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Influence of Bridge on Coupled Chaotic Circuit Network

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Abstract

How network topological structure influences its dynamical behaviors, is currently becoming a topic of great importan. In this study, we investigate synchronization phenomena of coupled chaotic circuit network with the bridge. By computer simulations, synchronization of the bridge is easy to break down and clustering of circuits is occurred from the bridge. Moreover, we statistically investigate the relation between synchronous rate and coupling strength.

1. Introduction

Recently, complex networks have attracted increasing attention from various field of science and engineering[1]–[4]. Additionally, how network topological structure influences its dynamical behaviors, is currently becoming a topic of great important. Many of complex networks can be described by node and edge, by its topological structure generates various synchronization phenomena. On the other hand, chaos synchronization is very interesting phenomena that two chaotic circuits are synchronized regardless of differences circuit parameters[5]. Also, as coupled systems of chaotic elements, Coupled Map Lattice (CML) and Globally Coupled Map (GCM) were proposed by Kaneko. These models generate many kinds of complexity phenomena such as spatiotemporal intermittency[6], clustering[7] and so on. However these models are very simple and discrete–time mathematical model, many of nonlinear phenomena generated in nature would be not so simple. Therefore, it is important to investigate the complex phenomena observed in continuous–time real physical system such as electrical circuits.

In this study, we focus on the relation between chaos synchronization and network topological structure. Therefore, we focus on the influence of the bridge on the network. The bridge is the edge which provide the only route between two nodes. Example of the bridge is shown in Fig. 1.

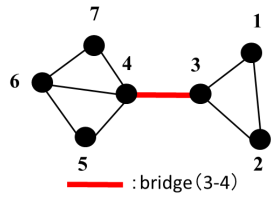


Figure 1: Example of the bridge.

In the Fig.1, the bridge between 3 and 4 provides the only route which can flow information or influence from any contact of 3 and 4. In order to analyze complex phenomena of the bridge on the network, chaotic circuits are applied to each node of the network and coupling elements between each circuit are used resistors. By computer simulations, we investigate the synchronization phenomena between each circuit on the coupled chaotic circuit network with bridge. Moreover, we statistically investigate the relation between synchronous rate and coupling strength. Thereby, we reveal the influence of the bridge on the network.

2. Network model

Figure 2 shows chaotic circuit which is three-dimensional autonomous circuit proposed by Shinriki et al.[8]. A proposed network model is shown in Fig. 3. In this study, chaotic circuits are applied to each node of the network and coupling elements between each circuit are used resistors. In this model, the bridge is the resistor between CC3 and CC4.

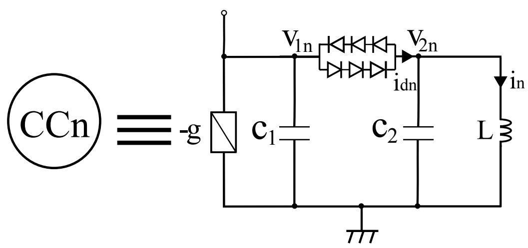


Figure 2: Chaotic circuit.

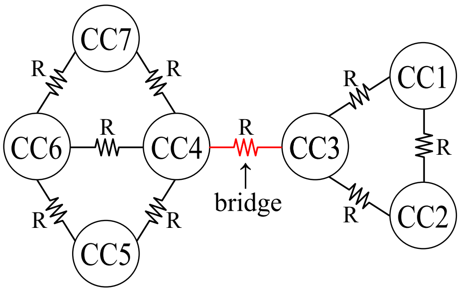


Figure 3: Model of coupled chaotic circuit network with bridge.

First, the circuit equations are given as follows:

$$\begin{cases} L\dfrac{di_n}{dt} = v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in C_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} = -i_n + i_{dn}, \end{cases} \quad (1)$$
$$(n = 1, 2, 3, ..., 7),$$

where C_n is set of nodes which are connected to CCn. We approximate the $i - v$ characteristics of the nonlinear resistors consisting of the diodes by the following 3–segment piecewise–linear function as shown in Fig.4.

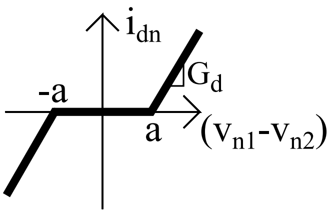


Figure 4: Approximation of the nonlinear resistor.

