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Investigation of Oscillation Death in Strongly Coupled Polygonal Oscillatory Networks

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Abstract

We have confirmed amplitude change and oscillation death in two coupled polygonal oscillatory networks with strong frustration. In order to understand the mechanism of these phenomena, in this study, theoretical analysis using power consumption is applied to solve the amplitude of the coupled oscillators.

1. Introduction

The synchronization phenomena of coupled oscillators are suitable model to express essensial behavior observed from the natural science. Therefore, many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena [1]-[4].

In our research group, we have focused on synchronization phenomena of coupled oscillators under a difficult situation for the circuit. Setou et al. have reported the synchronization phenomena in N oscillators coupled by resistors as a ring. The oscillation stop in some range of the coupling resistors was confirmed [5].

We have investigated the synchronization phenomena in the coupled polygonal oscillatory networks sharing branches [6]. In this system, van der Pol oscillators are connected to every corner of polygonal network. By using computer simulations and theoretical analysis, we confirm that the coupled oscillators tend to synchronize to minimize the power consumption of the whole system. The phase difference of the shared oscillators is solved by finding the minimum value of the power consumption function.

Recently, in order to investigate amplitude change of the oscillators with strong coupling parameter, we have proposed a new circuit model of coupled polygonal network which is inserted the earth resistances in all ground parts [7], [8]. In this circuit system, we confirm that the amplitude of the oscillators decreases by increasing the value of the coupling strength and oscillation death of the oscillators located farthest place from the shared oscillators is occurred. These phenomena are very interesting, however, we have observed such oscillation death from only simple coupled polygonal oscillatory networks as basic model.

In order to make clear the mechanism of these phenomena especially oscillation death, in this study, theoretical analysis using power consumption of the coupling resistance is applied to solve the amplitude of the coupled oscillators.

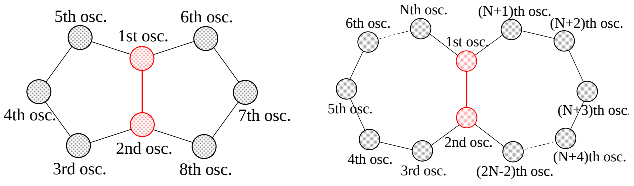
We expect that the results of this study contribute to understanding of synchronization phenomena observed in general complex networks.

2. Two Coupled Oscillatory Networks

2.1 Symmetric Model

Two identical polygonal oscillatory networks are coupled by sharing a branch as shown in Fig. 1. In this circuit model, we consider the coupling method which two adjacent oscillators are tend to synchronize at anti-phase state. We call the first and the second oscillators which are connected to both side of polygonal network "shared oscillators."

Figure 2 shows the circuit model of the 5-5 coupling networks.



(a) 5-5 coupling network.
(b) N-N coupling network.

Figure 1: Two Coupled Oscillatory Networks (Symmetric Model).

Next, we develop the expression for the circuit equations of N-N coupling oscillatory networks as shown in Fig. 2. The

