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Synchronization Phenomena of Two Chaotic Oscillators with Shifting Input Wave

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Abstract

In this study, we investigate synchronization phenomena observed in two coupled chaotic oscillators with shifting input wave. In this circuit system, two chaotic circuits are coupled by one resistor or capacitor. We investigate the synchronization state when one side's input wave is shifted. We carry out computer calculation to investigate in-phase and anti-phase using correlation value. Moreover, when we change the value of coupling strength between two oscillators, we observe the change of synchronization state.

1. Introduction

Recently, many researchers have shown their interests in chaotic systems [1]-[3]. In particular, chaos synchronization has attracted many researchers' attentions and their mechanism has been gradually made clear. This is because, chaotic systems observing chaos synchronization are good models to explain and describe the higher dimensional nonlinear phenomena in the field of natural science. Chaotic behavior has been observed in various fields of research. Hence, chaos has been investigated not only in engineering but also in various fields such as biology, sociology and economics. In the field of electrical and electronic engineering, many researchers has proposed many applications using chaos such as chaos communications, chaos cryptosystem and chaos neural networks. In order to realize chaotic engineering systems, it is important to understand simple coupled chaos-generating circuits.

In our previous study, a simple chaotic oscillator composed of two RC circuits were proposed [4]. When we changed the parameter, we could observe not only periodic attractor but also chaotic attractor in this simple oscillator. And, we investigated synchronization phenomena when two chaotic oscillator composed of RC circuits are coupled by one resister. Moreover, we found the cross correlation characteristics between neighboring oscillators. In addition, we have investigated synchronization phenomena when two input waves are in-phase and anti-phase [5].

In this study, we investigate synchronization phenomena observed in two coupled chaotic oscillators composed of RC circuits with shifting input wave. Two chaotic oscillators are coupled by one resistor or capacitor. We carry out computer calculation and investigate chaotic behavior when one side's input wave is shifted.

2. Circuit Model

Figures 1, 2 show the circuit model in this study. Two chaotic oscillators, which was proposed in [4], are coupled via one resistor or capacitor. An independent rectangular voltage sources V_{S1} and V_{S2} are connected of each oscillator's two comparators.

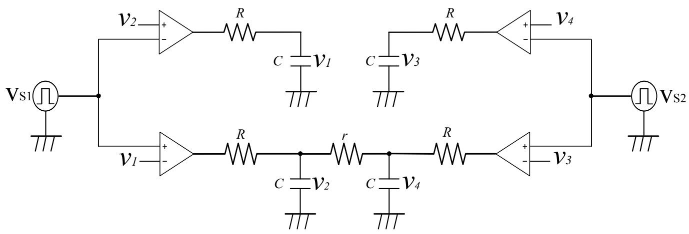


Figure 1: Circuit model 1.

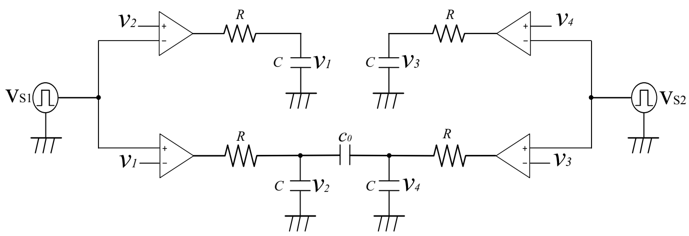


Figure 2: Circuit model 2.

Hence, the whole circuit consists of two rectangular voltage sources, four comparators, four resistors, one variable resistor and four capacitors. Figure 3(a) shows the rectangular voltage waveform $V_{Sn}(t)$ $(n = 1, 2)$. $E\alpha$ is the amplitude of the rectangular voltage and T is the period of the waveform. E is the output voltage of the comparators, namely the DC supply voltage of the operational amplifiers.

