2014 International Symposium on Nonlinear Theory and its Applications
NOLTA2014, Luzern, Switzerland, September 14-18, 2014

Synchronizations and Oscillations of Negative Coupled Neural Oscillators

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Abstract—Neurons transmit signals through the chemical and electrical synapses such as resistance coupling that act as a neurotransmitter. This electrical synapse functions such as the resistor coupling. By contrast, the coupling van der Pol oscillators generate complex synchronization phenomena. In this study, we focus on the negative coupled neural oscillator such as a ring and polygonal topology. We give the simulation results and investigate comparison with the coupled neural oscillators to van der Pol oscillators.

1. Introduction

The synchronization phenomena of coupled oscillators have been researched widely in the fields of communication systems, robot engineering, biological systems, social networks, and so forth, mainly using the van der Pol oscillator and the Kuramoto model. On the other hand, the brain network and other biological neural networks can also cause synchronization. The oscillation and synchronization phenomena of neural networks arise from neurons. However, coupled oscillator models based on neurons have not been investigated sufficiently. It is important to investigate oscillator models based on neurons in order to reveal various complex synchronization phenomena in the brain.

Artificial neural networks are calculation models that can replicate some functions of the human brain. In particular, a neural network with oscillation is applied to pattern recognition and walking rhythm outbreak circuits [1]-[2]. Meanwhile, neurons transmit signals through the synapses that act as a neurotransmitter each other. Synaptic connection methods are classified into two types: a chemical synapse and an electrical synapse, the so-called ”gap junction”. The chemical synapse denotes the general combination of a mathematic neuron model. By contrast, the electrical synapse functions such as a resistor coupling and promotes synchronization between the neurons.

Here the van der Pol circuit is one of the oscillation electric circuit. The coupled oscillator composed by some van der Pol circuits shows complex synchronization and oscillation phenomenon. In this paper, we propose negative coupled neural oscillators as a ring and polygonal topology. We discuss various synchronizations and compare with these phenomena of the coupled neural oscillators to the van der Pol oscillators [3][4].

2. Oscillation Circuit

2.1. Neural oscillator

First of all, we explain the general neural oscillator composed by two components. The general neuronal oscillator is shown in Fig. 1. This model is composed by the excitatory neuron N_1 and the inhibitory one N_2. The weight W_{21} is defined as the positive weight $W_{21} = W_{positive}$ and W_{12} is defined as the negative weight $W_{12} = W_{negative}$. Moreover, the neuron N_1 has a self-connection with the weight W_{11} of the positive weight $W_{11} = W_{positive}$. The dynamical system of this general model is described by Eq. (1). Here u is the internal value of each neuron and τ is the time constant. The output function is defined as the arctangent.

$$\begin{cases} \tau \dfrac{du_1}{dt} = -u_1 + W_{12} f(u_2) + W_{11} f(u_1) \\ \tau \dfrac{du_2}{dt} = -u_2 + W_{21} f(u_1) \\ f(u) = tan^{-1} u \end{cases} \tag{1}$$

Figure 2 shows oscillation of the general model with the weight $W_{positive} = 2.0$ and $W_{negative} = -2.0$. The neurons in this general model generates oscillation each.

The negative coupled neural oscillators are composed by some general oscillators coupled each other. In the coupled neural oscillator, each excitatory neuron N_{odd} is connected with the resister such as the electrical synapse. The equation of the electrical synapse is described by Eq. (2).

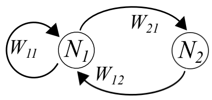


Figure 1: The general neural oscillator.

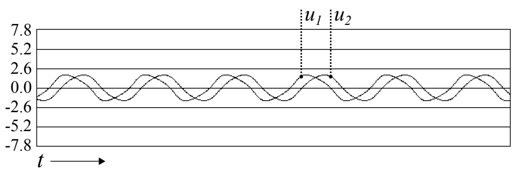


Figure 2: Oscillation of the general neural oscillator ($W_{positive} = 2.0$, $W_{negative} = -2.0$, $\tau = 2.0$).

Here the parameter W_{gap} denotes the weight of the electrical synapse such as a resistance value. V_{pre} is membrane potential of a previous neuron.

$$I_{gap} = -W_{gap}(U - U_{pre}) \tag{2}$$

The coupling with the general resister promotes synchronization. By contrast, we consider the negative coupled oscillators connected by the negative resistances. In the coupled neural oscillator, the excitatory neurons N_{odd} are connected each other to compose a topology. Figure 3 shows the negative coupling neural oscillator as a ring topology. In our simulations, the parameters of neural oscillators are defined as $W_{positive} = 3.0$, $W_{negative} = -3.0$, $W_{gap} = -0.18$, $\tau = 2.0$.