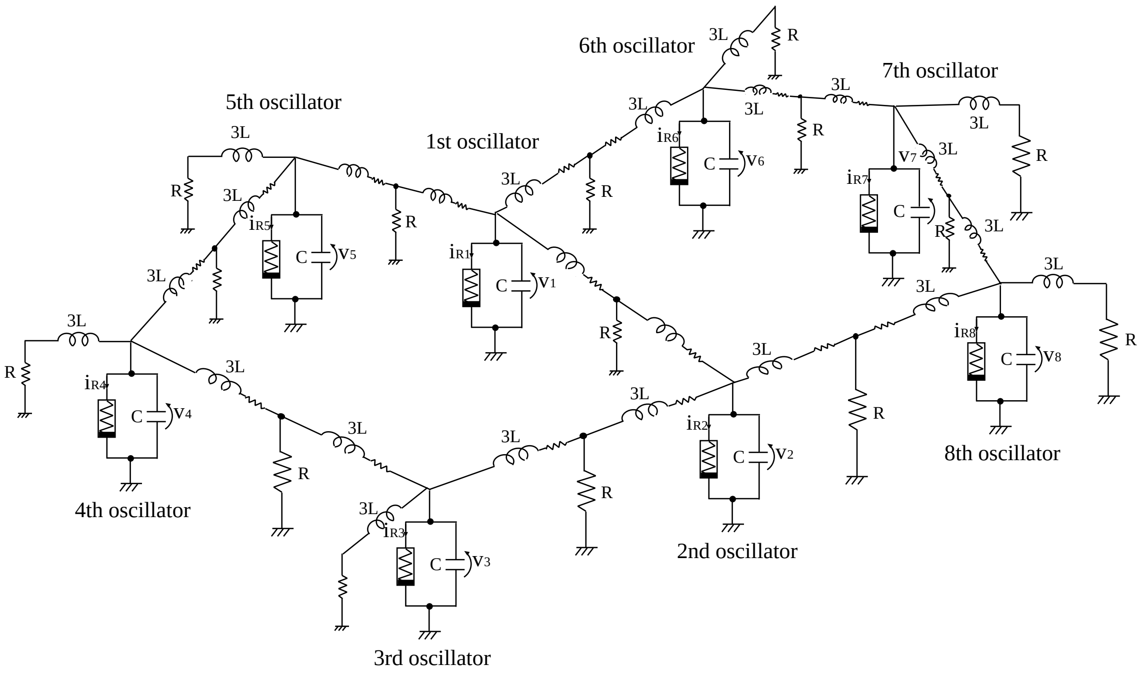


Figure 2: Coupling Model (5-5 coupling networks).

$v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third order polynomial equation,

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), (k = 1, 2, 3, 4). \tag{1}$$

The normalized circuit equations governing the circuit are expressed as
[kth oscillator]

$$\left\{\begin{array}{l} \dfrac{dx_k}{d\tau} = \varepsilon\Big(1 - \dfrac{1}{3}{x_k}^2\Big)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\Big\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\Big\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\Big\} \\ \qquad\qquad\qquad\qquad (k = 1, 2, 3, 4). \end{array}\right. \tag{2}$$

where

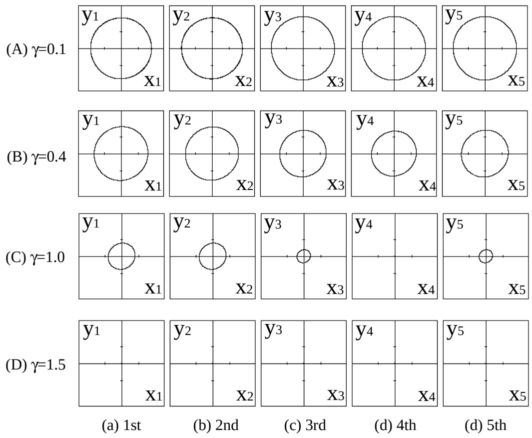
$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}}x_k,$$

$$i_{ak} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{ak}, \quad i_{bk} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{bk},$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}}, \gamma = R\sqrt{\frac{C}{L}}, \quad \eta = r_m\sqrt{\frac{C}{L}},$$

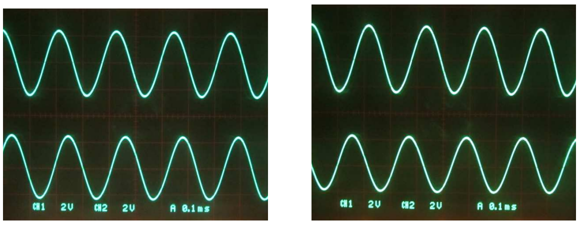
In this equations, γ is the coupling strength, ε denotes the nonlinearity of the oscillators and y_n denotes the current of neighbor oscillator on coupling resistor.

