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Bifurcation and Basin in Two Coupled Cubic Maps

Takuya Nishimoto*, Yoko Uwate*, Yasuteru Hosokawa**,
Yoshifumi Nishio*, Danièle Fournier-Prunaret† and Abdel-Kaddous Taha‡

*Dept. of Electrical and Electronic Eng., Tokushima University,
2-1 Minami-Josanjima, Tokushima, 770-8506 Japan
Email: {nishimoto, uwate, nishio}@ee.tokushima-u.ac.jp
**Dept. of Media and Information Systems, Shikoku University,
Furukawa, Ohjin, Tokushima, 771-1192, Japan
Email: hosokawa@keiei.shikoku-u.ac.jp
†LAAS-CNRS, INSA, University of Toulouse, France
Email: daniele.fournier@insa-toulouse.fr
‡ INSA, University of Toulouse, France
Email: taha@insa-toulouse.fr

Abstract—Some complex nonlinear phenomena have reported on the coupled chaotic system included double scroll family. It is considered that investigating bifurcation structures of the system is extremely important to clarify high-order complex nonlinear phenomena. In this study, we investigate bifurcation structures on two coupled cubic maps which belongs to double scroll family. By carrying out computation, bifurcation curves and basins are obtained.

1. Introduction

Chaotic phenomena observed on large-scale coupled chaotic systems have been attracted attentions in various science fields since it can be regarded as models of real physical system. For instance, chaos phenomena have been reported in engineering, biology, economics, astronomy and so on. Therefore investigation of chaotic system on high-order chaotic systems are very important to grasp essentials of the complex phenomena observed in natural system. On the other hand, some chaotic system generate multi-scroll attractor [1]. Complex nonlinear phenomena have reported in chaotic system which belongs to multi-scroll family. Moreover, the relationships between bifurcation structure and synchronization phenomena have reported [2] [3]. Therefore investigating bifurcation structures of multi-scroll family is useful to clarify complex nonlinear phenomena.

In this study, we investigate bifurcation structure and basins in a coupled cubic maps which generates double scroll attractor. By carrying out computation, complex bifurcation curves can be obtained. Be based on the bifurcation curves, we investigate the basins of multistability of fixed point. This paper is organized as follows. In Section 2, we propose coupled cubic maps. In Section 3, bifurcation curves on the system are investigated. Be based on Section 3, multistability of fixed point are shown. Finally, Section 4 concludes this work.

2. Coupled Cubic Map

In this study, we use a cubic map as the one-dimensional subsystem in the coupled chaotic system. A cubic map used in this study is described as:

$$x(n+1) = ax(n)(1 - x(n)^2). \tag{1}$$

Figure 1 shows chaotic orbit which can be observed in the system described with Eq. (1). The attractors are classified with colors. One-parameter bifurcation diagram and the Lyapunov exponent of the cubic map is obtained as shown in Fig. 2. By increasing the parameter a, pitchfork bifurcation and flip bifurcation can be observed in the one-dimensional map. The cubic map shown in Fig. 1 is cou-

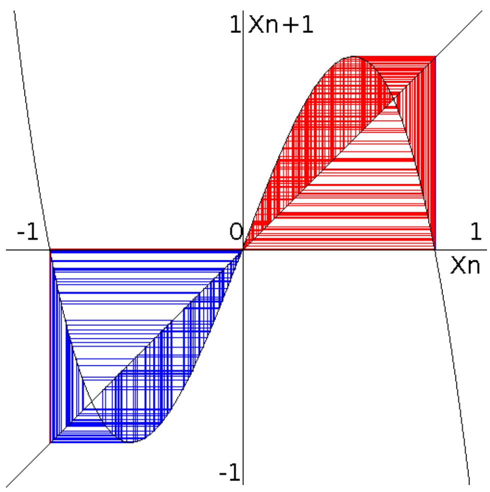
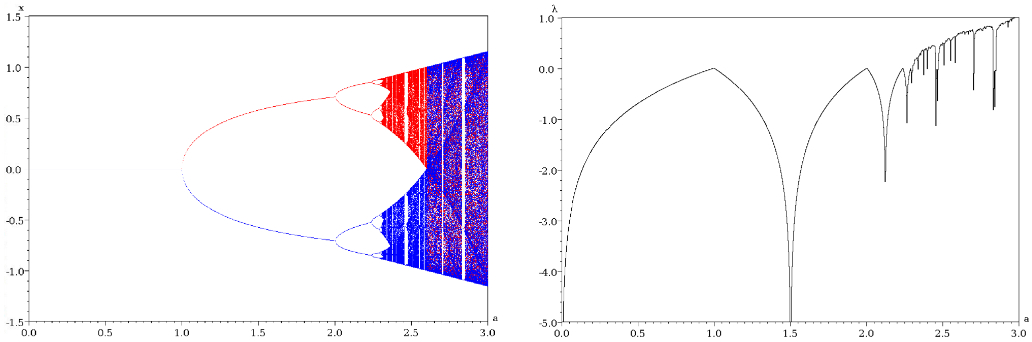


Figure 1: Chaotic orbit observed in a cubic map.



