Synchronization and Clustering in Coupled Parametrically Excited Oscillators with Small Mismatch

Kosuke Oi, Yoko Uwate and Yoshifumi Nishio
Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770–8506, Japan
Email: {ooi,uwate,nishio}@ee.tokushima-u.ac.jp

***Abstract*—In this study, we investigate synchronization of parametrically excited van der Pol oscillators with small mismatch. In the case of two subcircuits, we confirm that the two subcircuits are synchronized at in-phase state when the adding mismatch is small. By increasing the small mismatch, we observe unsynchronous phenomena. Furthermore, we apply this circuit model to ten coupled oscillators as random network model with hub.**

I. Introduction

Synchronization is one of the fundamental phenomena in nature and it is observed over the various fields. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. We consider that it is important to investigate the synchronization phenomena of coupled oscillators for the future engineering application. The coupled van der Pol oscillator is one of coupled oscillators, and synchronization generated in the system can model certain synchronization of natural rhythm phenomena. The van der Pol oscillator is studied well because it is expressed in simple circuit. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena of the parametric excitation circuits for future engineering applications. In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Refs. [4] and [5].

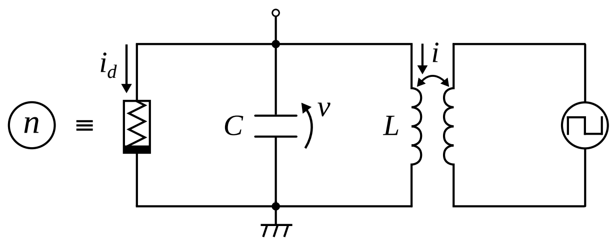
In our previous study, we have investigated synchronization of parametrically excited van der Pol oscillators [6]. By carrying out computer calculations for two or three subcircuits case, we have confirmed that various kinds of synchronization phenomena of chaos are observed. In the case of two subcircuits, the anti-phase synchronization is observed. In the case of three subcircuits, self-switching phenomenon of synchronization states is observed.

However, in the previous study, we have investigated the only simple network models. It is important to investigate more complex network for the broad-ranging future engineering application. In this study, we challenge to investigate more complex network using parametrically excited van der Pol oscillators with small mismatch.

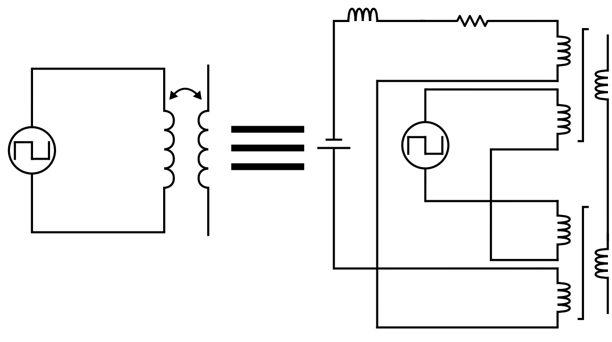
First, we investigate synchronization of parametrically excited van der Pol oscillators with small mismatch. The small mismatch is added to the amplitude of the function relating to parametrically excitation. In the case of two subcircuits, we confirm that the two subcircuits are synchronized at in-phase state when the adding mismatch is small. By increasing the small mismatch, we observe unsynchronous phenomena. Furthermore, we apply this circuit model to ten coupled oscillators as random network model with hub.

II. van der Pol oscillator under parametric excitation

The circuit model of van der Pol oscillator under parametric excitation is shown in Fig. 1.



(a) Parametrically excited van der Pol oscillator



(b) Time-varying inductor

Fig. 1. Circuit model.

978-1-4799-8391-9/15/$31.00 ©2015 IEEE

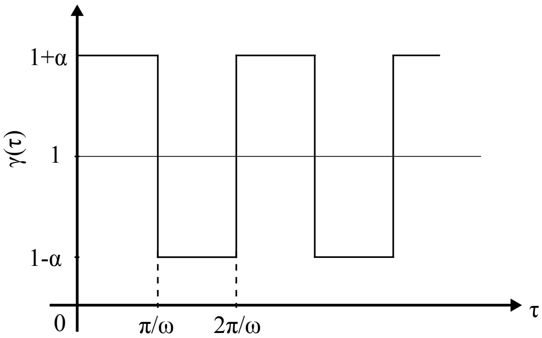


Fig. 2. Function relating to parametrically excitation.

