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Clustering and Synchronous Firing of Coupled Rulkov Maps with STDP

Naohiro Shibuya[1], Charles Unsworth[2], Yoko Uwate[1] and Yoshifumi Nishio[1]

[1] The University of Tokushima
2-1 Minami-Josanjima, Tokushima, Japan
Phone/FAX:+81-88-656-7470/+81-88-656-7471
E-mail: {shibuya, uwate, nishio}@ee.tokushima-u.ac.jp

[2] The University of Auckland
70 Symonds Street, Auckland, New Zealand
Phone/FAX:+64-09-373-7599/+64-09-373-7468
E-mail: c.unsworth@auckland.ac.nz

Abstract

Epilepsy of the neuropsychiatric disorder is provoked from an imbalance in the long-term potentiation (LTP) versus long-term depression (LTD) of the synapses in the hippocampus. The LTP and LTD are replicated by using the Spike Timing Dependent Plasticity (STDP). Additionally, the spiking activity of the synapses in the hippocampus can be expressed by using the Rulkov Maps. In this paper, we consider some easy simulation models which are constructed by using Rulkov Maps with STDP. We explore the effect of 2 and 3 connective arrangements on spiking activity, as basic simulation for constructing the approximate simulation model of epilepsy. From the result, the coupled Rulkov Maps show in-phase/anti-phase synchronization by relationship of pre-synaptic and post-synaptic neuron.

1. Introduction

Epilepsy is known as one of neuropsychiatric disorder and involved with a large part of the hippocampus. The hippocampus has the lowest seizure threshold in the brain, therefore it has indicated the beginning of most epilepsy seizures. Moreover, the seizure-related neuronal electrical activity has feature of synchrony arising regularly [1]. In the normal status, some neurons suppress the abnormal firing. If the neuronal firing property of neurons or potentiation and depression balance are altered slightly, the supranormal excitability is diffused and lead to seizure. We consider that the each potentiation and the depression corresponds approximately to each long-term potentiation (LTP) and long-term depression (LTD).

In this study, we apply Spike Timing Dependent Plasticity (STDP) to Rulkov Maps for constructing the small-scale simulation models. STDP is a generic model used to replicate the LTP and the LTD [2], and Rulkov Map produces two-dimensional spiking-bursting behavior like real biological neurons [3],[4]. By incorporating the STDP into the original Rulkov Map, we can explore the synchronous behavior like real biological spiking activity. We explore the effect of 2 and 3 connective arrangements on spiking activity, as basic simulation for constructing the approximate simulation model of epilepsy.

2. Spike Timing Dependent Plasticity

STDP is a temporally asymmetric form of Hebbian learning induced by tight temporal correlations between the spikes of pre-synaptic and post-synaptic neurons. STDP provokes the LTP of the synapses, if the pre-synaptic spike arrival a few milliseconds before post-synaptic spikes. Whereas, it provokes the LTD of the same synapse, if pre-synaptic spike arrival after post-synaptic spikes.

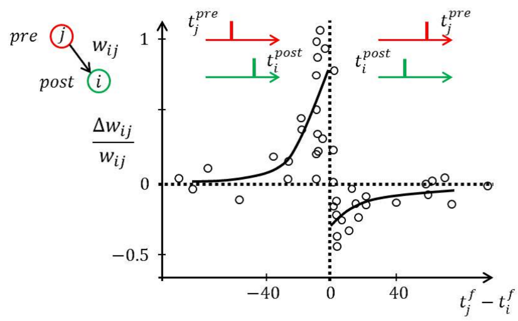

Figure 1: The STDP function of changing synaptic connection.

The weight change Δw_{ij} depends on the relative timing between pre-synaptic spike arrivals and post-synaptic spikes. The total weight change Δw_{ij} induced by a simulation protocol with pairs of pre-synaptic and post-synaptic spikes is described as following.

$$\Delta w_{ij} = \sum_{f=1}^{N} W(t_j^f - t_i^f) \tag{1}$$

where $W(x)$ denotes one of the STDP functions in Fig. 1. The method to choose for the STDP function $W(x)$ is shown as following.

$$W(x) = \begin{cases} A_+ exp(-x/\tau_+) & (x < 0) \\ -A_- exp(x/\tau_-) & (x > 0) \end{cases} \tag{2}$$

which has been used in fits to experimental data and models. The parameters A_+ and A_- may depend on the current value of the synaptic weight w_{ij}. In this article, the parameter A_+ and A_- are fixed each 0.05 and 0.055, and the time constants are on the order of $\tau_+ = 10ms$ and $\tau_- = 10ms$.