(1) (2)

Figure 2: (1) One-parameter bifurcation diagram. (2) The Lyapunov exponent. Horizontal axis: a.

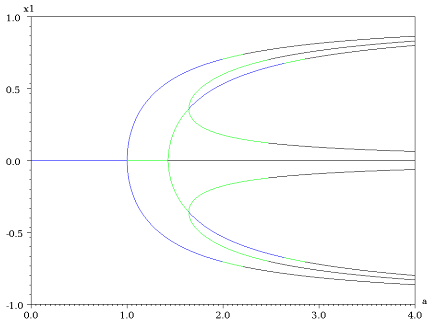


Figure 3: Parameter bifurcation diagram corresponds to the pitchfork bifurcation. Blue, green and black lines show stable node, saddle and unstable node respectively. $\varepsilon = 0.3$.

pled with the scheme of GCM which is described as:

$$x_i(n+1) = (1-\varepsilon)f[x_i(n)] + \frac{\varepsilon}{N}\sum_{j=1}^{2} f[x_j(n)] \qquad (2)$$

$$i = 1, 2$$

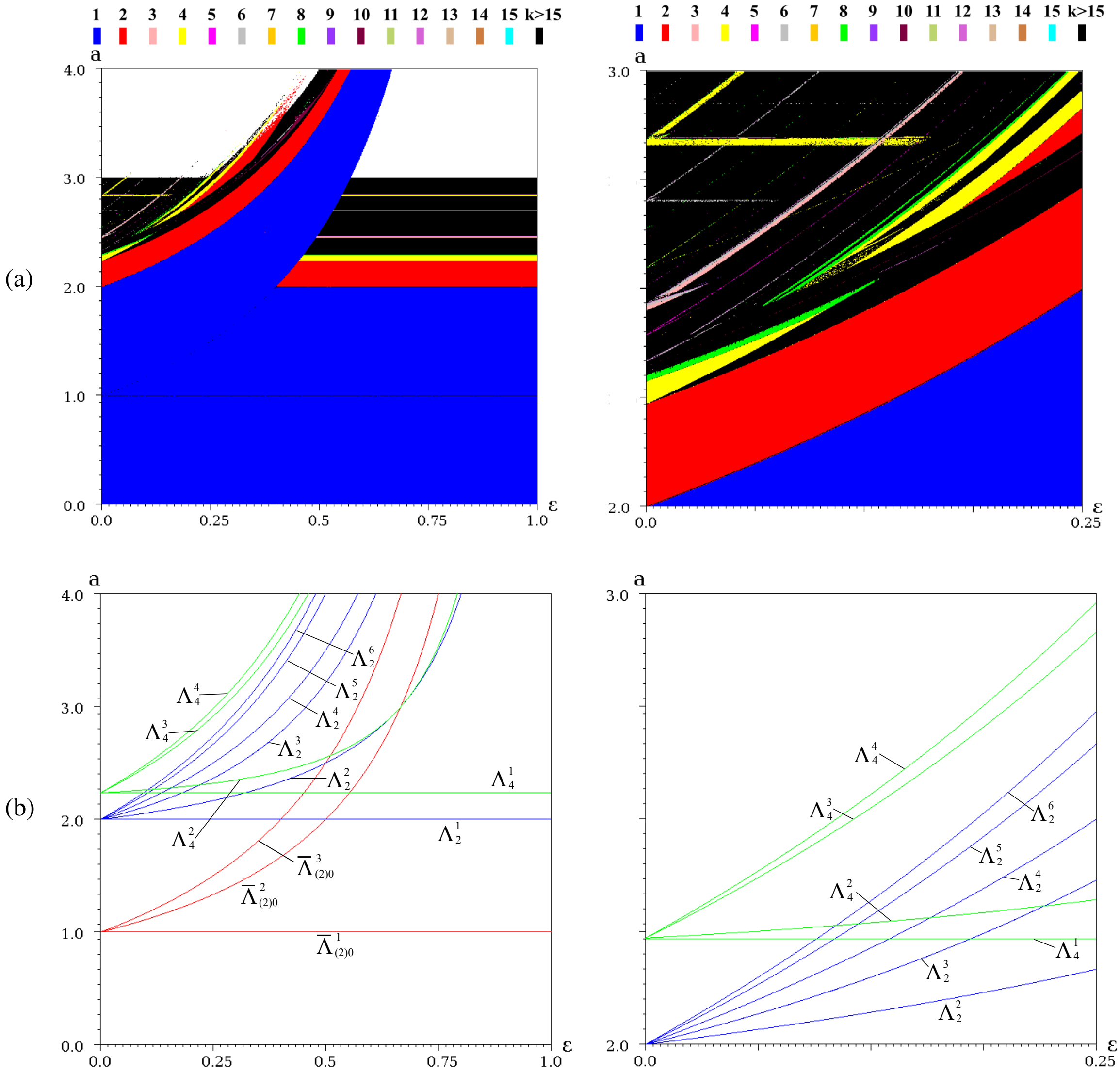
where $\varepsilon \in [0 : 1]$ is the coupling intensity. Then, the map T is described as:

