Synchronization State of Coupled Chaotic Circuits Containing Time Delay

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***Abstract*— In this study, we investigate synchronization states observed in coupled chaotic circuits containing time delay. We focus on relationships between synchronization state and chaotic strength. Coexisting synchronization states depending on initial values can be observed in the proposed system. Moreover, we investigate the effect of chaotic behavior on the subcircuit to synchronization states.**

I. Introduction

Generally, generation of chaos is reported self excited oscillation system containing time delay. This chaotic circuit can be easily realized by using simple electric circuit element and analyzed exactly [1]. Furthermore, a number of studies on synchronization of coupled chaotic circuits have been made [2]. In this study, we investigate synchronization state observed in some chaotic circuits containing time delay coupled by inductor. Especially, we focus on the transition of synchronization state by changing the chaotic strength in this system. By carrying out computer simulations, two types of synchronization state depending on initial values and parameters can be observed. Moreover, we investigate the relationship between parameters and synchronization state. In addition we consider synchronization state of changes due to the number of subcircuits.

II. Circuit Model

Figure 1 shows the chaotic circuit containing time delay. This circuit consists of one inductor L , one capacitor C , one linear negative resistor $-g$ and one linear positive resistor R of which amplitude is controlled by the switch containing time delay. The current flowing through the inductor L is i, and the voltage between the capacitor C is v. The circuit equations are normalized as Eqs. (1) (2) by changing the variables as below.
(A) In case of switch connected to $-g$,

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= 2\alpha y - x. \end{cases} \tag{1}$$

(B) In case of switch connected to G,

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= -2\beta y - x. \end{cases} \tag{2}$$

By changing the parameters and variable as follow:

$$i = \sqrt{\frac{C}{L}} V_{th} x,\ v = V_{th} y,\ t = \sqrt{LC}\tau,$$

$$g\sqrt{\frac{C}{L}} = 2\alpha \text{ and } G\sqrt{\frac{C}{L}} = 2\beta.$$

Figure 2 is chaotic attractor observed from the circuit. The switching operation is shown in Fig. 3, it controls the amplitude of the oscillator. This switching operation is included time delay. T_d denotes the time delay. First, the switch is connected to a negative resistor. In state of that, the voltage v is amplified up to while v is oscillating, the amplitude exceeds the threshold voltage V_{th} which is the threshold control loop. Second, the system memorize the time as T_{th} while v is exceeding the threshold voltage V_{th} and that state is remained for T_{th}. In subsequent the instant of exceeding threshold V_{th}, the switch stays the state for T_d. After that switch is connected to positive resistor during T_{th}. The switch does not immediately connect in the positive resistor however the switch is connected after T_d. A set of switching operations control the amplitude of v. In the computer simulation, T_d is

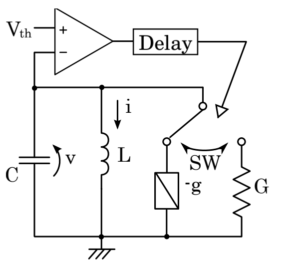


Fig. 1. Chaotic circuit containing time delay.

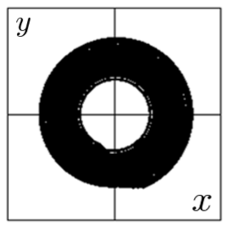


Fig. 2. Chaotic attractor obtained by computer simulation.

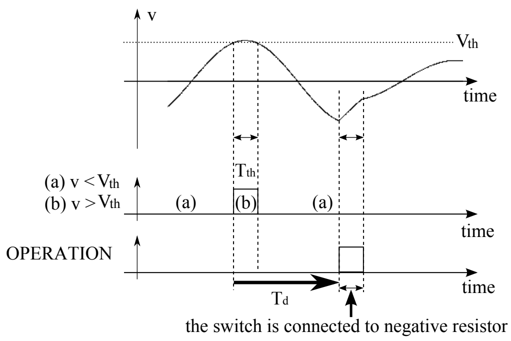


Fig. 3. Switching operation.

fixed as $\pi/(1-\alpha^2)$. By increasing the parameter value of β, the chaotic strength is qualitatively swelled since the solution set of attractor is enlarged accordance with the increase the parameter value.

