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Synchronization of Coupled Chaotic Circuits Arranged in One-dimensional Coordinate Depending on Network Structures

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Abstract

In this study, we investigate synchronization phenomena of coupled chaotic circuits. First, the chaotic circuits are combined by resisters in one-dimensional coordinate system. We change the distance between the circuits to adapt the coupling strength. We investigate the phase difference between the central circuits using computer simulation. From the computer simulation, the difference synchronization phenomena are obtained between the full coupled system and the ladder system.

1. Introduction

Synchronization phenomenon is one of the typical phenomena observed in nature. Recently, many studies have been investigated synchronization of chaotic circuits [1]∼[5]. It is focused how the differences of the network structure impact on the whole circuits. Additionally, it is applicable to the fields of medical science and biology and so on.

In our research group, we have investigated the clustering phenomena resulting from the synchronization phenomena observed in coupled chaotic circuits when the chaotic circuits are arranged in two-dimensional coordinate[6], [7]. We observed that the chaotic circuits arranged in the near distance are synchronized at in-phase state, and the coupled circuits with the far distance could not be synchronized. From the results we confirmed the relationship between clustering and synchronization phenomena.

The research about the system arranged in two-dimensional coordinate is complicated for more detailed researches. So in this study, we investigate the synchronization phenomena of coupled chaotic circuits arranged in one-dimensional coordinate. We combine chaotic circuits by resister, and the circuits are arranged in one-dimensional coordinate system. The number of the circuits is always an even number and we investigate symmetry systems. We investigate synchronization phenomena by changing the distance between the circuit and the number of coupling circuits. In this study, we use two systems. They are the full coupled system and the ladder system. In the full coupled system, chaotic circuits are connected to all chaotic circuits. Figure 1 shows the system model of the full coupled system. In the ladder system, chaotic circuits are connected to only adjacent circuits. Figure 2 shows the system model of the ladder system. We measure the phase difference between the central circuits using computer simulations. We compare the results of the full coupled system and the ladder system.

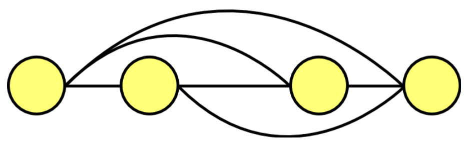


Figure 1: Full coupled system.

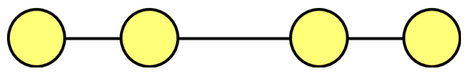


Figure 2: Ladder system.

2. Circuit Model

Figure 3 shows the circuit model. This is a chaotic circuit called Nishio-Inaba circuit [8]∼[10].

The circuit equations of this circuit are described as Eq. (1).

$$
\begin{aligned}
L_1\frac{di_1}{dt} &= v + ri_1 \\
L_2\frac{di_2}{dt} &= v - v_d(i_2) \\
C\frac{dv}{dt} &= -i_1 - i_2
\end{aligned}
\tag{1}
$$

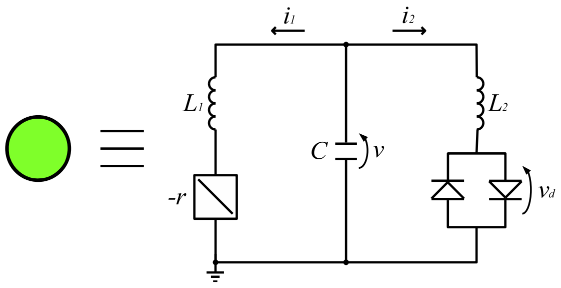


Figure 3: Circuit model.

The characteristic of nonlinear resistance is described as Eq. (2).

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right) \tag{2}$$

The circuit equations are normalized as Eq. (3) by changing the variables as below.

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1C}}{L_2}Vy;\ v = Vz;$$
$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1C}}{L_2} = \delta;$$
$$t = \sqrt{L_1C}\tau;\ \text{“}\cdot\text{”} = \frac{d}{d\tau};$$

$$\begin{aligned}\dot{x} &= \alpha x + z\\ \dot{y} &= z - f(y)\\ \dot{z} &= -x - \beta y\end{aligned} \tag{3}$$

The value of $f(y)$ is described as Eq. (4).

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right) \tag{4}$$

We use the chaotic attractor as shown in Fig. 4.