$$T = \begin{cases} x_1(n+1) = (1-\frac{\varepsilon}{2})ax_1(n)(1-x_1(n)^2) \\ \qquad + \frac{\varepsilon}{2}ax_2(n)(1-x_2(n)^2) \\ x_2(n+1) = (1-\frac{\varepsilon}{2})ax_2(n)(1-x_2(n)^2) \\ \qquad + \frac{\varepsilon}{2}ax_1(n)(1-x_1(n)^2) \end{cases} \qquad (3)$$

3. Bifurcation

In this section, bifurcations are investigated with respect to parameter plane (a, ε). In the map T, pitchfork bifurcations of supercritical case and subcritical case are coexisting as shown in Fig. 3. Blue, green and black lines are corresponding to stable node, saddle and unstable node respectively. Figure 4 (a) shows a representation of the parameter plane and its enlargement. Each colored part shows the existence of stable cycle. In the figure some discontinuous boundaries of bifurcation are observed. The phenomena are caused due to multistability behaviors and Neïmark-Sacker bifurcation. Different attractors can coexist, depending on the choice of different initial conditions in a certain parameter region. Figure 4 (b) shows bifurcation curves corresponding to pitchfork bifurcation curves and flip bifurcation curves of order 2 and 4 cycles which can be derived with the eigenvalue of Jacobian matrix. The bifurcation curves regarding order 2 and 4 cycles are obtained analytically and given by:

$$\begin{aligned}
\bar{\Lambda}^1_{2(0)} &: a(\varepsilon) = 1 \\
\bar{\Lambda}^2_{2(0)} &: a(\varepsilon) = -\frac{1}{\varepsilon - 1} \\
\bar{\Lambda}^3_{2(0)} &: a(\varepsilon) = -\frac{1}{2}\frac{\varepsilon + 2}{\varepsilon - 1} \\
\Lambda^1_2 &: a(\varepsilon) = 2 \\
\Lambda^2_2 &: a(\varepsilon) = -\frac{1}{2}\frac{3\varepsilon - 4}{\varepsilon - 1} \\
\Lambda^3_2 &: a(\varepsilon) = \frac{0.25\varepsilon - 0.5 - 0.25\sqrt{25\varepsilon^2 - 36\varepsilon + 36}}{\varepsilon - 1} \\
\Lambda^4_2 &: a(\varepsilon) = \frac{1}{2}\frac{\varepsilon - 4}{\varepsilon - 1} \\
\Lambda^5_2 &: a(\varepsilon) = -\frac{2}{\varepsilon - 1} \\
\Lambda^6_2 &: a(\varepsilon) = \frac{1}{2}\frac{\sqrt{16 - 2\varepsilon^2 + 4\varepsilon}}{\varepsilon - 1} \\
\Lambda^1_4 &: a(\varepsilon) = \sqrt{5} \\
\Lambda^2_4 &: a(\varepsilon) = -\frac{1}{2}\frac{\sqrt{18\varepsilon^2 - 36\varepsilon + 20}}{\varepsilon - 1} \\
\Lambda^3_4 &: a(\varepsilon) = -\frac{1}{2}\frac{\sqrt{20 - 4\varepsilon - 2\varepsilon^2}}{\varepsilon - 1} \\
\Lambda^4_4 &: a(\varepsilon) = -\frac{\sqrt{5}}{\varepsilon - 1}
\end{aligned} \qquad (4)$$