III. Two Coupled Chaotic Circuits

Figure 4 shows the coupled chaotic circuit. The normalized circuit equations of the system are given as follows:

(A) In case of that switch is connected to $-g$,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= 2\alpha y_n - x_n - \gamma(x_{n+1} - x_n). \end{cases} \tag{3}$$

(B) In case of that switch is connected to G,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -2\beta y_n - x_n - \gamma(x_{n+1} - x_n). \end{cases} \tag{4}$$

where $(n = 1, 2)$ and $x_3 = x_1$. By changing the parameters and variable as follow:

$$i_n = \sqrt{\frac{C}{L}} V_{th} x_n,\ v_n = V_{th} y_n,\ t = \sqrt{LC}\tau,$$

$$g\sqrt{\frac{C}{L}} = 2\alpha,\ G\sqrt{\frac{C}{L}} = 2\beta \text{ and } \gamma = \frac{L}{L_0}.$$

Figure 5 shows example of simulation results. Figures 5 (1) and (2) are in-phase synchronization and anti-phase synchronization states depending on the initial values. We investigate the basin of attraction regarding anti-phase and in-phase synchronization. Figure 6 shows the example of the basin. Red and blue colors denote the basin related to anti-phase and in-phase synchronization state respectively. By increasing the value of β, the proportion of basin regarding anti-phase synchronization state is increased. The relation of the parameter value of β and the population of anti-phase synchronization states show Fig. 7. By increasing β, the proportion of basin regarding anti-phase synchronization state is increased. Namely, it is considered that anti-phase synchronization state is induced by chaotic behavior of each subcircuits.

IV. A Ring of three Coupled Chaotic Circuit

Figure 8 shows schematic of a ring of N coupled chaotic circuits containing time delay coupled by the inductor. The normalized system equations are given as follows:

(A) In case of switch connected to $-g$,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= 2\alpha y_n - x_n - \gamma(2x_n - x_{n+1} - x_{n-1}). \end{cases} \tag{5}$$

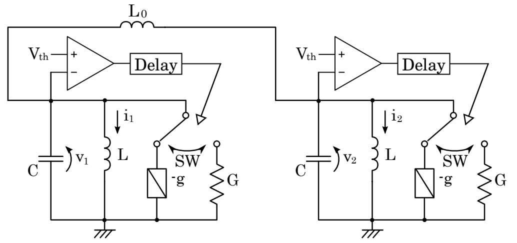
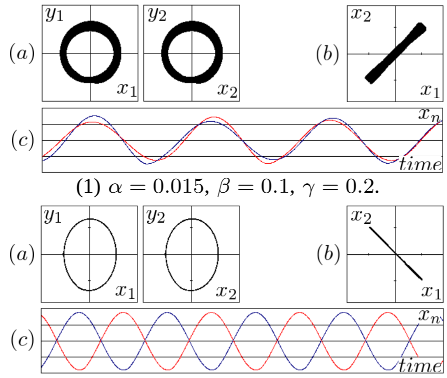


Fig. 4. Two coupled chaotic circuits containing time delay.

(B) In case of switch connected to G,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -2\beta y_n - x_n - \gamma(2x_n - x_{n+1} - x_{n-1}). \end{cases} \tag{6}$$

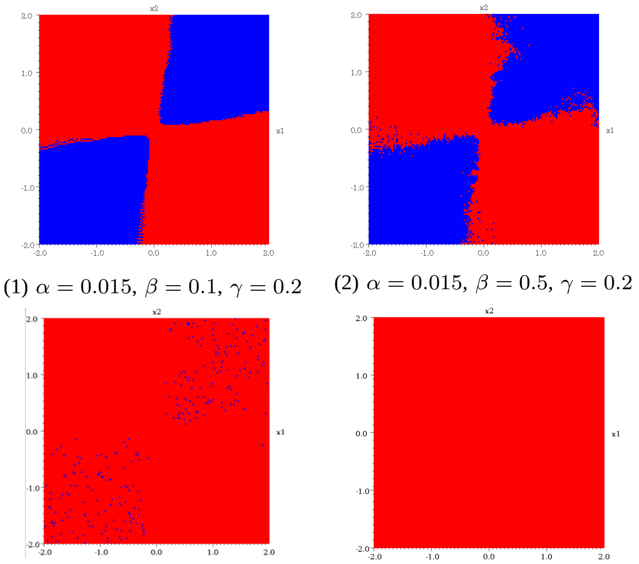
where $(n = 1, 2, ..., N)$, $x_0 = x_N$ and $x_{N+1} = x_1$.