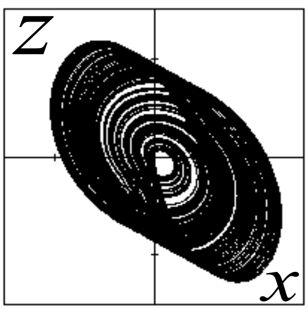


Figure 4: Chaotic attractor.

The parameters of this attractor are described as below.

$$\begin{aligned}\alpha &= 0.460\\ \beta &= 3.0\\ \delta &= 470\end{aligned}$$

In this study, we use two systems. The full coupled system and the ladder system. In the full coupled system, all circuits are connected to all circuits by resisters. In the ladder system, chaotic circuits are connected to only adjacent circuits.

When all circuits are connected to all circuits, the circuit equations are shown in Eq. (5).

$$\begin{aligned}\frac{dx_i}{d\tau} &= \alpha x_i + z_i\\ \frac{dy_i}{d\tau} &= z_i - f(y_i)\\ \frac{dz_i}{d\tau} &= -x_i - \beta y_i - \sum_{j=1}^{N}\gamma_{ij}(z_i - z_j)\\ &(i, j = 1, 2, \cdots, N)\end{aligned} \tag{5}$$

Where the parameter γ_{ij} represents the coupling strength between the circuits. The value of γ_{ij} reflects the distance between the circuits in an inverse way, described by the following equation:

$$\gamma_{ij} = \frac{g}{(d_{ij})^2}. \tag{6}$$

d_{ij} denotes the Euclidean distance between the i-th circuit and the j-th circuit. The parameter g is coupling coefficient that determines the coupling strengths. In this study, we set the parameter as $g = 1.0 \times 10^{-2}$.

3. Simulation Method

In this study, we use two systems arranged in one-dimensional coordinate. There are the full coupled system and the ladder system. We use ten circuits in all simulations. We divide into the two symmetric groups, and there are five circuits in one side of the group. The network structure of the ladder system is shown in Fig. 5.

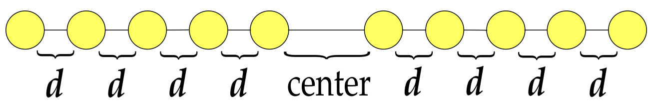


Figure 5: Network structure.

We change the distance between the circuits by changing the coupling strength. The value of d is the distance of the circuits in the group. After that, we change the distance between the central circuits. We define the distance between the central circuits as d_{center}. We change the value of center from 0.1 to 13.0. We investigate the phase difference between the central circuits. We compare synchronization phenomena of the full coupled system and ladder system.

4. Simulation result

Figures 6 ∼ 8 show the simulation results when d is fixed with 0.1, 0.3 and 0.5.

From the simulation results, the ladder system becomes asynchronous quickly in Figs. 6 ∼ 8. The central circuits become asynchronous around d_{center} = 2.0. However, in the full coupled system, the way of a change in the phase difference is different by the value of d. So we focus on the full coupled system as follows.

In the case of d = 0.1, the phase difference increases in small steps, and the central circuits become asynchronous around d_{center} = 11 ∼ 12. In the case of d = 0.3, the phase difference increases with the case of d = 0.1. Although the central circuits in the case of d = 0.3 become asynchronous around d_{center} = 9 ∼ 10. In the case of d = 0.5, the phase difference increases quickly. The central circuits become asynchronous around d_{center} = 6 ∼ 7.

From these results, the relationship between the network structure and synchronization phenomena on each coupled system is made clear. We show some examples of the graphic of phase difference in Fig. 9.

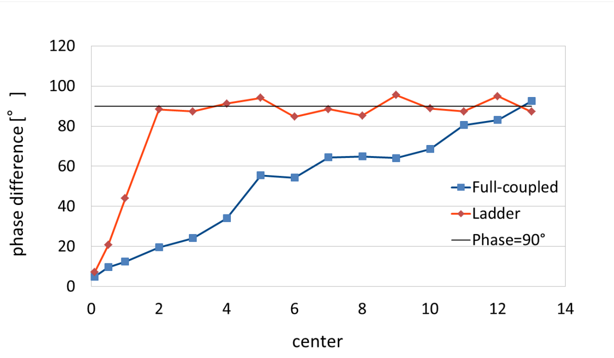


Figure 6: Phase difference (d=0.1).