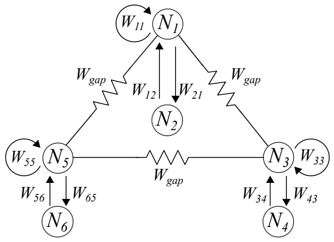


Figure 3: The negative coupled neural oscillator as a ring topology.

2.2. van der Pol oscillator

In this section, we introduce the van der Pol oscillator. As a characteristic, the van der Pol oscillator is similar to the neural oscillator. Figure 4 shows the negative coupled van der Pol oscillator as a ring topology. The resistance r is defined as a negative value. We assume that the $v_k - i_{rK}$ characteristics of the nonlinear resistor is given by the following equation

$$i_{rK} = -g_1 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0), (k = 1, 2, 3). \tag{3}$$

The normalized circuit equations governing the circuit in Fig. 4 are expressed as

$$\begin{cases} \dfrac{dx_k}{d\tau} = -y_k + \epsilon(1 - \dfrac{1}{3}x_k^2)x - \gamma(2x_k - x_{k+1} - x_{k-1}) \\ \dfrac{dy_{lk}}{d\tau} = x_k \qquad\qquad\qquad (k_0 = k_3, k_4 = k_1). \end{cases} \tag{4}$$

where

$$t = \sqrt{LC}\tau, v_k = \sqrt{\frac{g_1}{3g_3}}, i_{lk} = \sqrt{\frac{g_1}{3g_3}}\sqrt{\frac{L}{C}}y_l k,$$

$$\epsilon = g_1\sqrt{\frac{L}{C}}, \ \ \gamma = r\sqrt{\frac{C}{L}}.$$

Here γ denotes the nonlinearity of the oscillation and ϵ is the coupling strength. The resistor r is defined as the negative value. In our simulations, the parameters of van der pol oscillators are defined as $\epsilon = 0.1$, $\gamma = -0.1$.

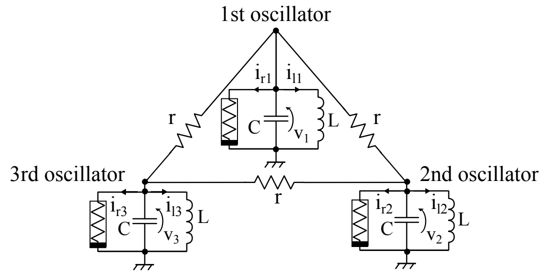


Figure 4: The negative coupled van der Pol oscillator as a ring topology.

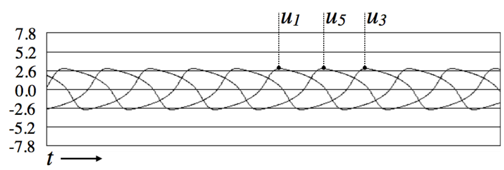
3. Simulation Results

In our simulation, we investigate dynamics of the models by the Runge-Kutta method of 4-order with the time distance $h = 0.05$.

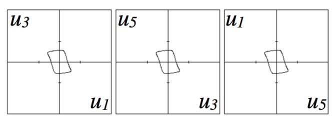
3.1. Ring topology

3.1.1. Neural oscillator

The negative coupled neural oscillator as a ring topology in Fig. 3 is composed by connecting the excitatory neurons N_1, N_3, and N_5. Figure 5(a) shows oscillations of the negative-coupled neural oscillator as a ring topology. The amplitude of each oscillation is larger and frequency is lower than one of the general neural oscillator. This neural oscillator generates oscillation with three-phase synchronization in the whole excitatory neurons.



(a) Oscillation (the excitatory neurons N_1, N_3 and N_5).



(b) Phase planes between the negative coupled neurons.

Figure 5: The behavior of the negative-coupled neural oscillator as a ring topology ($W_{gap} = -0.18$, $W_{positive} = 2.0$, $W_{negative} = -2.0$, $\tau = 2.0$).

3.1.2. van der Pol oscillator

The negative coupled van der Pol oscillator as a ring topology in Fig. 4 is composed by connecting van der Pol oscillator through the negative resisters. Figure 6 shows the oscillation of the this coupled oscillator. The oscillators generates oscillation with three-phase synchronization in the whole system. This synchronization phenomenon is common to the neural and the van der Pol oscillator. This synchronization phenomenon is similar to one of the coupled van der Pol oscillator in the references [3]. From the above, the negative coupled oscillators as a ring topology show three-phase synchronization in any models.

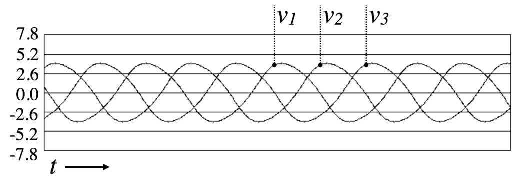


Figure 6: The oscillation of the van der Pol oscillator as a ring topology ($\epsilon = 0.1$, $\gamma = -0.1$).

