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Modified Capon Beamformer for High-Resolution Direction-of-Arrival Estimation

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Abstract—In this paper, a modified Capon beamformer (MCB) is proposed for estimating the direction-of-arrivals (DOAs) of multiple noncoherent narrowband signals, where the computationally expensive eigendecomposition is not required. Further the relation between this nonparametric estimator and the well-known parametric method called the multiple signal classification (MUSIC) is studied, where it is clarified that by increasing the power of the inverse of the array covariance matrix, the resolution of the MCB can be improved significantly compared with the standard Capon beamformer (SCB) and as better as that of MUSIC regardless of the signal-to-noise ratio (SNR). The effectiveness of the proposed MCB and the performance analysis are verified through numerical examples.

1. Introduction

Direction-of-arrival (DOA) estimation of multiple narrowband signals incoming on an array of sensors is of great importance in a variety of applications, such as radar, sonar, astronomy, and seismology (e.g., [1]–[2]). The beamforming is one of the oldest ideas for DOA estimation in array processing [3], and perhaps the most well-known one is the Capon beamformer [4]. The standard Capon beamformer (SCB) has the advantage that any assumption about the statistical properties of the array data is not required and hence it can be used in situations, where the information about these properties is unavailable (e.g., [2]), but it does not have the best resolution as the parametric methods such as the multiple signal classification (MUSIC) [5], which is a relatively simple and efficient subspace-based DOA estimation method and the large sample realization of the maximum likelihood (ML) method in the presence of uncorrelated incident signals [6]. Furthermore, the SCB can be applied to the array with arbitrary geometrical configurations and is computationally efficient than the (spectral) MUSIC method, where the computationally intensive eigendecomposition is involved besides the one-dimensional (1-D) searching (cf., [1], [12]), and it may become a tremendous computational burden in the practice applications of arrays with large numbers of sensors. However, the DOA estimation performance of the SCB degrades significantly and it is difficult to distinguish the closely spaced incident signals, when the signal-to-noise ratio (SNR) is low or the number of snapshots is small.

The relation between the Capon beamformer and the MUSIC method was studied in [7], where it was shown that the resolution of the Capon beamformer can be equal to that of MUSIC when the SNR tends to infinite. In an attempt to alleviate the limitations of the SCB, some modifications were proposed [8]–[12]. However, there are no existing techniques for selecting the weighting matrix [12] and the weight parameter [10], while the eigendecomposition is required in [9], and Cholesky decomposition and the selection of LP model order are needed in [11].

Therefore in this paper, we investigate the DOA estimation of multiple noncoherent narrowband signals in a computationally efficient way and propose a new modified Capon beamformer (MCB) to overcome the limitation of the SCB, where the eigendecomposition is avoided. Further the relation between the MCB and the MUSIC method is clarified explicitly, where it is shown that by increasing the power of the inverse of the array covariance matrix, the effect of signal components in the spatial spectrum can be reduced and hence the resolution of the MCB can be improved significantly compared with the SCB and as better as that of the MUSIC method regardless of the SNR. The effectiveness of the proposed MCB is verified through numerical examples.

2. Problem Formulation

We consider an array composed of M sensors with the intersensor spacing d and p ($p < M$) noncoherent narrowband signals $\{s_k(n)\}_{k=1}^{p}$ with the wavelength λ incoming from far-filed along the distinct directions $\{\theta_k\}_{k=1}^{p}$. The received array data at the nth snapshot can be expressed as

$$\boldsymbol{y}(n) = \boldsymbol{A}(\theta)\boldsymbol{s}(n) + \boldsymbol{w}(n) \quad (1)$$