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - a) & (v_{1n} - v_{2n} > a) \\ 0 & (|v_{1n} - v_{2n}| \le a) \\ G_d(v_{1n} - v_{2n} + a) & (v_{1n} - v_{2n} < -a). \end{cases} \quad (2)$$

By using the parameters and variables as follows:

$$i_n = \sqrt{\frac{C_2}{L}}ax_n, \; v_{1n} = ay_n, \; v_{2n} = az_n$$
$$t = \sqrt{LC_2}\tau, \; \text{“ · ”} = \frac{d}{d\tau}, \; \alpha = \frac{C_2}{C_1} \quad (3)$$
$$\beta = \sqrt{\frac{L}{C_2}}G_d, \; \gamma = \sqrt{\frac{L}{C_2}}g, \; \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}},$$

the normalized circuit equations are given as follows:

$$\begin{cases} \dot{x}_n = z_n \\ \dot{y}_n = \alpha\gamma y_n - \alpha\beta f(y_n - z_n) - \alpha\delta\displaystyle\sum_{k\in C_n}(y_n - y_k) \\ \dot{z}_n = \beta f(y_n - z_n) - x_n, \end{cases} \quad (4)$$

where the nonlinear function corresponding to the characteristics of the nonlinear resistor of the diodes and are described as follows:

$$f(y_n - z_n) = \begin{cases} y_n - z_n - 1 & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \le 1) \\ y_n - z_n + 1 & (y_n - z_n < -1). \end{cases} \quad (5)$$

This circuits generates chaotic attractor as shown in Fig. 5.

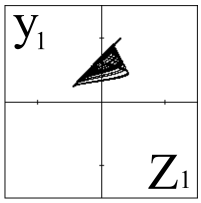


Figure 5: Example of chaotic attractors. $\alpha = 0.4, \beta = 0.5$ and $\gamma = 20$.

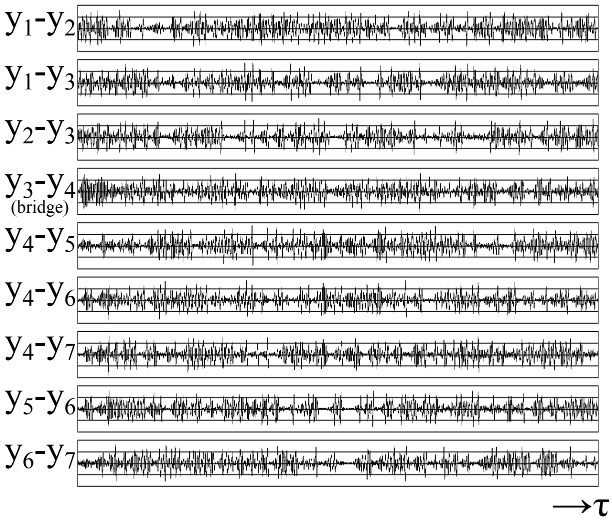
3. Simulation Result

In this study, we fix the same parameters as $\alpha = 0.4, \beta = 0.5, \gamma = 20$ and δ on all circuits. We focus on the coupling strength δ. Each circuit is given different initial values each other. Figure 6 shows the computer simulation results. In the Fig.6, the vertical axes are the differences between the voltage of the two chaotic circuits. Namely, if the two chaotic circuits synchronize, the value of the graph should be almost zero like $y_1 - y_2$ of Fig. 6(b). In the case of Fig. 6(a), all circuits behave asynchronously. In the case of Fig. 6(b), $y_1 - y_2$ shows full synchronization. In the case of Fig. 6(c), additionally $y_5 - y_6$ and $y_6 - y_7$ show full synchronization. In the Fig. 6(b)(c), the bridge $(y_3 - y_4)$ is bursting waveform compared with others. In other words, the bridge is easy to break down.