3.2. Polygonal topology

In this section, we focus on the negative coupled oscillator as a polygonal topology. We propose the symmetric $3-3$ and the asymmetric $3-5$ polygonal topology.

3.2.1. $3-3$ *neural oscillator*

Figure 7 shows the negative coupled neural oscillator as a $3-3$ polygonal topology. The excitatory neurons N_1, N_3, N_5, and N_7 are connected each other through the negative coupling without a connection between the neurons N_1 to N_5. The connection between the neurons N_3 to N_7 is shared with two ring topology.

Figure 8 shows the behavior of the neural oscillator as $3-3$ polygonal topology. The oscillation phenomenon on this condition are shown in Figs. 8(a) and phase planes are shown in 8(b). The neurons neighbored each other without the neurons N_3 and N_7 synchronize with anti-phase state. By contrast, the excitatory neurons N_3 and N_7 cause synchronization phenomenon with in-phase. These phenomena can be confirmed by phase planes in 8(b).

3.2.2. $3-3$ *van der Pol oscillator*

Figure 9 shows the negative coupled van der Pol oscillator as a $3-3$ polygonal topology. The 1st and 3rd oscillator is also shared with two ring topology such as neural oscillator as a $3-3$ polygonal topology.

The behavior of the van der Pol oscillator as $3-3$ polygonal topology is shows in Fig. 10. This coupled oscillator also cause synchronization with the neighbor oscillators. The 2nd oscillator synchronized only the 4th oscillator with in-phase state, and synchronizations in other couplings show anti-phase state. This synchronization phenomenon is similar to one of the coupled van der Pol oscillator such as the references [4]. The negative coupled oscillators as a $3-3$ polygonal topology also show same synchronization regardless of each model.

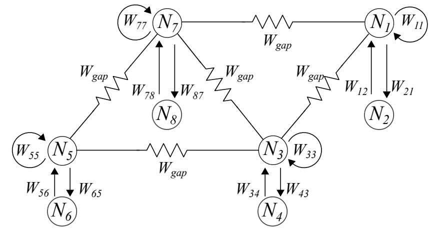
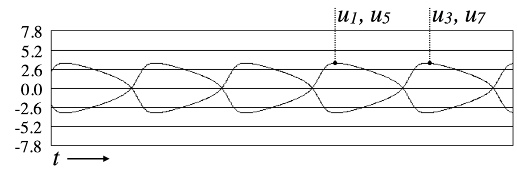
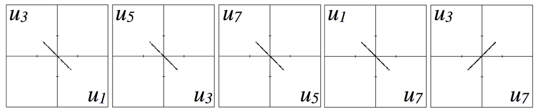


Figure 7: The negative coupled neural oscillator as $3-3$ polygonal topology.



(a) Oscillation (the excitatory neurons N_1, N_3, N_5 and N_7).



(b) Phase planes between the negative coupled neurons.

Figure 8: The behavior of the neural oscillator as a $3-3$ polygonal topology ($W_{gap} = -0.18$, $W_{positive} = 2.0$, $W_{negative} = -2.0$, $\tau = 2.0$).

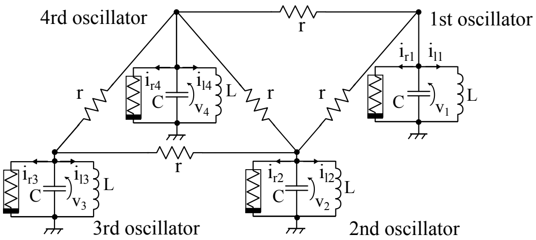


Figure 9: The van der Pol oscillator as a $3-3$ polygonal topology.

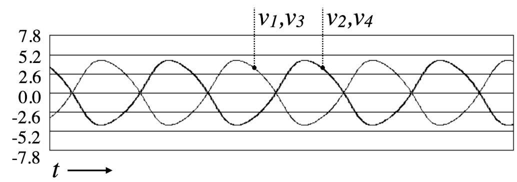


Figure 10: The oscillation of the van der Pol oscillator as a $3-3$ polygonal topology ($\epsilon = 0.1$, $\gamma = -0.1$).

3.2.3. $3-5$ *neural oscillator*

Figure 11 shows the negative coupled oscillator as a $3-5$ polygonal topology. We give the simulation results of the neural and van der Pol oscillator as this topology.

Figure 12 shows the behavior of the neural oscillator as a $3-3$ polygonal topology. As characteristics of the asymmetrical negative coupled neural oscillator as a polygonal topology, the shared neurons between two rings only synchronize with in-phase state. By contrast, other connections synchronize with anti-phase state.

In the case of a polygonal topology, coupled oscillator can synchronized with in-phase and anti-phase if both ring topologies are composed by odd neural oscillators. In other cases, each neural oscillator can not synchronize and increase the lineality.

