Multimode Oscillations Coupled with an Inductor with High-order Nonlinearities

Masaaki Kojima† ‡, Yoko Uwate† and Yoshifumi Nishio†

†Department of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima, 770-8506, Japan
‡Engineering Administration Department, NHK
2-2-1 Jinnan, Shibuya-ku, Tokyo, 150-8001, Japan
Email: kojima@ee.tokushima-u.ac.jp, uwate@ee.tokushima-u.ac.jp, nishio@ee.tokushima-u.ac.jp

Abstract—We propose two van der Pol oscillators with high-order nonlinearities coupled with an inductor and investigate various types of multimode oscillations. We confirm that these oscillations are stably excited by numerical calculations. In particular, we show the multimode oscillations in the thirteenth and fifteenth-power oscillator systems (the coupled oscillators with thirteenth and fifteenth-power nonlinear characteristics).

1. Introduction

Coupled oscillator systems have brought improvements to natural science and fundamental research. Various types of synchronization phenomena occur in the oscillator systems. Hence, some studies of synchronization phenomena have been reported in electrical systems.

It is known that two oscillators coupled with an inductor (or a capacitor) excite two different modes: In-phase single-mode and anti-phase single-mode which both coexist for some range of parameter values when each negative resistor within the oscillator has third-power nonlinear characteristics [1]. The fifth-power oscillator systems excite four different modes: Zero-mode (non-oscillation), in-phase single-mode, anti-phase single-mode and double-mode, which coexist for some range of parameter values [2]. The double-mode means that two single-modes (in-phase and anti-phase) oscillate, simultaneously and asynchronously.

We investigated multimode oscillations when two van der Pol oscillators with high-order (seventh, ninth and eleventh-power) nonlinear characteristics were coupled with an inductor [3]. As a result, we could confirm some interesting multimode phenomena:

1) 2 in-phase, 2 anti-phase single-modes and 2 double-modes
2) zero-mode, 2 in-phase, 2 anti-phase single-modes and 4 double-modes
3) 3 in-phase, 3 anti-phase single-modes and 6 double-modes

in the seventh, ninth and eleventh-power oscillator systems, respectively, coexisted and were stably excited.

From these results, we could show that by conjecture:

1) j in-phase, j anti-phase single-modes and j^2-j double-modes
2) zero-mode, j in-phase, j anti-phase single-modes and j^2 double-modes

for N_nth and N_pth-power oscillator systems, respectively (where j=1, 2, $\cdots$, N_n=4j-1, N_p=4j+1).

In this study, we investigate some phenomena in the thirteenth and fifteenth-power oscillator systems by numerical calculations. We confirm that the number of stable mode in the thirteenth and fifteenth-power oscillator systems corresponds to this conjecture.

2. Circuit Model

Figure 1 shows a proposed circuit model. In this model, two van der Pol oscillators with Nth-power nonlinear characteristics are coupled with an inductor. The equations governing the circuit in Fig. 1 are described as follows: The

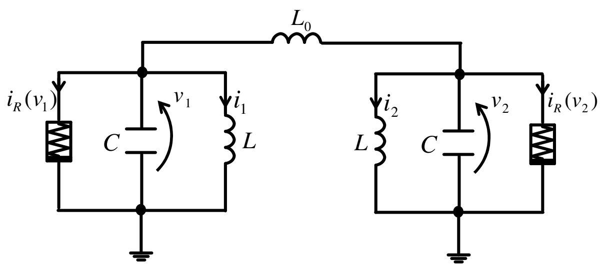

Figure 1: Circuit model.

equations governing the circuit in Fig. 1 are described as follows:

$$C\frac{dv_k}{dt} = \frac{L}{L_0}(i_1 - i_2)(-1)^k - i_k - i_R(v_k)$$
$$L\frac{di_k}{dt} = v_k. \qquad (k = 1, 2) \qquad (1)$$

$i_R(v_k)$ determines the $v - i$ characteristics of the Nth-power nonlinear resistor. It is approximated by the following function:

$$i_R(v_k) = (-1)^{(N-1)/2}(g_1 v_k - g_3 {v_k}^3 + g_5 {v_k}^5 - \cdots g_N {v_k}^N)$$
$$= (-1)^{(N-1)/2}\left(g_1 v_k + \sum_{m=1}^{(N-1)/2}(-1)^m g_{2m+1} {v_k}^{2m+1}\right).$$
$$(g_1, g_3, g_5, \cdots g_N > 0) \qquad (2)$$