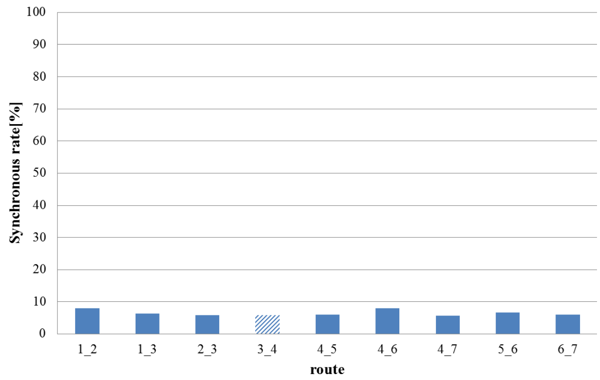
In order to analyze synchronization, we define the synchronization as following equation,

$$|y_n - y_k| < 0.01, \quad (6)$$

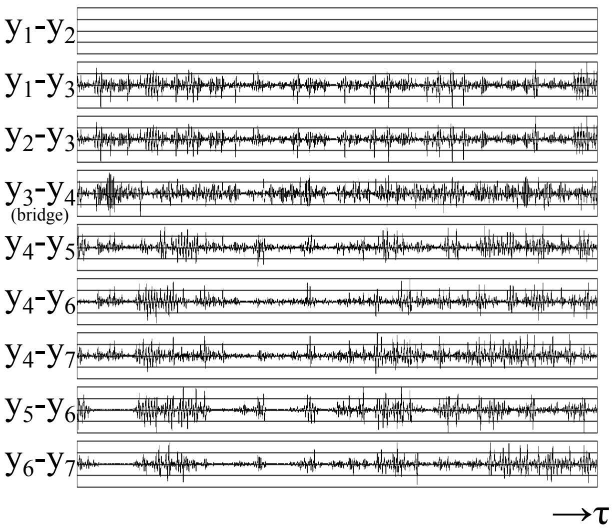
where n is number of circuits and k is circuits which connected circuit n. Figure 7 shows statistically investigated result of synchronous rate. The iteration time τ is fixed with



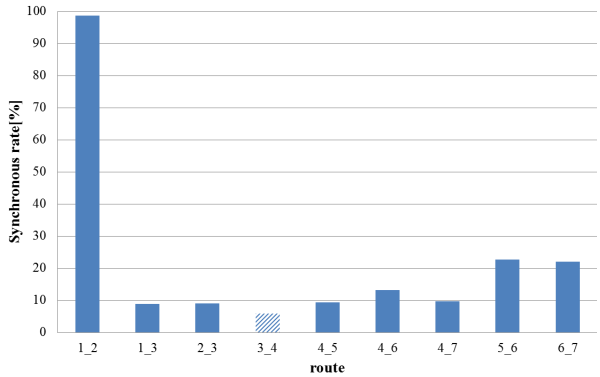
(a) δ = 0.10.



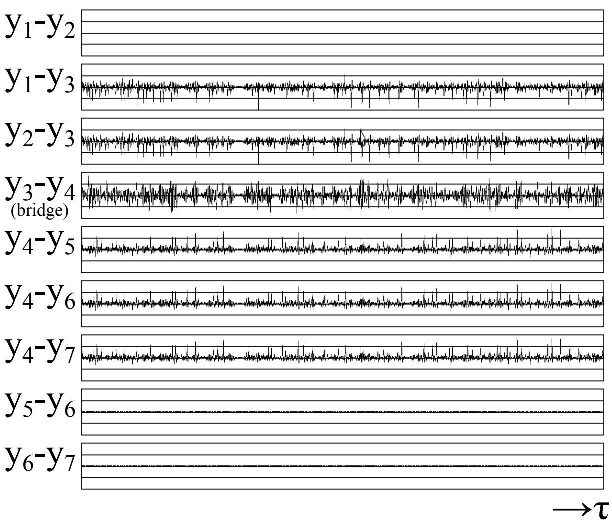
(a) δ = 0.10.



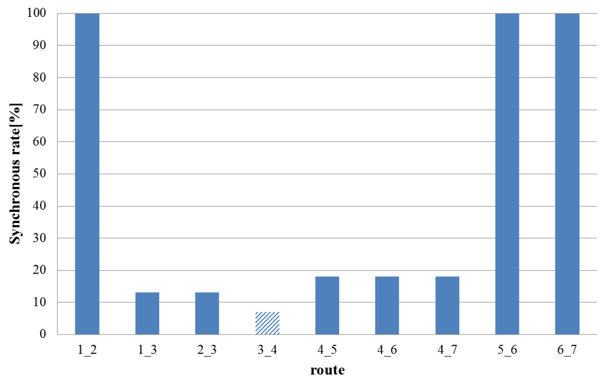
(b) δ = 0.25.