(1) $\alpha = 0.015$, $\beta = 0.1$, $\gamma = 0.2$.

(2) $\alpha = 0.015$, $\beta = 0.1$, $\gamma = 0.2$.

Fig. 5. Simulation results of N = 2. (a) Attractor. (b) Lissajous figure. (c) Timewaveform. Red: x_1, Blue: x_2.



(1) $\alpha = 0.015$, $\beta = 0.1$, $\gamma = 0.2$ (2) $\alpha = 0.015$, $\beta = 0.5$, $\gamma = 0.2$

(3) $\alpha = 0.015$, $\beta = 0.7$, $\gamma = 0.2$ (4) $\alpha = 0.015$, $\beta = 0.8$, $\gamma = 0.2$

Fig. 6. Basin of attraction. Red and blue colors denote anti-phase synchronization state and in-phase synchronization state respectively. $y_1 = 0.1112$, $y_2 = -0.1113$.

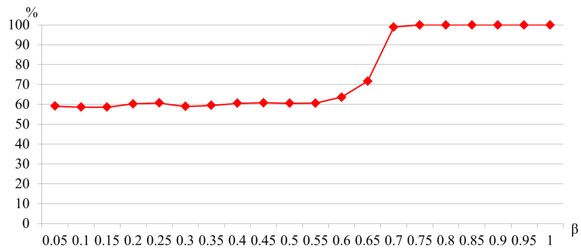


Fig. 7. Proportion of anti-phase synchronization state.

A. In case of $N = 3$

Figure 9 shows the simulation results in case of $N = 3$. We observed the two types of synchronization state same with $N = 2$. In-phase synchronization and three-phase synchronization states depending on initial values can be observed as shown in Fig. 9 (1) and (2). Also, by increasing the parameter value β, in-phase synchronization state is gradually changed and finally the system generates only three-phase synchronization as shown in Fig 9 (3). This behavior implies the basin of attraction regarding to enlargement of three-phase synchronization for increase of the parameter. In state of three-phase synchronization, the phase difference is almost $120°$ with respect to neighboring subcircuit.

B. In case of $N = 4$

Figure 10 shows the simulation results in case of $N = 4$. Red, blue green and black colors denote x_1, x_2 x_3 and x_4 respectively. In this case, similar behavior with the case of $N = 2$ can be observed. For instance, in-phase synchronization and anti-phase synchronizations state can be observed in case of $N = 4$. It seems that anti-phase synchronization tend to cause the stable state for the system inasmuch as the orbit is exactly periodic. Moreover, from the results of $N = 2$ and $N = 4$, the system generates in-phase and anti-phase synchronization in case of that the number of coupled chaotic circuit is even number in any number of coupled circuit.

C. In case of $N = 5$

Similar synchronization state in $N = 3$ can be observed in $N = 5$ as shown in Fig. 11. Red, blue and green colors denote x_1, x_2 and x_3 respectively. Except for x_1, x_2, x_3, time waveforms are described with black color. In the parameter of weak chaotic region, the system generates coexisting synchronization state. And only five-phase synchronization is induced in certain chaotic. The two types Lissajous figures depending initial value can be observed the parameter value of β as shown in Fig. 11 (3) and (4). In state of five-phase synchronization, there is phase difference of $72°$ with respect to neighboring subcircuit. We compare the value of β when coupled circuits are attracted to N-phase synchronization state. β in case of $N = 5$ is smaller than $N = 3$.

V. Consideration

Table I summarizes results from this study. The synchronization state can be classified by the number of coupled chaotic circuit whether the number is even or odd. When the number of subcircuits is large, synchronization state is easily attracted anti-phase synchronization or N-phase synchronization.