Grouping constants and simplifying gives:

$$v_k = \left(\frac{g_1}{N g_N}\right)^{\frac{1}{N-1}} x_k, \ \ i_k = \sqrt{\frac{C}{L}}\left(\frac{g_1}{N g_N}\right)^{\frac{1}{N-1}} y_k,$$
$$(k = 1, 2)$$
$$\alpha = \frac{L}{L_0}\sqrt{\frac{L}{C}}, \ \ t = \sqrt{LC}\tau, \ \ \text{“}\cdot\text{”} = \frac{d}{d\tau}, \ \ \varepsilon = g_1\sqrt{\frac{L}{C}},$$
$$\varepsilon_{2m+1} = \begin{cases} 0 & \cdots (N < 2m+1) \\ 1 & \cdots (N = 2m+1) \\ \dfrac{(2m+1)g_{2m+1}}{g_1}\left(\dfrac{g_1}{N g_N}\right)^{\frac{m}{(N-1)/2}} & \cdots (N > 2m+1) \end{cases} \qquad (3)$$

where α is the coupling factor. ε, ε_3, $\cdots$, ε_{2m+1} are the coefficients of the nonlinear characteristics. Eq. (1) is normalized as follows:

$$\dot{x_k} = \alpha(y_1 - y_2)(-1)^k - y_k - f_R(x_k)$$
$$\dot{y_k} = x_k. \qquad (k = 1, 2) \qquad (4)$$

The function f_R corresponds to i_R and becomes:

$$f_R(x_k) = \varepsilon(-1)^{(N-1)/2}\left(x_k + \sum_{m=1}^{(N-1)/2}(-1)^m \varepsilon_{2m+1} {x_k}^{2m+1}\right).$$
$$(k = 1, 2) \qquad (5)$$

Eqs. (4) and (5) are the normalized equations of the circuit model and calculated by the Runge-Kutta method. The Runge-Kutta step size is 0.001.

3. Numerically Calculations

Figures 2 and 3 show numerically calculated results of the thirteenth and fifteenth-power oscillator systems, respectively, for different mode attractors and time waveforms. Each mode coexists at the same parameter values and is determined by the different initial values.

3.1. Thirteenth-power Oscillator Systems (N=13)

The zero-mode means that neither of the two oscillators is excited. The zero-mode always exists at the small initial values.

The in-phase single-mode has three different (small, medium and large amplitude) amplitudes at the same parameter values as shown in Figs. 2 (1)-(3). The thirteenth-power nonlinear resistor has three positive regions of symmetry about the origin. These regions dictate the different amplitudes.

The anti-phase single-mode also has three different amplitudes as shown in Figs. 2 (4)-(6).

The double-mode means that the above two single-mode oscillations (in-phase and anti-phase) are excited, simultaneously and asynchronously. The types of double-modes are determined by the combinations of their single-modes with different amplitudes, which have nine types ((in-phase, anti-phase) = (small, small), (medium, medium), (large, large), (medium, small), (large, medium), (large, small), (small, medium), (medium, large), (small, large)) [Figs. 2 (7)-(15)].

3.2. Fifteenth-power Oscillator Systems (N=15)

The zero-mode does not exist in the fifteenth-power nonlinearities since the nonlinear characteristics near the origin are negative.

The in-phase single-mode has four different (A_1 - A_4) amplitudes at the same parameter values as shown in Figs. 3 (1)-(4). The fifteenth-power nonlinear resistor has four positive regions of symmetry about the origin.

The anti-phase single-mode also has four different amplitudes as shown in Figs. 3 (5)-(8).

The double-mode has twelve types ((in-phase, anti-phase) = (A_2, A_1), (A_3, A_1), (A_4, A_1), (A_3, A_2), (A_4, A_2), (A_4, A_3), (A_1, A_2), (A_1, A_3), (A_1, A_4), (A_2, A_3), (A_2, A_4), (A_3, A_4)) ($A_1 < A_2 < A_3 < A_4$) [Figs. 3 (9)-(20)].

4. Conclusions

As numerically calculated results of this study, we confirmed that
1) zero-mode, 3 in-phase, 3 anti-phase single-modes and 9 double-modes
2) 4 in-phase, 4 anti-phase single-modes and 12 double-modes
in the thirteenth and fifteenth-power oscillator systems, respectively, coexisted and were stably excited.

Therefore, we could confirm that our conjecture corresponded to the actual calculated results in the the thirteenth and fifteenth-power oscillator systems.