3. Coupled Rulkov Maps

In recent years, a simple model which replicates the dynamics of spiking and spiking-bursting activity of real biological neurons has proposed by N. F. Rulkov. The model is a two-dimensional map that produces chaotic spiking-bursting neural behavior. It is demonstrated that the results of this model are in agreement with the synchronization of chaotic spiking-bursting behavior experimentally found in real biological neurons. The expressions of the Coupled Rulkov maps are shown as following.

$$\begin{aligned} x_{m,n+1} &= f(x_{m,n}, x_{m,n-1}, y_{m,n}) \\ &\quad + \frac{1}{2} w_{ij}(x_{m+1,n} - 2x_{m,n} + x_{m-1,n}) \\ y_{m,n+1} &= y_{m,n} - \mu(x_{m,n} + 1) + \mu\sigma + \mu\sigma_{m,n} \\ &\quad + \frac{1}{2} w_{ij}(x_{m+1,n} - 2x_{m,n} + x_{m-1,n}) \end{aligned} \quad (3)$$

where x is the fast and y is the slow dynamical variables and the parameter w_{ij} shows the coupling weight of the connection between the map. The coupling weights w_{ij} are updated by STDP function. If STDP provokes the LTP, the coupling weights are updated to positive direction. Whereas, if it provokes LTD, the coupling weights are updated to negative direction. The nonlinear function $f(x_n, x_{n-1}, y_n)$ is shown as following:

$$\begin{aligned} & f(x_{m,n}, x_{m,n-1}, y_{m,n}) \\ &= \begin{cases} \alpha/(1 - x_{m,n} + u), & (x_{m,n} \le 0) \\ \alpha + u, & (0 < x_{m,n} < \alpha + u \ and \ x_{m,n-1} \le 0) \\ -1, & (x_{m,n} \ge \alpha + u \ or \ x_{m,n-1} > 0) \end{cases} \end{aligned} \quad (4)$$

α and μ, σ shows the parameters of the maps. In this paper, we set parameter α and σ with arbitrary value, and μ is fixed to 0.001. Here, these parameters are the control the dynamics parameters and the behavior shows the typical of the neurons. Specifically, the amplitude of waveform is changing by parameter α.

4. Network Models and Simulations

In this section, we consider some easy simulation models to analyze CA3 hippocampus network model.

4.1 CA3 Hippocampus Network Model

The network model of CA3 hippocampus is shown in Fig. 2. CA3 hippocampus network consist of pyramidal cells (excitability) and intercalated cells (inhibitory). The pyramidal cells are interconnected with 8 pyramidal cells of neighborhood. Moreover, the intercalated cells are interconnected with 16 pyramidal cells of neighborhood.

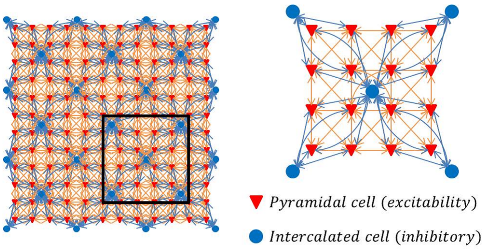

Figure 2: Network structure of CA3 hippocampus.

Additionally, the pyramidal cells and the intercalated cells exhibit spiking/bursting activity as shown in Fig 3. We set the parameter of Rulkov Maps for replicating the spiking/bursting activity of the pyramidal cells and intercalated cells. The parameter α is set to 4.7 ˜ 5.5, and σ is set to 0.1 for the pyramidal cells. Moreover, the parameter α is set to 4.5, and σ is set to 0.33 for the intercalated cells.

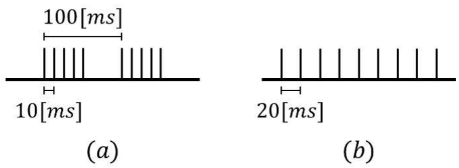

Figure 3: (a) Spiking activity of pyramidal cells. (b) Spiking activity of intercalated cells.

4.2 2 Coupled Maps

In this section, we observe fundamental synchronization phenomena of the coupled Rulkov Map. Moreover, we define the synchronization probability to provide quantitative analysis of synchronization accuracy. The synchronization probability is defined as follows.

$$I = \frac{Y}{X} \quad (5)$$

where X is the number of burst waveform, and Y is the number of synchrony burst waveform. The coupled Rulkov Maps shows inphase synchronization, if the synchronization probability closes in the value to 1. Whereas, it shows antiphase synchronization, if the synchronization probability closes in the value to 0.