where $\boldsymbol{y}(n)$, $\boldsymbol{s}(n)$ and $\boldsymbol{w}(n)$ are the vectors of the received noisy data, incoming signals and additive noise given by $\boldsymbol{y}(n) \triangleq [y_1(n), y_2(n), \cdots, y_M(n)]^T$, $\boldsymbol{s}(n) \triangleq [s_1(n), s_2(n), \cdots, s_p(n)]^T$ and $\boldsymbol{w}(n) \triangleq [w_1(n), w_2(n), \cdots, w_M(n)]^T$, $\boldsymbol{A}(\theta)$ is the array response matrix given by $\boldsymbol{A}(\theta) \triangleq [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_p)]$, while the array response vector $\boldsymbol{a}(\theta_k)$ is related with the array configuration, and $(\cdot)^T$ denotes the transpose. For the uniform linear array (ULA), $\boldsymbol{a}(\theta_k) = [1, e^{j\tau(\theta_k)}, \cdots, e^{j(M-1)\tau(\theta_k)}]^T$, and $\tau(\theta_k) \triangleq 2\pi d \sin(\theta_k)/\lambda$.

We make the basic assumptions that the incoming signals $\{s_k(n)\}_{k=1}^p$ are temporally complex Gaussian random process with zero-mean and mutually uncorrelated. The additive noises $\{w_m(n)\}_{m=1}^M$ are temporally and spatially complex white Gaussian random process with zero-mean and variance σ^2 and uncorrelated with the incoming signals. The number of incoming signals p is known or estimated in advance by using the detection methods (cf., [13]).

Under the basic assumptions, from (1), we easily obtain the covariance matrix $\boldsymbol{R}$ of the received array data

$$\boldsymbol{R} \triangleq E\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\} = \boldsymbol{A}(\theta)\boldsymbol{R}_s\boldsymbol{A}^H(\theta) + \sigma^2\boldsymbol{I}_M. \quad (2)$$

In practice, where the finite snapshots of array data are available, $\boldsymbol{R}$ is unavailable and usually replaced by its sample estimate $\hat{\boldsymbol{R}}$ given by

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{y}(n)\boldsymbol{y}^H(n) \quad (3)$$

where N is the number of snapshots.

3. Modified Capon Beamformer for DOA Estimation

3.1. Standard Capon Beamformer [4]

The SCB [4] is a nonparametric DOA estimator, which selects the weight vector $\bar{\boldsymbol{w}}$ to maintain a fixed gain at a "look direction" while to maximally reject the signals coming from other directions by using the following constrained quadratic problem [4]

$$\min_{\bar{\boldsymbol{w}}} \bar{\boldsymbol{w}}^H\boldsymbol{R}\bar{\boldsymbol{w}} \quad \text{subject to} \quad \bar{\boldsymbol{w}}^H\boldsymbol{a}(\theta) = 1. \quad (4)$$

By exploiting some techniques such as Lagrange optimization method, when the number of snapshots is finite, we can obtain the weight vector $\bar{\boldsymbol{w}}_{\text{SCB}}$ as

$$\bar{\boldsymbol{w}}_{\text{SCB}} = \frac{\hat{\boldsymbol{R}}^{-1}\boldsymbol{a}(\theta)}{\boldsymbol{a}^H(\theta)\hat{\boldsymbol{R}}^{-1}\boldsymbol{a}(\theta)}. \quad (5)$$

Noting that the presence of additive noise will ensure $\hat{\boldsymbol{R}}$ is invertible, the DOAs $\{\theta_k\}_{k=1}^p$ can be estimated from the locations of the p highest peaks of the Capon spatial spectrum (i.e., array output power) $P_{\text{SCB}}(\theta)$ given by

$$P_{\text{SCB}}(\theta) = \frac{1}{f_{\text{SCB}}(\theta)} \quad (6)$$

where $f_{\text{SCB}}(\theta) \triangleq \boldsymbol{a}^H(\theta)\hat{\boldsymbol{R}}^{-1}\boldsymbol{a}(\theta)$.