The circuit includes a time-varying inductor L whose characteristics are given as the following equation. The time-varying inductor is shown as Fig. 2.

$$L = L_0 \gamma(\tau). \tag{1}$$

$\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 2, and its amplitude and angular frequency are termed α and ω, respectively. The $v - i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v_k + g_3 v_k. \tag{2}$$

By changing the variables and the parameters,

$$\begin{cases} t = \sqrt{L_0 C}\tau, \ v_n = \sqrt{\dfrac{g_1}{g_3}} x_n \\ \omega = \omega_0 \sqrt{L_0 C} \\ i_n = \sqrt{\dfrac{g_1}{g_3}} \sqrt{\dfrac{C}{L_0}} y_n \\ \varepsilon = g_1 \sqrt{\dfrac{L_0}{C}}, \ \delta = \dfrac{1}{R} \sqrt{\dfrac{L}{C}} \end{cases} \tag{3}$$

The normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon(x_n - x_n^3) - y_n + \delta \displaystyle\sum_{k \in S_n} (x_k - x_n) \\ \dfrac{dy_n}{d\tau} = \dfrac{1}{\gamma(\tau)} x_n \end{cases} \tag{4}$$

where $n = 1, 2, 3, ..., 10$. S_n is the set of nodes which are directly connected to the node n.

III. Two Subcircuit Case

In this section, we consider the case of $n = 2$. Only two parametrically excited van der Pol oscillators are coupled by one resistor (see Fig. 3). We investigate synchronization state when two oscillators have different parametrically excitations. The parameter α of the first oscillator is fixed with 0.95, and the value of α of the second oscillator is changed. For the computer simulations, we fix the circuit parameter as follows; $\varepsilon = 1.00$, $\alpha = 0.95$, $\omega = 1.00$ and $\delta = 0.50$.

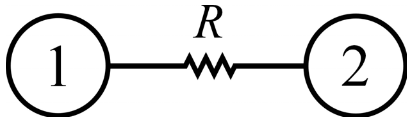
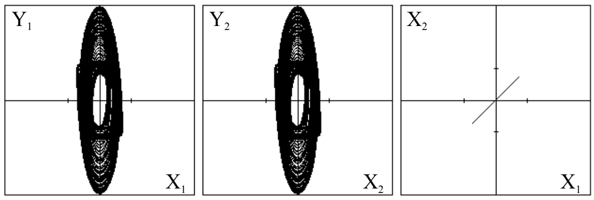
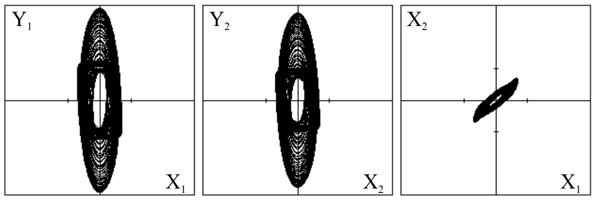


Fig. 3. Two coupled oscillators.

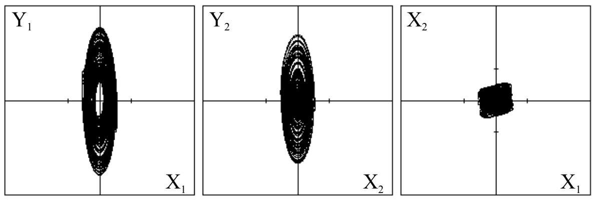
Figure 4 shows the computer calculated results when the parameter α is changed. In the case of two parametrically excitations are same, two oscillators are fully synchronized at in-phase state as shown in Fig. 4(a). If the second oscillator has little bit different value of α with the first oscillator, fully synchronization state changes to quasi synchronization state as shown in Fig. 4(b). Figure 4(c) shows that the state of two oscillators becomes to unsynchronus state with large difference between α of the first and the second oscillators.