VI. Conclusions

In this study, we have investigated synchronization state observed in two coupled chaotic circuits containing time delay. As a result, induction of anti-phase synchronization or N-phase synchronization state caused by increasing the chaotic strength of subcircuit has confirmed. Furthermore, we have investigated synchronization state observed on a ring of coupled chaotic circuits containing time delay. As a result, induction of anti-phase or N-phase synchronization caused by chaotic strength of subcircuit have clarified. Moreover, synchronization state can be classified by the number of coupled chaotic circuit whether the number is even or odd. In future work, We will investigate the parameter region of coexisting synchronization and the mechanism of induction of

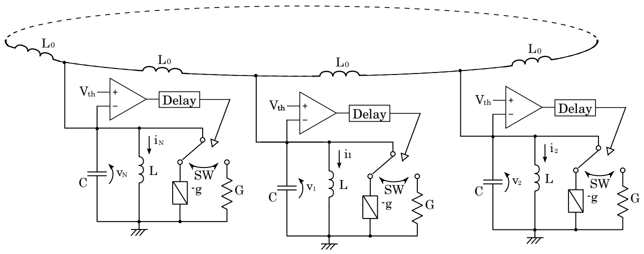


Fig. 8. A ring of coupled chaotic circuits containing time delay.

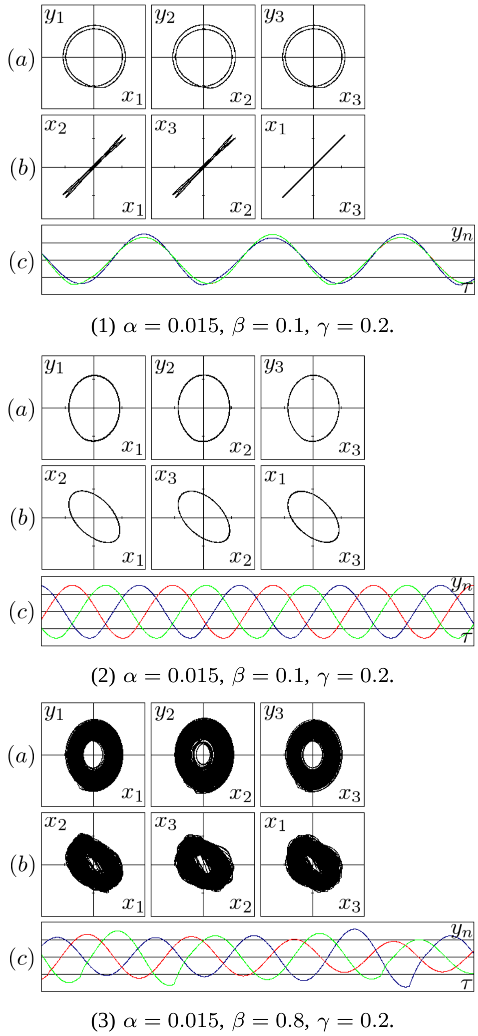
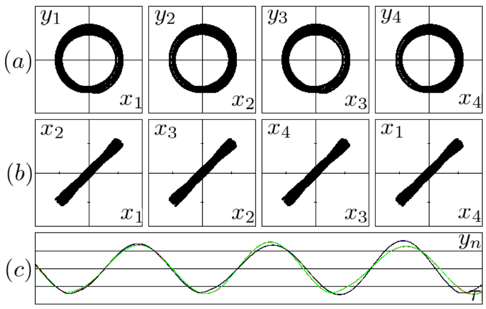


Fig. 9. Simulation results of $N = 3$. (a) Attractor. (b) Lissajous figure. (c) Timewaveform. Red, blue and green colors denote x_1, x_2 and x_3 respectively.

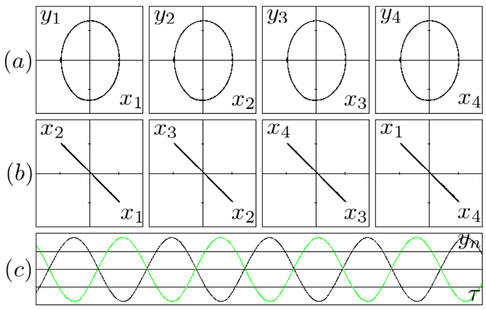
TABLE I

SYNCHRONIZATION STATE.