References

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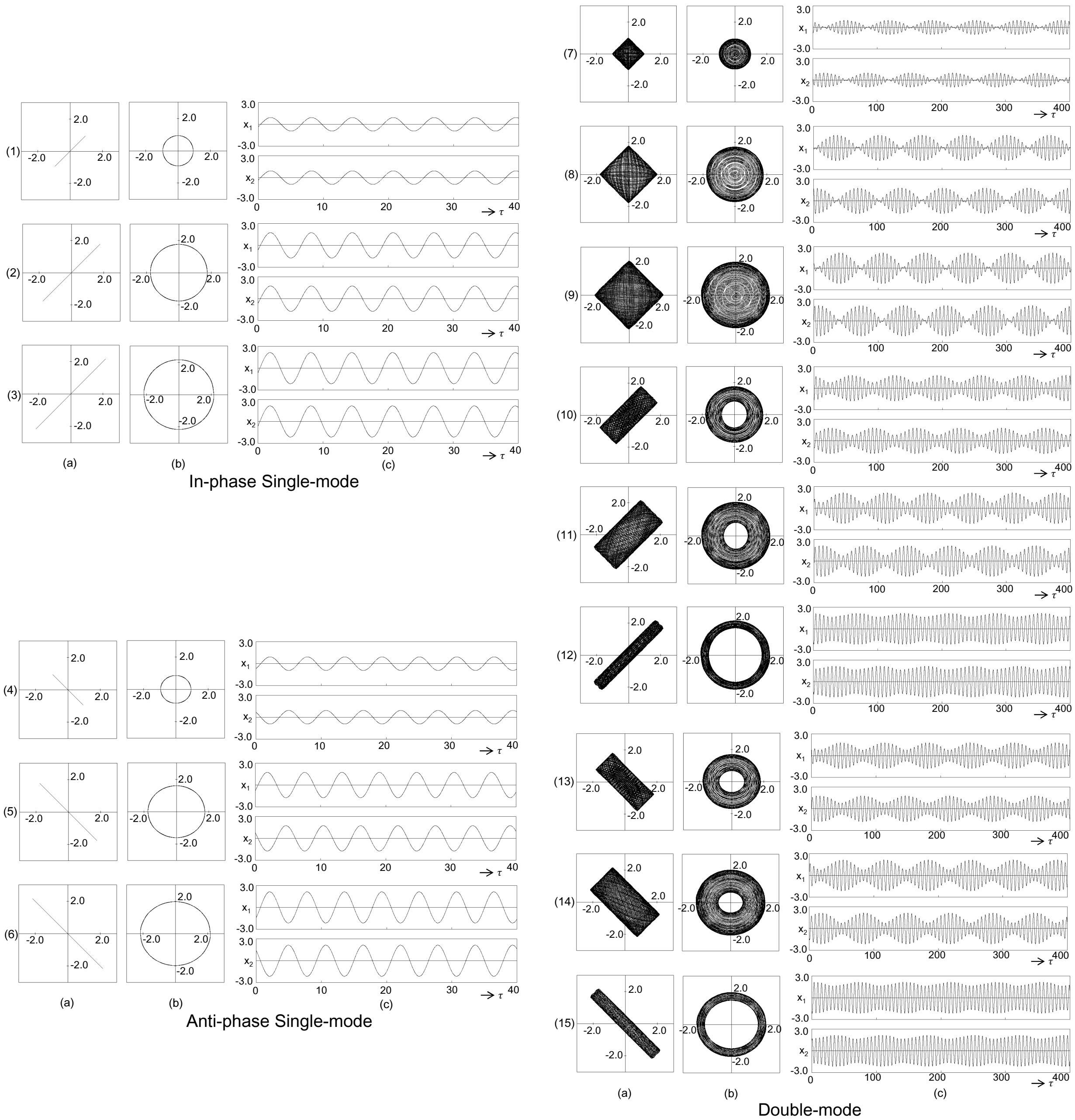

