Oscillation Death and Amplitude Change in Coupled van der Pol Oscillators with Strong Frustrations

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***Abstract*— In this study, we investigate synchronization phenomena in coupled polygonal oscillatory networks with strong frustrations. We focus on the amplitude of each oscillator when the coupling strength is increased. By using the computer simulations, we confirm that the amplitude of the oscillators decreases by increasing the coupling strength and oscillation death frustrated oscillators is occurred. Furthermore, we observe the amplitude change in the asymmetric network model.**

I. Introduction

Synchronization phenomena with coupled oscillators are suitable model to analyze the natural phenomena. Therefore, many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena [1]-[4].

On the other hand, there are several types of polygonal network structures (e.g. honeycomb, soup bubbles and cracks of drying mud structure) in the natural science. Generally, for the studies of large-scale network using coupled oscillators, a ring, a ladder and a two dimensional array structure are often investigated. However, there are not many discussions about coupled polygonal oscillatory networks by using electrical oscillators.

In our research group, we have focused on synchronization phenomena of coupled oscillators under a difficult situation for the circuit. Setou et al. have reported the synchronization phenomena in N oscillators coupled by resistors as a ring. The oscillation stop in some range of the coupling resistors was confirmed [5]. We have investigated synchronization phenomena in coupled polygonal oscillatory networks sharing branches [6]. In this system, van der Pol oscillators are connected to every corner of polygonal network. By using computer simulations and theoretical analysis, we confirm that coupled oscillators tend to synchronize to minimize the power consumption of the whole system. The phase difference of the shared oscillators is solved by finding the minimum value of the power consumption function. Furthermore, we have discussed synchronization state of the proposed oscillatory networks by changing the coupling parameter [7]. In this circuit system, we confirm that the amplitude of the shared and the other oscillators obtains different value. The torus attractors can be also observed in the certain parameter region. However, we could not observe oscillation death even if the coupling strength is set to very large value. In the circuit model of Ref. [7], the earth resistance is missing in the 3rd and the 4th oscillators. Namely, the network topology changes to just ring oscillators coupled by the inductors if the coupling strength becomes large.

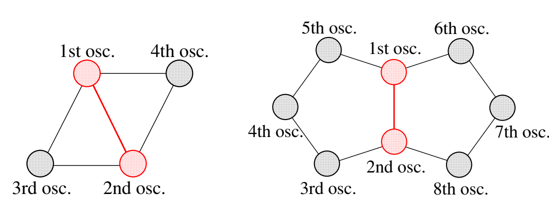
In this study, we propose the new circuit model which is including the earth resistance in all ground parts. Synchronization phenomena in coupled polygonal oscillatory networks with strong frustration are investigated. We confirm that the amplitude of the oscillators decreases by increasing the value of the coupling strength and oscillation death frustrated oscillators is occurred. Furthermore, we observe the amplitude change when the network topology is asymmetric.

II. Two Coupled Oscillatory Networks

In this study, we investigate the synchronization phenomena in two coupled oscillatory systems.

A. Symmetric Model

Two identical polygonal oscillatory networks are coupled by sharing a branch as shown in Fig. 1. The new part of this circuit system is that the earth resistances are connected to ground parts in the 3rd and the 4th oscillators. We consider the coupling method which two adjacent oscillators are tend to synchronize at anti-phase state. We call the first and the second oscillators which are connected to both side of polygonal network "shared oscillators."



(a) 3-3 coupling network. (b) 5-5 coupling network.

Fig. 1. Two Coupled Oscillatory Networks (Symmetric Model).

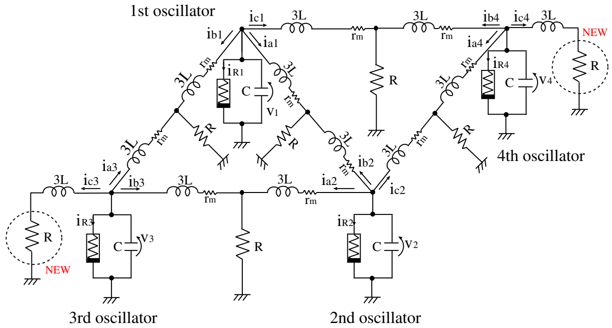


Fig. 2. Coupling Model $(3-3$ coupling networks).