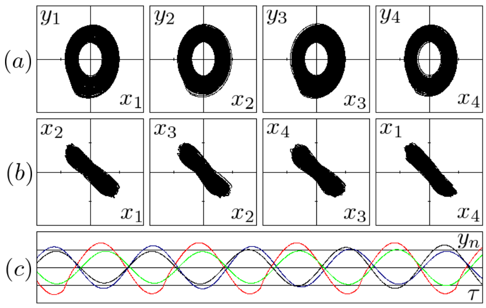
Coupled circuits		A ring of coupled circuits					
$N = 2$		$N = 3$	$N = 4$	$N = 5$		$N = n$ odd	$N = k$ even
in-phase $\theta_d \simeq 0[°]$	...	in-phase $\theta_d \simeq 0[°]$	in-phase $\theta_d \simeq 0[°]$	in-phase $\theta_d \simeq 0[°]$	...	in-phase $\theta_d \simeq 0$ [rad]	in-phase $\theta_d \simeq 0$ [rad]
anti-phase $\theta_d \simeq 180[°]$		3-phase $\theta_d \simeq 120[°]$	anti-phase $\theta_d \simeq 180[°]$	5-phase $\theta_d \simeq 72[°]$		N-phase $\theta_d \simeq \frac{2\pi}{n}$ [rad]	anti-phase $\theta_d \simeq \pi$ [rad]



(1) $\alpha = 0.015$, $\beta = 0.1$, $\gamma = 0.2$.

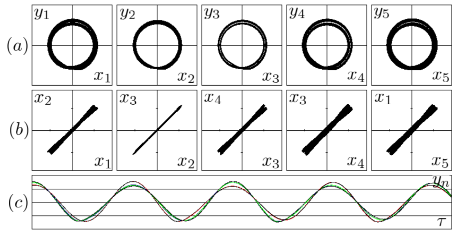


(2) $\alpha = 0.015$, $\beta = 0.1$, $\gamma = 0.2$.

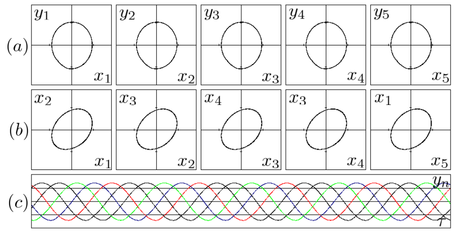


(3) $\alpha = 0.015$, $\beta = 0.8$, $\gamma = 0.2$.

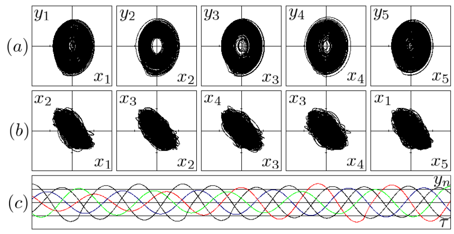
Fig. 10. Simulation results of N = 4. (a) Attractor. (b) Lissajous figure. (c) Timewaveform. Red, blue green and black colors denote x_1, x_2 x_3 and x_4 respectively.



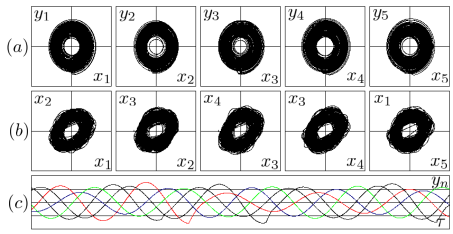
(1) $\alpha = 0.015$, $\beta = 0.1$, $\gamma = 0.2$.



(2) $\alpha = 0.015$, $\beta = 0.1$, $\gamma = 0.2$.



(3) $\alpha = 0.015$, $\beta = 0.8$, $\gamma = 0.2$. type1



(4) $\alpha = 0.015$, $\beta = 0.8$, $\gamma = 0.2$. type2

Fig. 11. Simulation results of N = 5. (a) Attractor. (b) Lissajous figure. (c) Timewaveform. Red, blue and green colors denote x_1, x_2 and x_3 respectively. Black color shows the $x_n (n > 3)$.

anti-phase or N-phase synchronization state.

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[2] L. M. Pecora and T. L. Carroll, “Synchronization in Chaotic Systems,” Physical Review Letters, vol. 64, no.8, pp. 821-824, Feb. 1990.