(a) $\alpha_1 = 0.950$ and $\alpha_2 = 0.950$



(b) $\alpha_1 = 0.950$ and $\alpha_2 = 0.946$



(c) $\alpha_1 = 0.950$ and $\alpha_2 = 0.936$

Fig. 4. Attractor and phase differences.

Next, we investigate synchronization area when the coupling strength δ is changed. In this simulation, α of the first oscillator is fixed with 0.95. The computer simulated results are shown in Fig. 5. The horizontal axis denotes the coupling strength δ, and the vertical axis denotes α of the second oscillator. In this figure, synchronous area is shown between the upper and the lower lines. We can see that synchronous area becomes bigger by increasing the coupling strength.

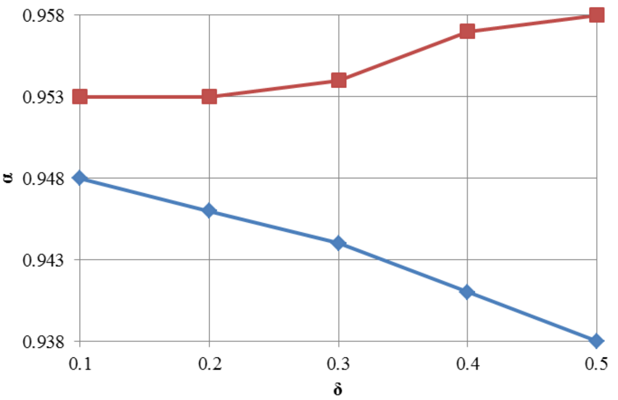


Fig. 5. Synchronization area by changing δ.

IV. Random Network Case

Next, we apply the previous circuit model to ten coupled oscillators as random network model. The ten coupled oscillator model is shown in Fig. 6. In this circuit system, there is a hub (1st oscillator) which is connected to many oscillators. Here, the small mismatch is added to α which is corresponding to the amplitude of the function relating to parametrically excitation. The mismatch is generated by random and the range of the mismatch is set to [-0.01:0.01]. Table I shows the pattern of five types of the small mismatch used in this computer simulations.

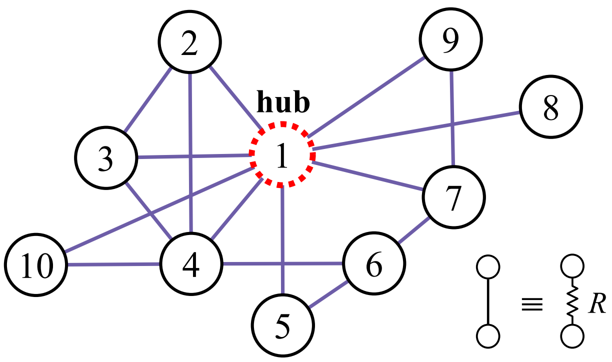


Fig. 6. A random network model with a hub.

Figure 7 shows the simulation results. The horizontal axis denotes the coupling strength δ, and the vertical axis denotes α of the second oscillator. In this graph, the upper area of each line denotes synchronous area and the lower area of each line denotes unsynchronous area. By increasing the value of the coupling strength, the synchronous area becomes large. We can see that the synchronous area is determined by the small mismatch pattern. Namely, several types of synchronization states can be observed from ten coupled oscillators with the small mismatch.