Next, we develop the expression for the circuit equations of $3-3$ coupling oscillatory networks as shown in Fig. 2. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are approximated by the following third order polynomial equation,

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0), (k = 1, 2, 3, 4). \quad (1)$$

The normalized circuit equations governing the circuit are expressed as
[kth oscillator]

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\Big\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\Big\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{3}\Big\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\Big\} \\ \qquad\qquad\qquad\qquad (k = 1, 2, 3, 4). \end{cases} \quad (2)$$

where

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}}x_k, \quad i_{ak} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{ak},$$

$$i_{bk} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{C}{L}}y_{bk}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}},$$

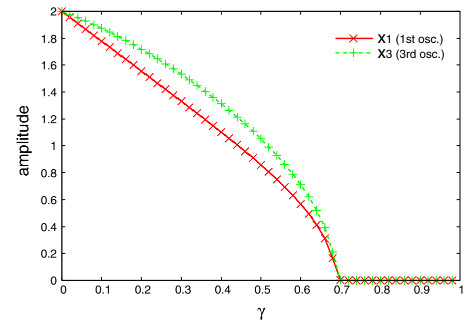
$$\gamma = R\sqrt{\frac{C}{L}}, \quad \eta = r_m\sqrt{\frac{C}{L}},$$

$$(k = 1, 2, 3, 4).$$

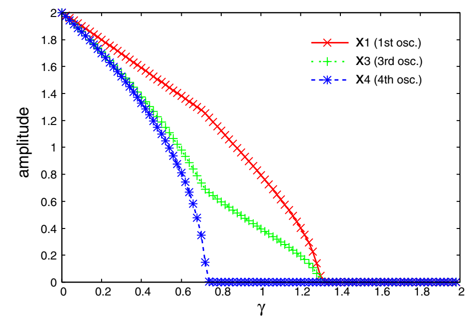
In this equations, γ is the coupling strength, ε denotes the nonlinearity of the oscillators and y_n denotes the current of neighbor oscillator on coupling resistor.

Figure 3 shows the change of the amplitude observed from each network. In the case of $3-3$ coupling network, oscillation death of all oscillators is occurred at same time. While, in the case of $5-5$ coupling network, first the oscillation death of the oscillators located farthest place from the shared oscillators is occurred. After that, the other oscillators stop to oscillate at same time.

Figure 4 shows the phase difference with the coupling strength.

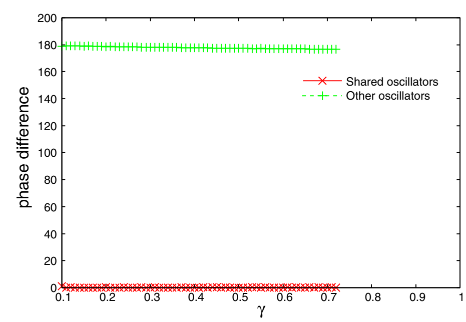


(a) 3-3 coupling network.

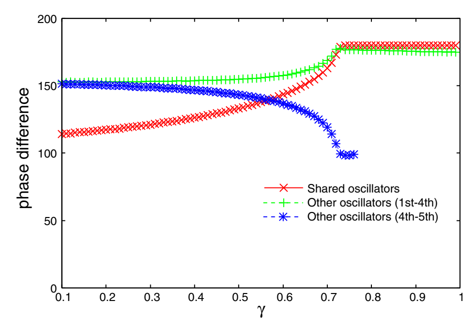


(b) 5-5 coupling network.

Fig. 3. Amplitude (Symmetric Model).



(a) 3-3 coupling network.

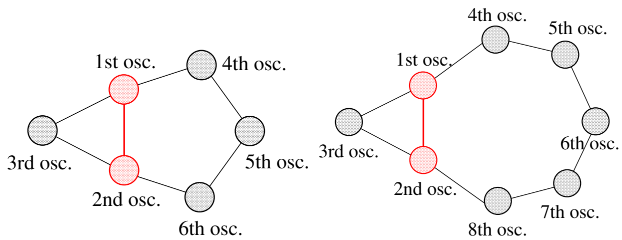


(b) 5-5 coupling network.

Fig. 4. Phase Difference (Symmetric Model).