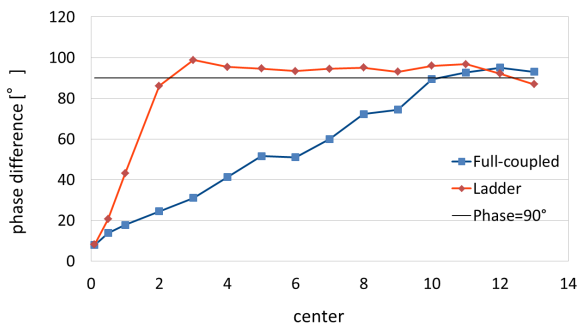


Figure 7: Phase difference (d=0.3).

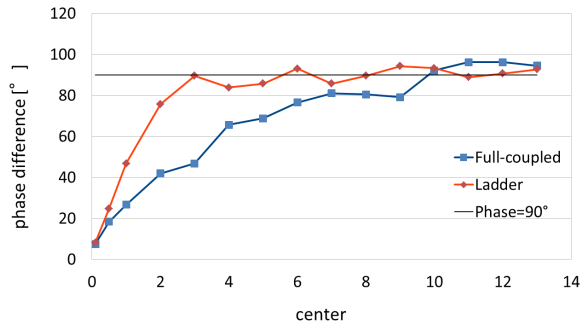
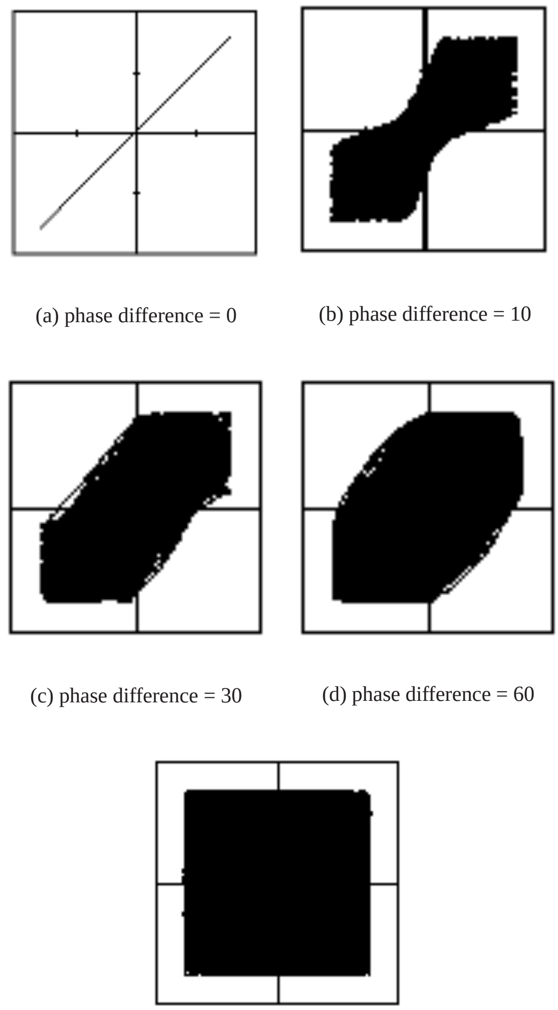


Figure 8: Phase difference (d=0.5).



(a) phase difference = 0

(b) phase difference = 10

(c) phase difference = 30

(d) phase difference = 60

(e) phase difference = 90

Figure 9: Graphic of phase difference.

5. Conclusions

In this study, we have investigated the synchronization phenomena in coupled chaotic circuits networks. We also investigated the phase difference between the central circuits. From computer simulation, we obtained the various results by changing the distance between the central circuits and the value of d.

In the ladder system, we obtained the result that the central circuits become asynchronous around d_{center} = 2.0 in the all results. After that, the central circuits are asynchronous even by increasing the value of d_{center}. From this result, in the ladder system, we can see that the central circuits are not impacted from other circuits.

On the other hand, in the full coupled system, the way of a change in the phase difference is different by the value of d. As enlarging the value of d, the value of d_{center} is gradually close until the state of asynchronous. From this result, the difference synchronization phenomena are obtained between the full coupled system and the ladder system.

In our future works, we would like to investigate asymmetric system and more large scale system.

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