Figure 3 shows the observed attractors of $5-5$ coupling network by changing the coupling strength. In this circuit model, the amplitude of fourth oscillator (which is located farthest place from the shared oscillators) decreases with the coupling strength. Then, we observe oscillation death of the fourth oscillator when the coupling strength is set to $\gamma = 1.0$. We also obtain the same synchronization states from the circuit experiment as shown in Fig. 4. The circuit experiments parameters are set to $L = 50mH$ and $C = 68nF$. The nonlinear resistor is realized by op-amp (TL082). The power supply of the op-amp is fixed as $\pm 12V$.

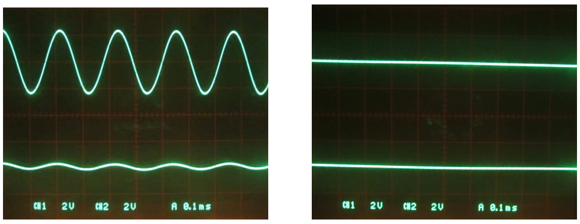


(a) 1st (b) 2nd (c) 3rd (d) 4th (d) 5th

Figure 3: Attractor (5-5 coupling network).

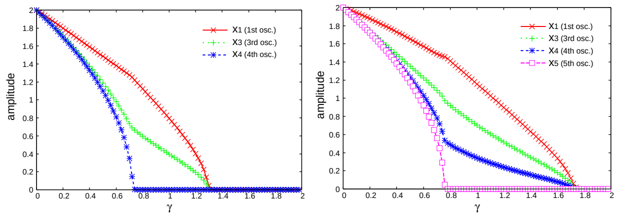


(a) $R = 150\Omega$. (b) $R = 200\Omega$.

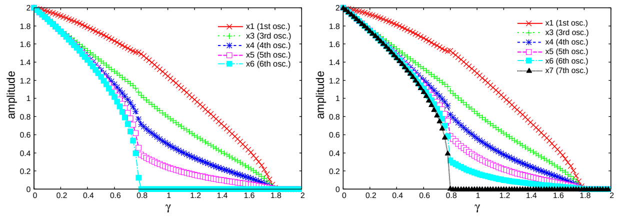


(c) $R = 300\Omega$. (d) $R = 390\Omega$.

Figure 4: Circuit experiments results for 5-5 coupling network. upper:v_1 (1st oscillator), lower:v_4 (4th oscillator), The horizontal axis: [0.1ns/div]. The vertical axis: [2V/div]



(a) 5-5 coupling network. (b) 7-7 coupling network.



(c) 9-9 coupling network. (d) 11-11 coupling network.

Figure 5: Amplitude (Symmetric Model).

Figure 5 shows the change of the amplitude observed from 5-5, 7-7, 9-9 and 11-11 coupling networks. From these figures, we can see that first, the oscillation death of the oscillators located farthest place from the shared oscillators is occurred. After that, the other oscillators stop to oscillate at same time.

We explain the oscillation death as physical phenomena. The earth resistance is not inserted in two shared oscillators, namely the shared oscillators do not tend to oscillation death by controling the phase difference to minimize enegy. Then next oscillators from the shared oscillators try to oscillate to synchronize with the shared oscillators. Furthermore, after oscillation death is occurred, the network topolgy is changed from ring to two ladders. How is the amplitude of the coupled oscillators determined? In order to make clear the mechanism of the oscillation death, we focus on the power consumption of the coupling resistors in the whole system. We assume the current of the inductor ($3L$) as the following equation.

$$i_k(t) = \rho_k \sin(\omega t + \varphi_k) \quad (k = 1, 2, 3...O_N), \tag{3}$$

where ω is the natural frequency of van der Pol oscillator as $\omega = \frac{1}{\sqrt{LC}}$. When the coupling resistance is assumed as $R = 1$, the average power consumption of the coupling resistor between kth and $(k+1)$st oscillators are described as Eq. (5).

$$P = \frac{1}{2\pi}\int_0^{2\pi} \{i_k(t) + i_{k+1}(t)\}^2 dt \tag{4}$$

Here, we try to derive each amplitude theoretically with the following asummptions. First, let the phase difference between the oscillators be π. Second,the amplitude of each oscillator has different value. So, we fix each amplitude depending on the located place of the oscillators. Figure 6 shows the amplitude setting for 5-5 coupling network as one example. The amplitude of the shared oscillators is set to ρ_0 and the next oscillators of the shared oscillators is set to ρ_1 and ρ_2.