3.2. Modified Capon Beamformer

Now by replacing $\boldsymbol{R}$ in the standard Capon constraint function in (4) with $\boldsymbol{R}^m$, where m is a positive integer (i.e., $m \geq 1$), we can obtain a MCB to design the optimal weight vector $\bar{\boldsymbol{w}}$ by solving the following problem

$$\min_{\bar{\boldsymbol{w}}} \bar{\boldsymbol{w}}^H\boldsymbol{R}^m\bar{\boldsymbol{w}} \quad \text{subject to} \quad \bar{\boldsymbol{w}}^H\boldsymbol{a}(\theta) = 1. \quad (7)$$

Similarly for the case of finite snapshots, the solutions to the weight vector and the MCB spatial spectrum are given by

$$\bar{\boldsymbol{w}}_{\text{MCB}} = \frac{\hat{\boldsymbol{R}}^{-m}\boldsymbol{a}(\theta)}{\boldsymbol{a}^H(\theta)\hat{\boldsymbol{R}}^{-m}\boldsymbol{a}(\theta)} \quad (8)$$

$$P_{\text{MCB}}(\theta) = \frac{1}{f_{\text{MCB}}(\theta)} \quad (9)$$

where $f_{\text{MCB}}(\theta)$ is the MCB cost function defined by

$$f_{\text{MCB}}(\theta) \triangleq \boldsymbol{a}^H(\theta)\hat{\boldsymbol{R}}^{-m}\boldsymbol{a}(\theta). \quad (10)$$

Apparently the DOAs $\{\theta_k\}_{k=1}^p$ can be estimated by maximizing the spectrum $P_{\text{MCB}}(\theta)$ in (9) without the procedure of eigendecomposition. Note that the proposed MCB in (9) reduces to the SCB in (6) when $m = 1$, while it turns into the modified Capon method considered in [8] and the improved Capon method with $\boldsymbol{W} = \boldsymbol{I}_M$ suggested in [12] when $m = 2$.

4. Mathematical Link between MCB and MUSIC

Here we focus on examining the relationship between the standard and modified Capon beamformers and the subspace-based MUSIC method [5].

4.1. MUSIC Method [5]

From (2), the eigenvalue decomposition (EVD) of the covariance matrix $\boldsymbol{R}$ is given by

$$\boldsymbol{R} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{U}^H = \boldsymbol{U}_s\boldsymbol{\Sigma}_s\boldsymbol{U}_s^H + \boldsymbol{U}_n\boldsymbol{\Sigma}_n\boldsymbol{U}_n^H \quad (11)$$

where $\boldsymbol{U} \triangleq [\boldsymbol{U}_s, \boldsymbol{U}_n]$, $\boldsymbol{U}_s \triangleq [\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_p]$, $\boldsymbol{U}_n \triangleq [\boldsymbol{u}_{p+1}, \boldsymbol{u}_{p+2}, \cdots, \boldsymbol{u}_M]$, $\boldsymbol{\Sigma} \triangleq \text{diag}[\boldsymbol{\Sigma}_s, \boldsymbol{\Sigma}_n]$, $\boldsymbol{\Sigma}_s \triangleq \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_p)$, $\boldsymbol{\Sigma}_s \triangleq \text{diag}(\lambda_{p+1}, \lambda_{p+2}, \cdots, \lambda_M)$, and $(\boldsymbol{u}_i, \lambda_i)$ is the ith eigenpair of $\boldsymbol{R}$ with $\lambda_1 \geq \cdots \geq \lambda_p \geq \lambda_{p+1} = \cdots = \lambda_M = \sigma^2$, while $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ correspond to the signal and noise subspaces, and $\boldsymbol{U}\boldsymbol{U}^H = \boldsymbol{I}_M$. By utilizing the properties that $\boldsymbol{U}_s$ and $\boldsymbol{A}$ have the same column space and the noise subspace $\boldsymbol{U}_n$ is their orthogonal complement, i.e., $\boldsymbol{U}_n^H\boldsymbol{A} = \boldsymbol{O}_{(M-p)\times p}$, when the number of snapshots is finite, the (spectral) MUSIC method estimates the DOAs by searching the p highest peaks of the following "spatial spectrum" given by