Figure 2: Numerically calculated results of the thirteenth-power oscillator systems (N=13). $\alpha = 0.1$, $\varepsilon = 0.1$, $\varepsilon_3 = 23.13$, $\varepsilon_5 = 81.42$, $\varepsilon_7 = 98.1$, $\varepsilon_9 = 50.99$, $\varepsilon_{11} = 11.81$. (a) $x_1 - x_2$ attractors. (b) $x_1 - y_1$ attractors. (c) x_1 & x_2 time waveforms. (1) In-phase single-mode (small amplitude) (initial values: $x_1 = 0.5$, $x_2 = 0.6$, $y_1 = 0.5$, $y_2 = 0.6$). (2) In-phase single-mode (medium amplitude) (initial values: $x_1 = 1.0$, $x_2 = 1.3$, $y_1 = 1.5$, $y_2 = 1.4$). (3) In-phase single-mode (large amplitude) (initial values: $x_1 = 2.0$, $x_2 = 2.2$, $y_1 = 2.0$, $y_2 = 2.1$). (4) Anti-phase single-mode (small amplitude) (initial values: $x_1 = 0.5$, $x_2 = -0.6$, $y_1 = 0.5$, $y_2 = -0.6$). (5) Anti-phase single-mode (medium amplitude) (initial values: $x_1 = 1.0$, $x_2 = -1.0$, $y_1 = 1.4$, $y_2 = -1.4$). (6) Anti-phase single-mode (large amplitude) (initial values: $x_1 = 2.0$, $x_2 = -2.2$, $y_1 = 2.0$, $y_2 = -2.1$). (7) Double-mode (small amplitudes) (initial values: $x_1 = -0.4$, $x_2 = 0.5$, $y_1 = 0.6$, $y_2 = 0.4$). (8) Double-mode (medium amplitudes) (initial values: $x_1 = -0.9$, $x_2 = 1.0$, $y_1 = 0.8$, $y_2 = 1.1$). (9) Double-mode (large amplitudes) (initial values: $x_1 = -2.0$, $x_2 = 2.1$, $y_1 = 2.0$, $y_2 = 2.1$). (10) Double-mode (in-phase-medium and anti-phase-small amplitudes) (initial values: $x_1 = -0.3$, $x_2 = 0.3$, $y_1 = 1.3$, $y_2 = 1.3$). (11) Double-mode (in-phase-large and anti-phase-medium amplitudes) (initial values: $x_1 = -0.5$, $x_2 = 0.5$, $y_1 = 1.8$, $y_2 = 1.8$). (12) Double-mode (in-phase-large and anti-phase-small amplitudes) (initial values: $x_1 = -0.6$, $x_2 = 0.4$, $y_1 = 2.1$, $y_2 = 2.0$). (13) Double-mode (in-phase-small and anti-phase-medium amplitudes) (initial values: $x_1 = -1.3$, $x_2 = 1.3$, $y_1 = 0.3$, $y_2 = 0.3$). (14) Double-mode (in-phase-medium and anti-phase-large amplitudes) (initial values: $x_1 = -2.5$, $x_2 = 2.5$, $y_1 = 0.8$, $y_2 = 0.8$). (15) Double-mode (in-phase-small and anti-phase-large amplitudes) (initial values: $x_1 = -2.1$, $x_2 = 2.0$, $y_1 = 0.6$, $y_2 = 0.4$).