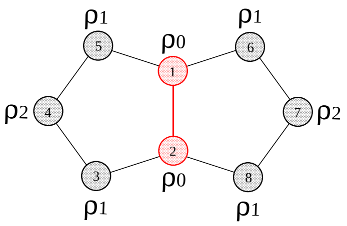


Figure 6: Amplitude setting for calculating the power consumption.

By using Eq. (4), we can obtain the power consumption of the whole systems for 5-5 coupling network as follows.

$$P = 4\rho_0^2 + 8\rho_1^2 + 4\rho_2^2 - 8\rho_0\rho_1 - 8\rho_1\rho_2. \tag{5}$$

We calculate the power consumption of the whole system by using the amplitude (ρ_0 and ρ_2) obtained from the computer simulations. Figure 7 (a) shows the theoretical results of the amplitude (ρ_1) of 5-5 coupling networks. The results of the theoretical analysis match well with simulation results.

The amplitude of other types of the coupling network is also solved by usinv previous theoretical approch. The results are shown in Fig. 7 (b), (c) and (d). From these results, we can see that the theoretical analysis can express the characteristics of the amplitude change.

3. Conclusions

In this study, we have investigated synchronization phenomena in coupled polygonal oscillatory networks with strong frustrations. We focused on the amplitude of each oscillator when the coupling strength is increased. By using the computer simulations, we confirmed that the amplitude of the oscillators decreases by increasing the coupling strength and oscillation death is occurred at un-frustrated oscillators.

Acknowledgment

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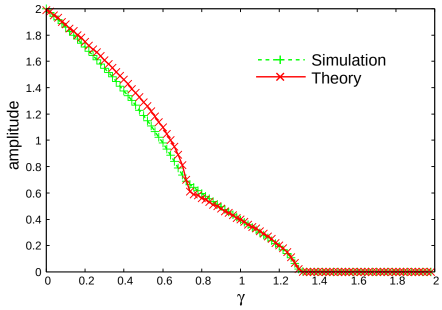
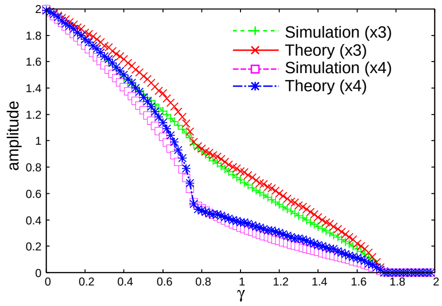
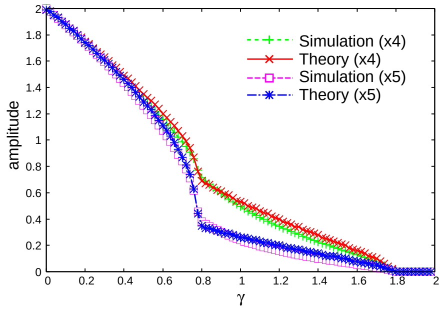


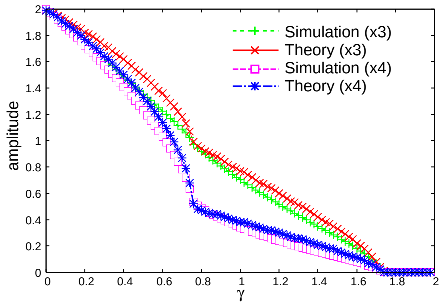
(a) 5-5 coupling network.
(ρ_1 by using ρ_0, ρ_2)



(b) 7-7 coupling network.
($\rho_{1,2}$ by using $\rho_{0,3}$)



(c) 9-9 coupling network.
($\rho_{2,3}$ by using $\rho_{0,1,4}$)



(d) 11-11 coupling network.
($\rho_{3,4}$ by using $\rho_{0,1,2,5}$)

Figure 7: Theoretical and simulation results.