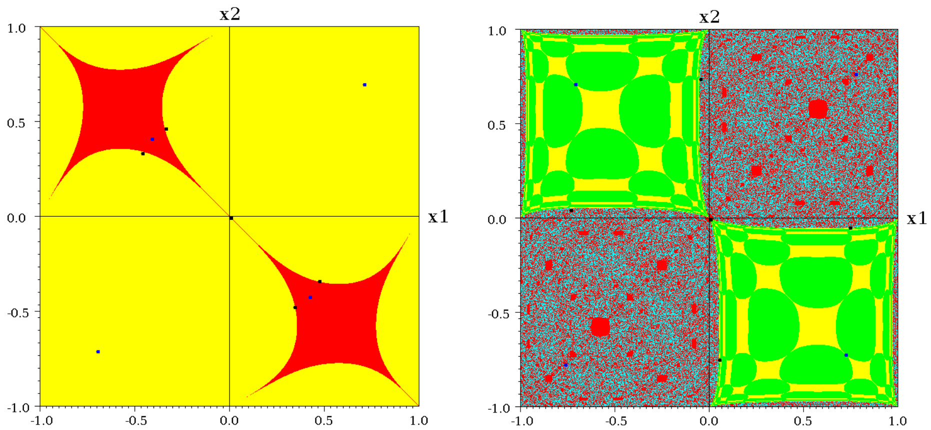
(1) (2)

Figure 4: Representation of the parameter plane (a, ε). Each colored part corresponds to the existence of a stable cycle (periodic point), the order k (period) of which is given by the upper side colored squares. The black color corresponds to $k > 15$ or to chaotic behavior. The white region corresponds to the nonexistence of attracting set, with chaotic transient toward infinity. Horizontal axis: ε. Vertical axis: a. (a) Existence of stable periodic orbit. (b) Bifurcation curves of (1). (2) Enlargement of (1).

where a pitchfork bifurcation curve is noted $\bar{\Lambda}_k^j$, and a flip bifurcation curve is noted Λ_k^j. Note that the k is the order of cycle and j differentiates cycles of same order Λ_k^j denotes flip bifurcation curves [5] [6]. From the representation and bifurcation curves, we found that a lot of multistability region are coexisting. In the next section, the basin which is an initial value set converging to a given orbit is investigated for parameter values corresponding to multistability.

4. Basin

Figure 5 shows basin when T has multistability. Each basin of attraction represented by different colors are corresponding to the stable periodic orbit. In Fig. 5 (1), two stable period 1 orbits are coexisting. Blue dots and black dots are corresponding to stable fixed point and unstable fixed point respectively. In another parameter sets, 2 stable period 2 orbits, a stable period 3 orbit, and another stable period are coexisting as shown in Fig. 5 (2). Similar shape of basin as shown in Fig. 5 (1) and complicated basin with fractal structure can be observed in the parameter set. The stable orbits observed in Fig. 5 (2) are shown in Fig. 6.



(1) (2)

Figure 5: Basin regarding to T. (1) $a = 1.99$, $\varepsilon = 0.39$. (2) $a = 2.46$, $\varepsilon = 0.16$.

5. Conclusion

In this study, we investigated a two-dimensional coupled cubic map which belongs to double scroll family. In the investigation of bifurcation curves, coexisting supercritical and subcritial pitchfork bifurcation are confirmed. Moreover, bifurcation curves regarding to order 2 pitchfork bifurcation and flip bifurcation curves and the basins are obtained on the parameter plane. In our future work, we will investigate Neïmark-Sacker bifurcation curve and foliated structure of the coupled cubic map.

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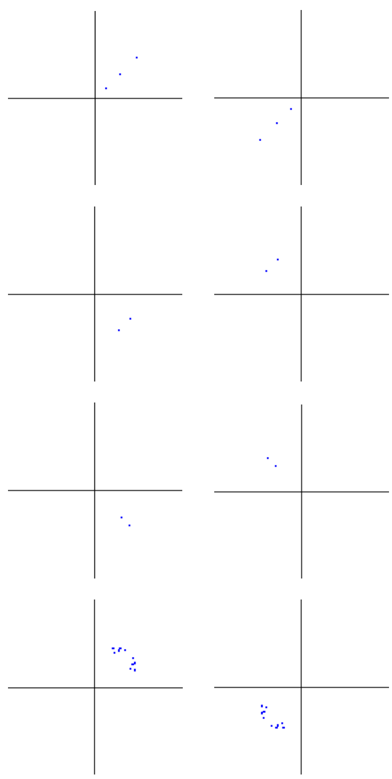

Figure 6: Coexisting stable period. The horizontal axis: x_1. The vertical axis: x_2. (1) $a = 1.99$, $\varepsilon = 0.39$ (2) $a = 2.46$, $\varepsilon = 0.16$.