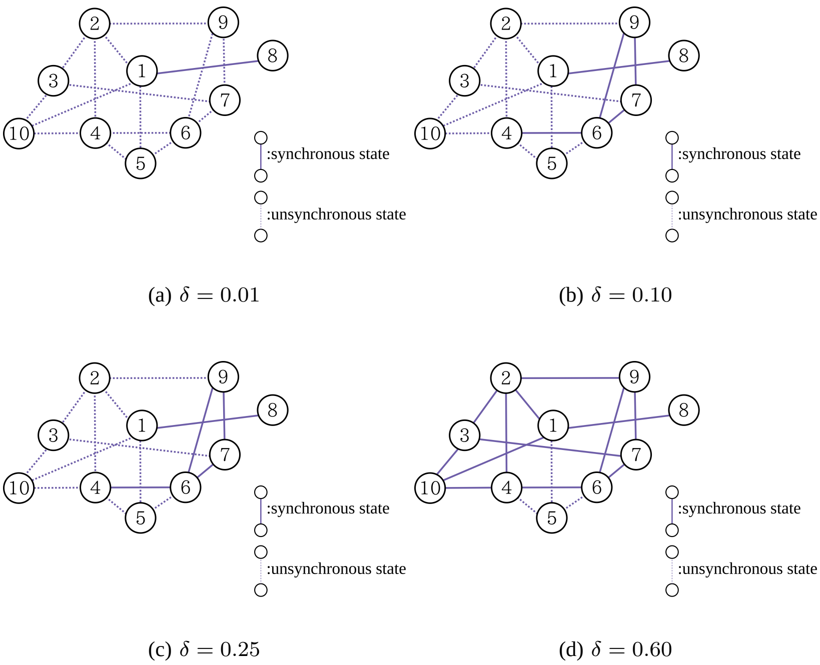


(a) $\delta = 0.01$ (b) $\delta = 0.10$

(c) $\delta = 0.25$ (d) $\delta = 0.60$

Fig. 9. Formation process of cluster (with hub).



(a) $\delta = 0.01$ (b) $\delta = 0.10$

(c) $\delta = 0.25$ (d) $\delta = 0.60$

Fig. 10. Formation process of cluster (without hub).

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