We consider 2 coupled Rulkov Maps with STDP as shown in Fig. 4. We explore the spiking activity and synchronization probability, if the parameters α are changed. The spiking activity of P-P model is shown in Fig. 5. Moreover, Fig. 6 shows the synchronization probability if the parameter α_{pre} and α_{post} are changed from 4.7 to 5.5 each. The red area of cubic graph shows synchronization probability, if it is bigger than 0.5. Whereas, the blue area of cubic graph shows synchronization probability, if it is smaller than 0.5.

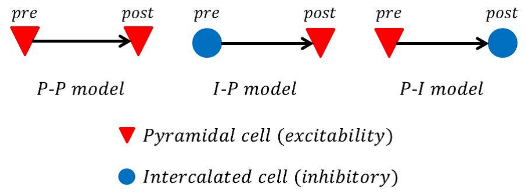

Figure 4: Connective arrangements of 2 coupled maps.

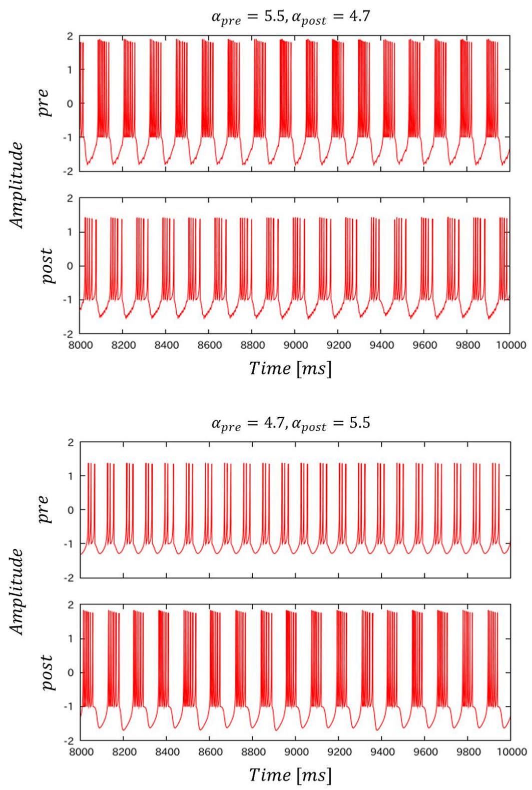

Figure 5: Spiking activity of P-P model.

Figure 5 shows spiking activities of P-P model between 8000 and $10000[ms]$. The upside of Fig. 5 shows anti-phase synchronization, and downside one shows in-phase synchronization. From Fig. 6, the coupled Rulkov Maps show anti-phase synchronization, if the parameter α_{pre} is bigger than α_{post}. Whereas, the coupled Rulkov Maps show in-phase synchronization, if the parameter value of α_{pre} is smaller than α_{post}. If α_{pre} is same as α_{post}, the coupled Rulkov Maps show perfect synchronism.

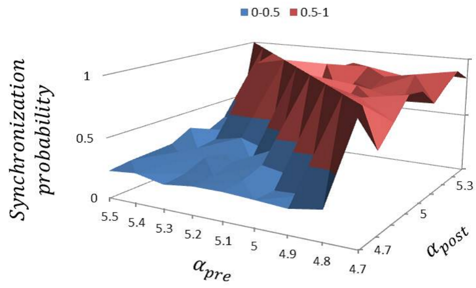

Figure 6: Synchronization probability of P-P model.

Figure 7 shows the synchronization probability of I-P and P-I models if the parameter $\alpha_{pyramidal\ cell}$ is changed from 4.7 to 5.5.

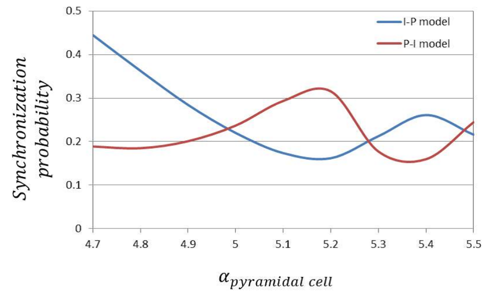

Figure 7: Synchronization probability of I-P and P-I models.