TABLE I
SMALL MISMATCH

small mismatch	w=1	w=2	w=3	w=4	w=5
α_1	0	0	0	0	0
α_2	-0.009	0.008	0.008	-0.01	-0.009
α_3	-0.006	-0.006	0.009	0.009	0.01
α_4	-0.001	-0.005	-0.01	-0.003	-0.007
α_5	0.009	0.007	-0.01	0.001	-0.001
α_6	-0.002	-0.01	-0.004	-0.002	0.004
α_7	0	-0.001	-0.003	0.007	0.009
α_8	0	-0.004	0.005	-0.006	0.007
α_9	-0.001	-0.01	0.009	-0.01	0.005
α_{10}	0.005	0.009	-0.009	-0.009	-0.001

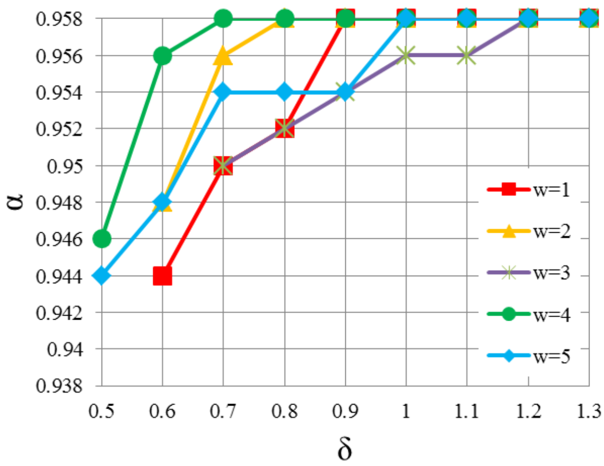


Fig. 7. Synchronous state with δ.

V. Clustering Phenomena

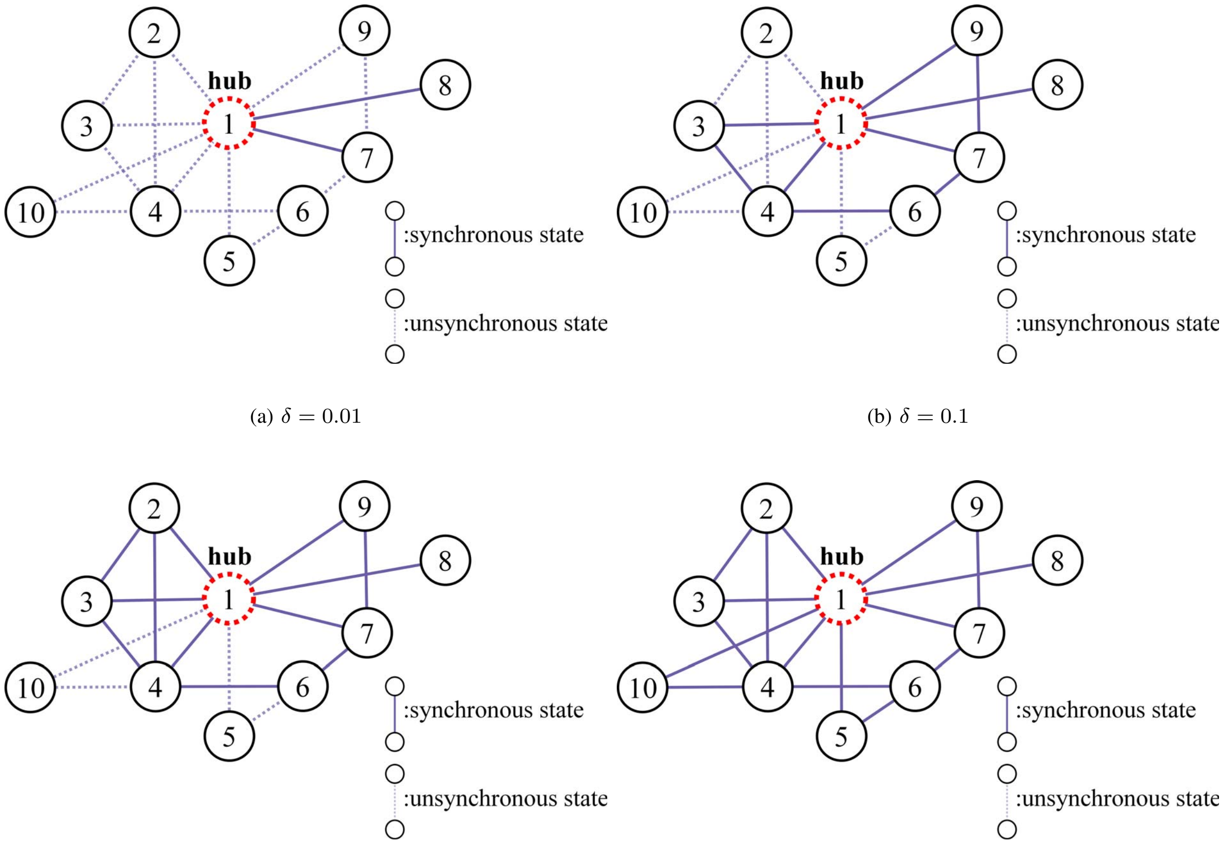
Finally, we focus on clustering phenomena of ten coupled oscillators as random network with hub. Figure 8 shows the clustering phenomena (how do the oscillators change the fully synchronization state) when the coupling strength is changed. The horizontal axis is the coupling strength, and the vertical axis denotes the node number between the oscillators.