B. *Asymmetric Model*

Next, we consider the asymmetric models such as $3-5$ coupling network and $3-7$ coupling network. The asymmetric models are shown in Fig. 5.

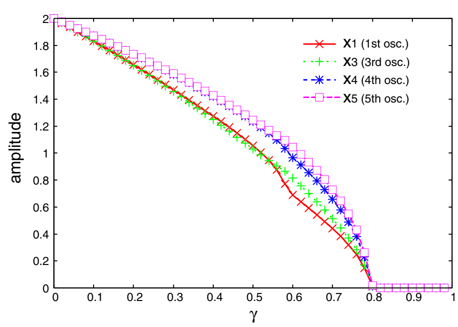


(a) 3-5 coupling network. (b) 3-7 coupling network.

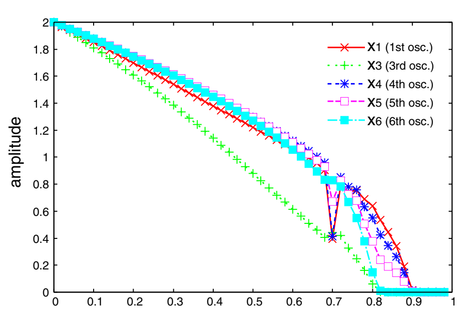
Fig. 5. Two Coupled Oscillatory Networks (Asymmetric Model).

Figure 6 shows the change of the amplitude observed from the asymmetric networks. In the case of $3-5$ coupling network, oscillation death of all oscillators is occurred at same time. While, in the case of $3-7$ coupling network, first the oscillation death of the oscillators located farthest place from the shared oscillators is occurred. After that, the other all oscillators stop to oscillate at same time. These results are similar with the results with the symmetric network models. However, we observe the amplitude change in $3-7$ coupling network when the coupling strength is set to around $\gamma = 0.75$. Figure 7 shows one example or the oscillation of the amplitude.

Figure 8 shows the phase difference with the coupling strength.



(a) 3-5 coupling network.



(b) 3-7 coupling network.

Fig. 6. Amplitude (Asymmetric Model).

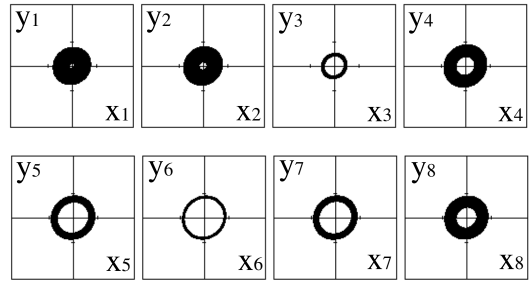
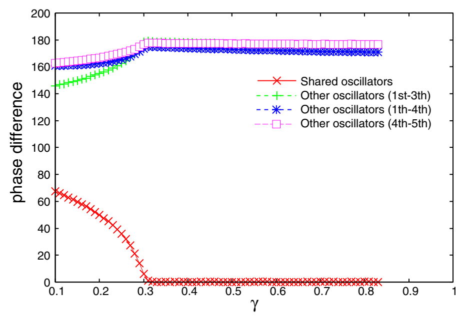
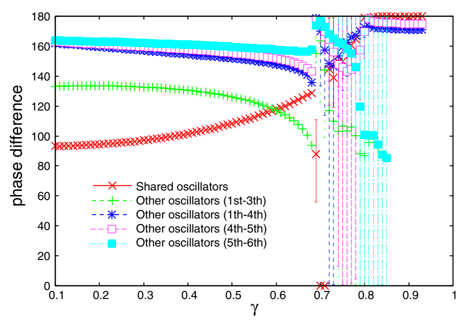


Fig. 7. Oscillation of Amplitude (3-7 coupling network, γ=0.75).



(a) 3-5 coupling network.



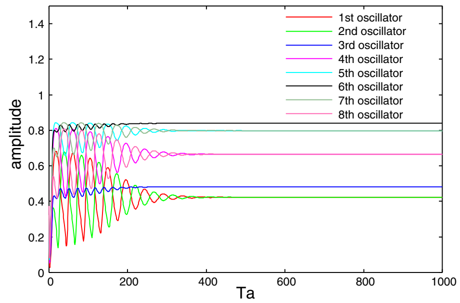
(b) 3-7 coupling network.