From Fig 7, the coupled Rulkov Maps show a anti-phase synchronization to all of the value of α. Thus, the intercalated cells provoke the anti-phase synchronization, or decrease the synchronization probability.

4.3 3 Coupled Maps

In this section, we consider 3 coupled Rulkov Maps with STDP as shown in Fig. 8. We offer an explanation of phenomena of 3 coupled maps from the result of 2 coupled maps. Figures 9-12 show the synchronization probability between the neuron 1 and the neuron 3.

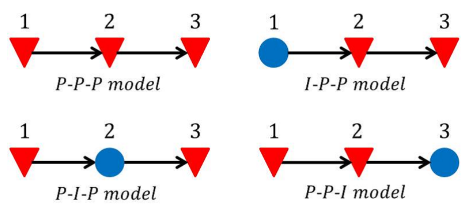

Figure 8: Connective arrangements of 3 coupled maps.

Figure 9 shows synchronization probability of P-P-P model, if the parameter α_1 is fixed 4.7 and α_2, α_3 are changed from 4.7 to 5.5 each. From this result, we can see that the in-phase place of synchronization probability is increased as compared with Fig. 6. The neuron 1 and 2 show in-phase synchronization because α_1 is fixed 4.7. Thus, the neuron 3 gets involved in in-phase synchronization of the neuron 1 and 2.

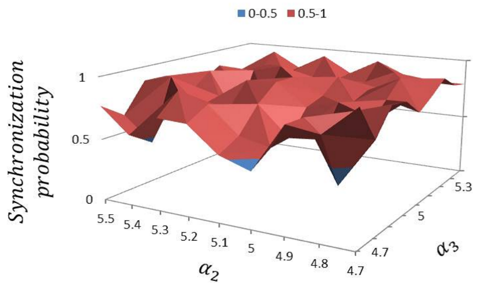

Figure 9: Synchronization probability of P-P-P model.

Figures 10-12 show the synchronization probability of I-P-P, P-I-P and P-P-I models if the parameter α of pyramidal cells changed from 4.7 to 5.5. From these result, the in-phase place of synchronization probability is decreased as compared with Fig. 6. Moreover, the peak values of synchronization probability are changed by position of intercalated cell. Because synchronization probability represent the different values if the intercalate cell is pre-neuron or post-neuron, from the result of Fig. 7.

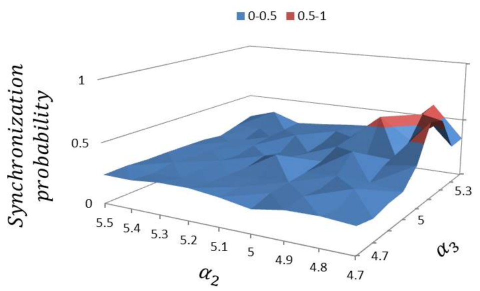

Figure 10: Synchronization probability of I-P-P model.

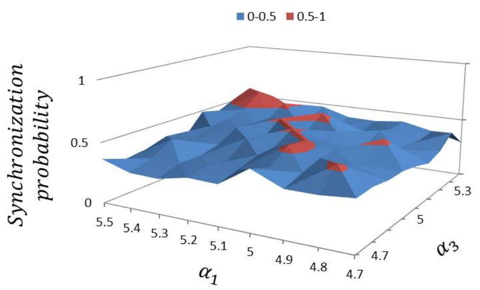

Figure 11: Synchronization probability of P-I-P model.

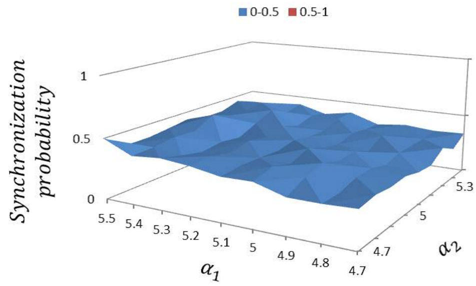

Figure 12: Synchronization probability of P-P-I model.

5. Conclusions

In this paper, we considered the simulation model which is constructed by using Rulkov Maps with STDP. We have explored the effect of 2 and 3 connective arrangements on spiking activity. From the result, the coupled Rulkov Maps show anti-phase synchronization, if the parameter α_{pre} is bigger than α_{post}. Whereas, the coupled Rulkov Maps show in-phase synchronization, if the parameter value of α_{pre} is smaller than α_{post}. In case of 3 coupled Rukov Maps, the spiking activity and synchronization probability become more complex as compared with 2 coupled Rulkov Maps.

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