$$P_{\text{MUSIC}}(\theta) = \frac{1}{f_{\text{MUSIC}}(\theta)} \quad (12)$$

where $f_{\text{MUSIC}}(\theta)$ is the MUSIC cost function defined by

$$f_{\text{MUSIC}}(\theta) \triangleq \boldsymbol{a}^H(\theta)\hat{\boldsymbol{U}}_n\hat{\boldsymbol{U}}_n^H\boldsymbol{a}(\theta) = \sum_{i=p+1}^{M}|\boldsymbol{a}^H(\theta)\hat{\boldsymbol{u}}_i|^2 = \sum_{i=p+1}^{M} f_i(\theta) \quad (13)$$

where $f_i(\theta) \triangleq |\boldsymbol{a}^H(\theta)\hat{\boldsymbol{u}}_i|^2$.

The performance of the MUSIC method has been studied extensively (e.g., [14]), and it is known that the MUSIC method is statistically efficient in cases when either the number of snapshots or the SNR is sufficiently large, but its computational cost is high due to the procedure of the eigendecomposition. The MUSIC method has higher resolution than the SCB does mainly because the utilization of the property that the noise subspace is orthogonal to the columns of $\boldsymbol{A}(\theta)$.

4.2. Relation Between MCB and MUSIC

According to (11), the inverse of the array covariance matrix $\boldsymbol{R}$ can be obtained

$$\boldsymbol{R}^{-1} = \boldsymbol{U}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^H = \boldsymbol{U}_s\boldsymbol{\Sigma}_s^{-1}\boldsymbol{U}_s^H + \sigma^{-2}\boldsymbol{U}_n\boldsymbol{U}_n^H. \quad (14)$$

Hence from (14), we easily have

$$\boldsymbol{R}^{-m} = \boldsymbol{U}\boldsymbol{\Sigma}^{-m}\boldsymbol{U}^H = \boldsymbol{U}_s\boldsymbol{\Sigma}_s^{-m}\boldsymbol{U}_s^H + \sigma^{-2m}\boldsymbol{U}_n\boldsymbol{U}_n^H. \quad (15)$$

Then when the number of snapshots is finite, the EVD of the sample estimate $\hat{\boldsymbol{R}}$ in (3) is given by $\hat{\boldsymbol{R}} = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{U}}^H$, where $\hat{\boldsymbol{\Sigma}} = \text{diag}(\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_M)$, and $\hat{\lambda}_1 \geq \cdots > \hat{\lambda}_p \geq \hat{\lambda}_{p+1} \geq \cdots \geq \hat{\lambda}_M > 0$, and hence the MCB cost function $f_{\text{MCB}}(\theta)$ in (10) can be expressed as

$$\begin{aligned} f_{\text{MCB}}(\theta) &= \boldsymbol{a}^H(\theta)\Big(\hat{\boldsymbol{U}}_s\hat{\boldsymbol{\Sigma}}_s^{-m}\hat{\boldsymbol{U}}_s^H + \sum_{i=p+1}^{M} \hat{\lambda}_i^{-m}\hat{\boldsymbol{u}}_i\hat{\boldsymbol{u}}_i^H\Big)\boldsymbol{a}(\theta) \\ &= \boldsymbol{a}^H(\theta)\hat{\boldsymbol{U}}_s\hat{\boldsymbol{\Sigma}}_s^{-m}\hat{\boldsymbol{U}}_s^H\boldsymbol{a}(\theta) + \sum_{i=p+1}^{M} \hat{\lambda}_i^{-m} f_i(\theta). \quad (16) \end{aligned}$$