At the beginning, first, two oscillators becomes synchronous state. By increasing the coupling strength, the number of synchronized oscillators becomes large.

VI. Conclusions

In this study, we have investigated synchronization phenomena of parametrically excited van der Pol oscillators with small mismatch. In the case of two subcircuits, we have confirmed that the two subcircuits are synchronized at in-phase state when the adding mismatch is small. By increasing the small mismatch, we observe unsynchronous phenomena. Furthermore, we have applied this circuit model to ten coupled oscillators as random network model with hub.

For the future work, we would like to consider the influence of the small mismatch for synchronization state of more large



(a) $\delta = 0.01$

(b) $\delta = 0.1$

(c) $\delta = 0.25$

(d) $\delta = 0.80$

Fig. 8. Circuit model.

scale networks.

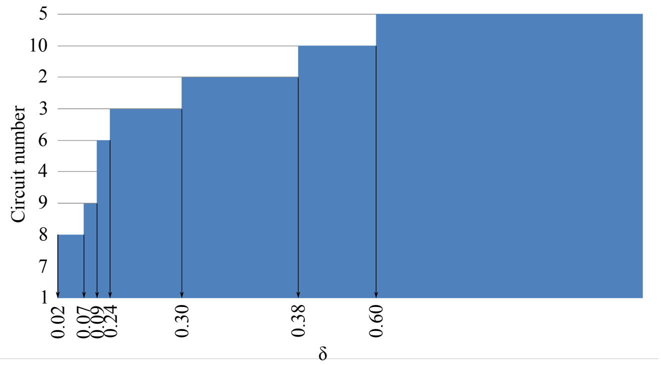


Fig. 9. Clustering phenomena with δ (w=1).

REFERENCES

[1] I. Belykh, M. Hasler, M. Lauret and H. Nijmeijer, "Synchronization and graph topology," Int. J. Bifurcation and Chaos, vol.15, no.11, pp.3423-3433, Nov. 2005.

[2] J. Cosp, J. Madrenas, E. Alarcon, E. Vidal and G. Villar, "Synchronization of nonlinear electronic oscillators for neural computation," IEEE Trans. Neural Networks, vol.15, no.5, pp.1315-1327, Sep. 2004.

[3] C. Hayashi, "Nonlinear Oscillations in Physical Systems," Chap. 11, McGraw-Hill, New York (1964).

[4] C. Hayashi, M. Abe, K. Oshima and H. Kawakami, "The method of mapping as applied to the solution for certain types of nonlinear differential equations," Ninth International Conference on Nonlinear Oscillations, Kiev (Aug.-Sept.1981).

[5] M. Inoue, "A Method of Analysis for the Bifurcation of the Almost Periodic Oscillation and the Generation of Chaos in a Parametric Excitation Circuit," Trans. of IEICE, vol. J68-A, no. 7, pp. 621-626, 1985.

[6] H. Kumeno, Y. Nishio, "Synchronization Phenomena in Coupled Parametrically Excited van der Pol Oscillators," *Proc. NOLTA'08*, pp. 128-131, Sep. 2008.