Fig. 8. Phase Difference (Asymmetric Model).

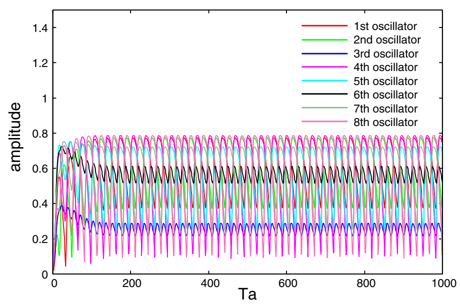
III. Amplitude Change

In this section, we focus on the phenomena of the amplitude change which are observed from $3-7$ coupling network. First, we confirm that the amplitude change can be observed around $\gamma = 0.75$. Figure 9 shows one example of the amplitude change by changing the coupling strength. The horizontal axis denotes the computer simulation time $T_a = 10,000\tau$.

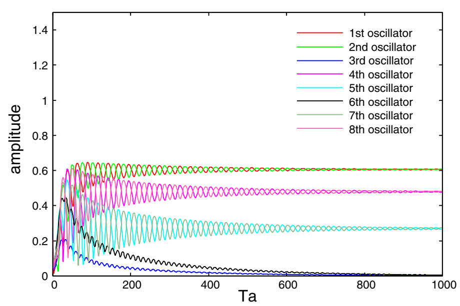
Figure 9(a) shows the result of the amplitude when the coupling strength is fixed with $\gamma = 0.72$. We observe oscillation of the amplitude at the beginning of the simulation. When T_a is larger than 300, the amplitude converges to constant. There are three pairs (1st-2nd, 4th-8th and 5th-7th) of the amplitude values and the 3rd and the 6th oscillators have own amplitude value. In the case of $\gamma = 0.75$, we confirm that the amplitude change can be observed constantly as shown in Fig. 9(b). There are two pairs (1st-4th-5th and 2nd-7th-8th) of oscillation types of the amplitude and these two pairs oscillate with anti-phase state. While the other two amplitudes (3rd and 6th) oscillate with in-phase state. The enlarged illustration of Fig. 9(b) is shown in Fig. 10. By increasing the coupling strength, the oscillation death of the 3rd and 6th oscillators is occurred. Then, we confirm that the network type depending on the amplitude returns to three pairs again (see Fig. 9(c)). Finally, we summarize the network types depending on the amplitude change as shown in Fig. 11.



(a) $\gamma = 0.72$.



(b) $\gamma = 0.75$.



(c) $\gamma = 0.80$.

Fig. 9. Amplitude Change ($3-7$ coupling network).

IV. Conclusions

In this study, we have investigated synchronization phenomena in coupled polygonal oscillatory networks with strong frustrations. We focused on the amplitude of each oscillator when the coupling strength is increased. By using the computer simulations, we confirmed that the amplitude of the oscillators decreases by increasing the coupling strength and oscillation death frustrated oscillators is occurred. Furthermore, we observed the amplitude change in the asymmetric network model.

Acknowledgment

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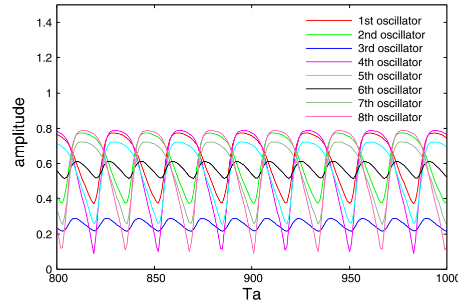
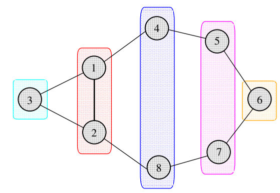
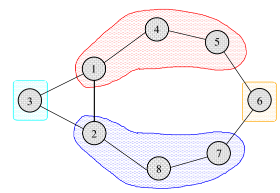


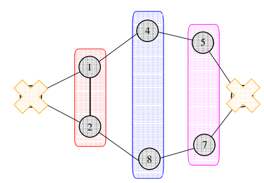
Fig. 10. Enlarged Illustration of Fig. 9(b).



(a) Type-A (3 pairs).



(a) Type-B (2 pairs).



(c) Type-C (3 pairs).

Fig. 11. Network Types Depending on Oscillation of Amplitude.

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