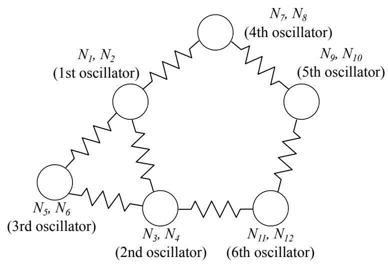
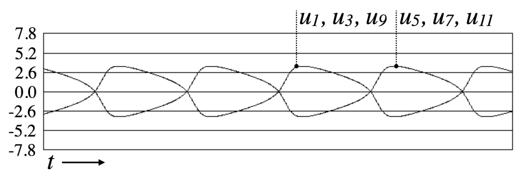
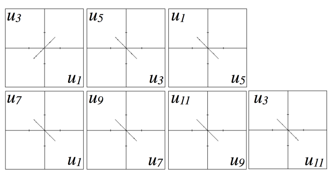


Figure 11: The negative coupled oscillator as a $3-5$ polygonal topology.



(a) Oscillation (the excitatory neurons N_1, N_3 and N_5).



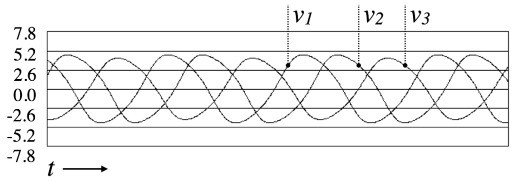
(b) Phase planes between negative-coupled neurons.

Figure 12: The behavior of the neural oscillator as a $3-5$ polygonal topology ($W_{gap} = -0.18$, $W_{positive} = 2.0$, $W_{negative} = -2.0$, $\tau = 2.0$).

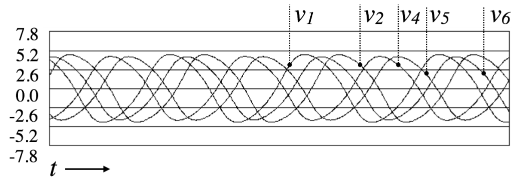
3.2.4. $3-5$ *van der Pol oscillator*

Figure 13 shows the behavior of the van der Pol oscillator as a $3-5$ polygonal topology. In the contrast to the previous neural oscillator, the van der Pol oscillator in this topology can not cause synchronization with in-phase and anti-phase. Each oscillation has various phase differences. These synchronized states close to the one of the coupled polygonal oscillatory networks in the references [4]. In this reference, it is introduced that the phase differences are caused in order to decrease the power consumption of the coupling resistors. These negative coupled neural oscillators are able to say the same thing.

In Ref [5], the power consumption in the whole system is introduces as a cause of the phase differences. Specifically, the phase difference at the shared connection and others are varied in order to make this power consumption minimum. However, the shared neurons are synchronized with in-phase in negative coupled neural oscillator as a polygonal topology synchronization, nevertheless the power consumption is minimum if it is 80 degrees. From the above, synchronizations of this coupled neural oscillator are not depended on the power consumption.



(a) Oscillation (the 1st, 2nd and 3rd oscillators).



(b) Oscillation (the 1st, 2nd, 4th, 5th and 6th oscillators).

Figure 13: The behavior of the van der Pol oscillator as a $3-5$ polygonal topology ($\epsilon = 0.1$, $\gamma = -0.1$).

4. Conclusion

In this study, we have focused on oscillations and synchronizations of negative coupled neural oscillators. Some negative coupled neural oscillators as a polygonal topology cause synchronization that can not be confirmed in the coupled van der Pol oscillators. On the other hand, neural oscillators as a ring topology shows similar synchronizations. In future, we will investigate general rules for the synchronization of each neuron and the oscillation conditions in detail.

Acknowledgment

This work was partly supported by JSPS Grant-in-Aid for Challenging Exploratory Research 26540127.

References

[1] D. Wang: Emergent synchrony in locally coupled neural oscillators, *IEEE Transactions on Neural Networks*, Vol.6, No.4, pp. 941-948, 1995.

[2] G. Endo, J. Morimoto, J. Nakanishi and G. Cheng: An empirical exploration of a neural oscillator for biped locomotion control, *Robotics and Automation*, Proceedings of ICRA, Vol.3, pp. 3036-3042, 2004.

[3] Y. Uwate, Y. Nishio, R. Stoop: Synchronization in three coupled van der Pol oscillators with different coupling strength. *2010 International Workshop on Nonlinear Circuits, Communication and Signal Processing*, pp.109-112, 2010.

[4] Y. Uwate, Y. Nishio: Synchronization in several types of coupled polygonal oscillatory networks. *IEEE Transactions on Circuits and Systems*, Vol.59, No. 5, pp.1042-1050, 2012.