Further when the number of snapshots is sufficiently large, we have $\lim_{N\to\infty} \hat{\lambda}_i = \sigma^2$ $(i = p+1, \cdots, M)$, and in the case of SNR $\neq \infty$ (i.e., $\sigma^2 \neq 0$), from (13), (15) and (16), we easily obtain

$$\begin{aligned} \lim_{N\to\infty} f_{\text{MCB}}(\theta) &= \boldsymbol{a}^H(\theta) \lim_{N\to\infty} \Big(\hat{\boldsymbol{U}}_s\hat{\boldsymbol{\Sigma}}_s^{-m}\hat{\boldsymbol{U}}_s^H\Big)\boldsymbol{a}(\theta) \\ &\quad + \sum_{i=p+1}^{M} \Big(\lim_{N\to\infty} \hat{\lambda}_i^{-m}\Big)\Big(\lim_{N\to\infty} f_i(\theta)\Big) \\ &= \sigma^{-2m}\Big(\boldsymbol{a}^H(\theta)\boldsymbol{U}_s\bar{\boldsymbol{\Sigma}}_s^{-m}\boldsymbol{U}_s^H\boldsymbol{a}(\theta) + \lim_{N\to\infty} f_{\text{MUSIC}}(\theta)\Big) \end{aligned} \quad (17)$$

where $\bar{\boldsymbol{\Sigma}}_s \triangleq \text{diag}(\bar{\lambda}_1, \bar{\lambda}_2, \cdots, \bar{\lambda}_p)$, and $\bar{\lambda}_i \triangleq \lambda_i/\sigma^2$. Evidently from (16) and (17), we can find that the MCB cost function (and hence that of the SCB with $m = 1$) consists of two terms, where the first one involves the information of signal subspace, while the latter one corresponds to the MUSIC cost function. Consequently the existence of the term associated with the signal subspace causes the estimation performance of the Capon method in (6) worse than that of the MUSIC method in (12).

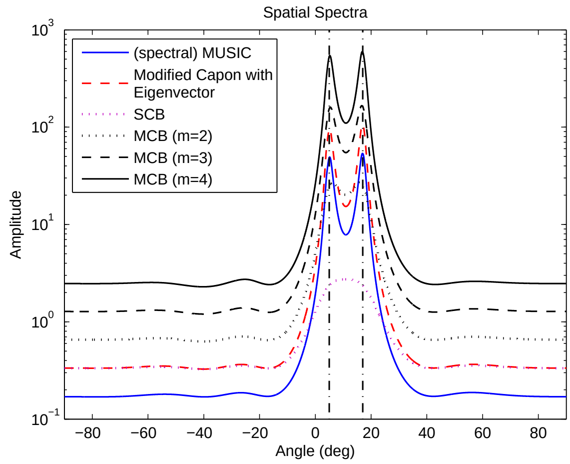

Figure 1: Spatial spectra of the conventional and modified Capon beamformers and the MUSIC method for Example 1. (SNR=0dB, $N = 200$)

Furthermore by considering the fact that the signal eigenvalues $\lambda_1, \cdots, \lambda_p$ are larger than the noise eigenvalues σ^2, i.e., $\bar{\lambda}_i^{-1} < 1$ for $i = 1, 2, \cdots, p$ and $\lim_{m\to\infty} \bar{\lambda}_i^{-m} = 0$, it is obvious that the increasing of the power of $\boldsymbol{R}^{-1}$ (i.e., m) can effectively weaken the proportion associated with the signal subspace. Hence from (17), we easily have

$$\lim_{N,m\to\infty} f_{\text{MCB}}(\theta) = \sigma^{-2m} \lim_{N,m\to\infty} f_{\text{MUSIC}}(\theta). \quad (18)$$

As a result, the MCB spatial spectrum becomes equal to that of the MUSIC method except for a scaled amplitude, when the power m and the number of snapshots tend to large enough. Therefore by increasing the power of $\boldsymbol{R}^{-1}$ (i.e., m) in (10) to eliminate the influence involved the signal subspace, the DOA estimation performance of the MCB can be improved significantly, where the procedure of computationally intensive eigendecomposition is not required.