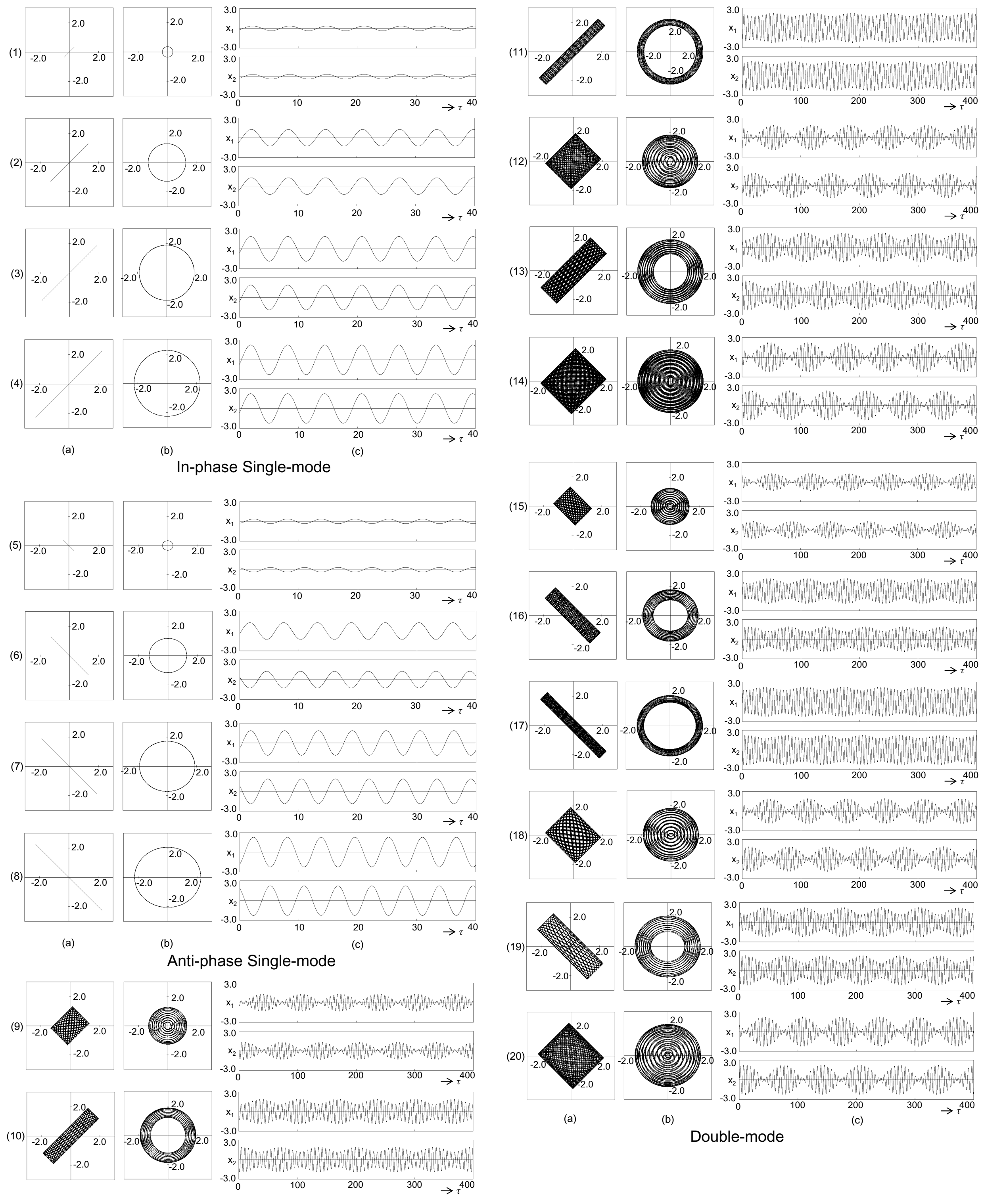

Figure 3: Numerically calculated results of the fifteenth-power oscillator systems (N=15). α = 0.1, ε = 0.01, ε_3 = 42.45, ε_5 = 203.48, ε_7 = 328.6, ε_9 = 239.04, ε_{11} = 86.073, ε_{13} = 14.96. (1) IM (A_1) (initial values: x_1 = 0.5, x_2 = 0.6, y_1 = 0.5, y_2 = 0.6) (2) IM (A_2) (1.0, 1.0, 1.1, 1.1) (3) IM (A_3) (1.3, 1.3, 1.5, 1.4) (4) IM (A_4) (2.0, 2.2, 2.0, 2.1) (5) AM (A_1) (0.5, −0.6, 0.5, −0.6) (6) AM (A_2) (1.0, −1.0, 1.1, −1.1) (7) AM (A_3) (1.3, −1.3, 1.5, −1.4) (8) AM (A_4) (2.0, −2.2, 2.0, −2.1) (9) DM (in-phase-A_2 and anti-phase-A_1) (−0.9, 0.5, 1.0, 1.0) (10) DM (A_3 and A_1) (−0.6, 0.3, 1.4, 1.5) (11) DM (A_4 and A_1) (−0.5, 0.3, 2.1, 2.0) (12) DM (A_3 and A_2) (−0.9, 0.5, 1.4, 1.0) (13) DM (A_4 and A_2) (−0.6, 0.4, 2.1, 2.0) (14) DM (A_4 and A_3) (−1.4, 1.5, 2.2, 2.2) (15) DM (A_1 and A_2) (−0.5, 0.6, 0.5, 0.6) (16) DM (A_1 and A_3) (0.6, 0.3, −1.4, 1.5) (17) DM (A_1 and A_4) (0.6, 0.4, −2.1, 2.0) (18) DM (A_2 and A_3) (0.9, 0.5, −1.4, 1.0) (19) DM (A_2 and A_4) (0.7, 0.5, −2.1, 2.0) (20) DM (A_3 and A_4) (1.4, 1.5, −2.2, 2.2) (IM:In-phase single-mode, AM:Anti-phase single-mode, DM:Double-mode) ($A_1 < A_2 < A_3 < A_4$: amplitude).