(b) δ = 0.25.



(c) δ = 0.40.



(c) δ = 0.40.

Figure 6: Computer simulation result.

Figure7: Synchronous rate.

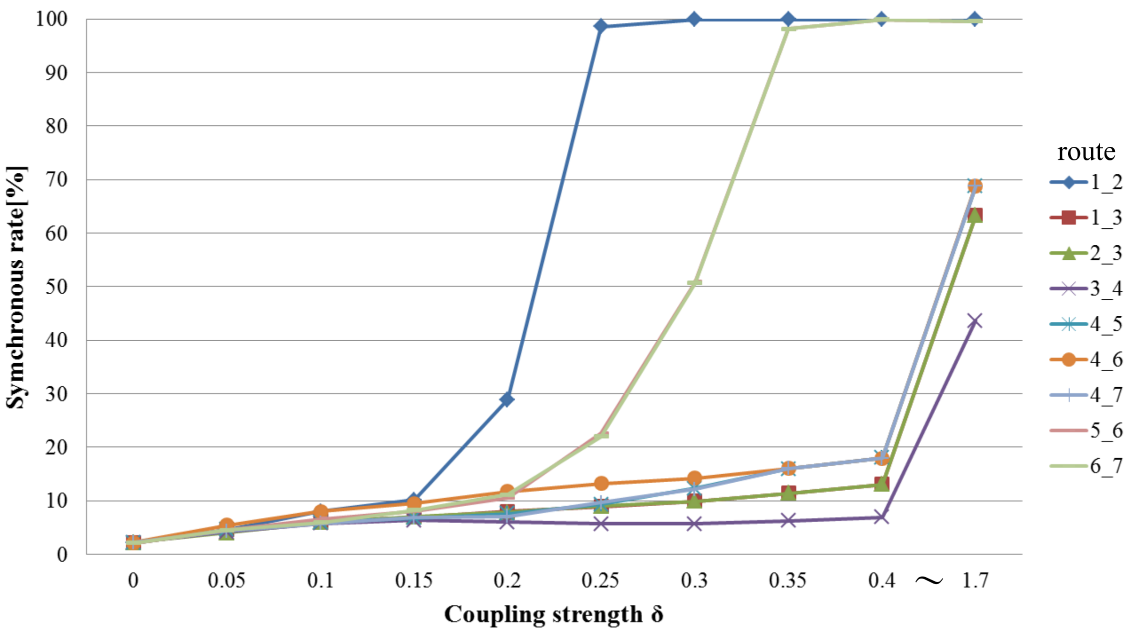


Figure 8: The relation between synchronous rate and coupling strength.

10,000,000. Synchronous rate of the Fig. 7 corresponds to the Fig. 6. In the Fig.7(c), full synchronization is shown as 100 % numerically like routes 1–2, 5–6 and 6–7. On the other hands, other routes show low synchronous rate. Additionally, bridge (3-4) is lowest synchronous rate of all routes.

Moreover, we statistically investigate the relation between synchronous rate and the coupling strength δ. Figure 8 shows the relation between synchronous rate and the coupling strength δ. By increasing the coupling strength δ, some combination of chaotic circuits become synchronous state. CC1 and CC2 are synchronized completely (around δ=0.3). CC5 and CC6, CC6 and CC7 are also synchronized completely after CC1 and CC2. The other combinations also become synchronous state, however the other combinations are not synchronized completely. Even if the differences between the initial value of each circuit are set very small, we can not be observed full synchronization of all circuits. Thereby, clustering of circuits is occurred from the bridge. Some results depending on the initial values can be observed since each circuit generate asymmetric attractor.

4. Conclusion

In this study, we have investigated influence of the bridge on the network via synchronization phenomena of coupled chaotic circuits. By computer simulations, synchronization of the bridge is easy to break down and clustering of circuits is occurred from the bridge. In our future work, we investigate the other network topological structure with the bridge and carry out circuit experiment.

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