5. Numerical Examples

Now we evaluate the performance of the proposed MCB method for estimating DOAs of uncorrelated narrowband signals with some numerical examples. Two uncorrelated signals with equal power and distinct angles $\theta_1 = 5°$ and $\theta_2 = 17°$ imping on a ULA composed of $M = 6$ sensors spaced half-wavelength apart (i.e., $d = \lambda/2$).

Example 1—Improved Resolution through Spatial Spectra Searching: Firstly we examine the spatial spectra of the SCB [4], the modified Capon beamformer with eigenvector method [9], [3], and the proposed MCB with different powers of $\hat{\boldsymbol{R}}^{-1}$ (i.e., m). The results are obtained from 100 independent trials and plotted in Fig. 1, where that of the (spectral) MUSIC [5] is also shown for comparison. We notice that the SCB and the MCB with $m = 2$ cannot provide evident peaks at the incoming directions, hence they fail to distinguish this two incident signals. However, by increasing the power m which can efficiently weaken the proportion associated with the signal subspace in (17), the

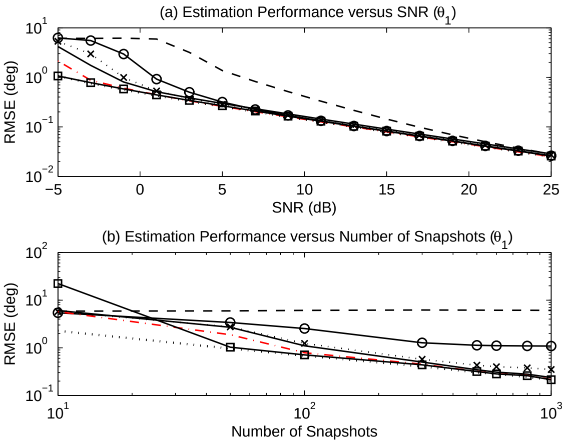

Figure 2: RMSEs of the estimation of $\hat{\theta}_1$ verse (a) SNR and (b) the number of snapshots (dashed line: SCB; solid line with "o": MCB with $m = 2$; dotted line with "×": MCB with $m = 3$; solid line: MCB with $m = 4$; dash-dotted line: (spectral) MUSIC; solid line with "□": root-MUSIC; dotted line: CRB) for Example 2 ($N = 200$) and Example 3 (SNR=0dB).

MCB with $m = 3$ and $m = 4$ succeeds in estimating these two DOAs and its resolution gradually approximate that of the (spectral) MUSIC exactly as it is derived in (18) .

Example 2—Performance versus SNR: Now we verify the performance of the MCB in (16) against SNR, while the SCB, the (spectral) MUSIC are also included for a comparison. The root-MUSIC [15] and the stochastic Cramer-Rao lower bound (CRB) [14] are plotted for reference. The results are obtained from 1000 independent trials. The empirical root-mean-square errors (RMSEs) of the estimates $\hat{\theta}_1$ against SNR are shown in Fig. 2(a). Obviously the MCB completely outperforms the SCB, and especially its performance becomes better with the increase of the power m even at low SNRs. For high SNRs, the proposed method and the (spectral) MUSIC provide similar small estimation errors which are very close to CRBs.

Example 3—Performance versus Number of Snapshots: Here we test the performance of the proposed method in terms of the number of snapshots. Other simulation conditions are similar to those in Example 2. From the results depicted in Fig. 2(b), it is clear that the RMSEs of the MCB decrease significantly with the increase of the number of snapshots, while the RMSEs of the SCB ($m = 1$) remain large for all numbers of snapshots. In addition, the RMSEs of the MCB get closer to that of the (spectral) MUSIC with the increase of m though the number of snapshots is small.

6. Conclusions

In this paper, a new modified Capon estimator called MCB was proposed for DOA estimation of noncoherent signals, where the computationally expensive procedure of eigendecomposition is avoided, and the connection between the proposed MCB and the MUSIC was studied. The resolution of the MCB can be improved significantly compared with the SCB and as better as that of the MUSIC method regardless of the SNR. The simulation results examined the effectiveness and validity of the proposed method.

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