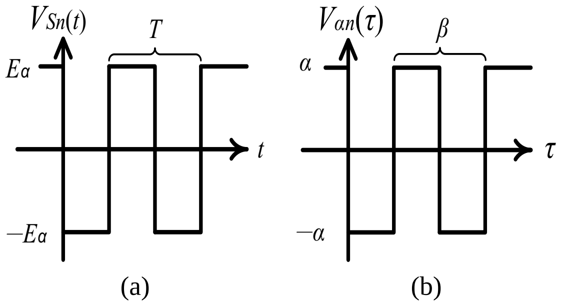


Figure 3: Rectangular voltage waveform.

Figure 1 is circuit model which composed two chaotic oscillators coupled by one resistor. Normalized system equations are given as follows:

$$x_1 = \begin{cases} (x_{10}-1)e^{-\tau}+1 & (x_2 > V_{\alpha 1}) \\ (x_{10}+1)e^{-\tau}-1 & (x_2 < V_{\alpha 1}) \end{cases}$$

$$x_2 = \begin{cases} \left(\frac{x_{20}+x_{40}}{2}+1\right)e^{-\tau} \\ \quad + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-(1+2\gamma)\tau}-1 \;\; (x_1 > V_{\alpha 1}) \\ \left(\frac{x_{20}+x_{40}}{2}-1\right)e^{-\tau} \\ \quad + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-(1+2\gamma)\tau}+1 \;\; (x_1 < V_{\alpha 1}) \end{cases}$$

$$x_3 = \begin{cases} (x_{30}-1)e^{-\tau}+1 & (x_4 > V_{\alpha 2}) \\ (x_{30}+1)e^{-\tau}-1 & (x_4 < V_{\alpha 2}) \end{cases}$$

$$x_4 = \begin{cases} \left(\frac{x_{20}+x_{40}}{2}+1\right)e^{-\delta\tau} \\ \quad + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-(1+2\gamma)\delta\tau}-1 \;\; (x_3 > V_{\alpha 2}) \\ \left(\frac{x_{20}+x_{40}}{2}-1\right)e^{-\delta\tau} \\ \quad + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-(1+2\gamma)\delta\tau}+1 \;\; (x_3 < V_{\alpha 2}) \end{cases} \tag{1}$$

where $V_{\alpha n}$ is a parameter corresponding to V_{Sn} $(n = 1, 2)$, β is parameter corresponding to T and γ is a coupling strength. x_{10}, x_{20}, x_{30} and x_{40} are initial values.

Figure 2 is circuit model which composed two chaotic oscillators coupled by one capacitor. Normalized system equations are given as follows:

$$x_1 = \begin{cases} (x_{10}-1)e^{-\tau}+1 & (x_2 > V_{\alpha 1}) \\ (x_{10}+1)e^{-\tau}-1 & (x_2 < V_{\alpha 1}) \end{cases}$$

$$x_2 = \begin{cases} \left(\frac{x_{20}+x_{40}}{2}+1\right)e^{-\tau} \\ \quad + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-\gamma\tau}-1 \;\; (x_1 > V_{\alpha 1}) \\ \left(\frac{x_{20}+x_{40}}{2}-1\right)e^{-\tau} \\ \quad + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-\gamma\tau}+1 \;\; (x_1 < V_{\alpha 1}) \end{cases}$$

$$x_3 = \begin{cases} (x_{30}-1)e^{-\tau}+1 & (x_4 > V_{\alpha 2}) \\ (x_{30}+1)e^{-\tau}-1 & (x_4 < V_{\alpha 2}) \end{cases}$$

$$x_4 = \begin{cases} \left(\frac{x_{20}+x_{40}}{2}+1\right)e^{-\delta\tau} \\ \quad + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-\gamma\delta\tau}-1 \;\; (x_3 > V_{\alpha 2}) \\ \left(\frac{x_{20}+x_{40}}{2}-1\right)e^{-\delta\tau} \\ \quad + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-\gamma\delta\tau}+1 \;\; (x_3 < V_{\alpha 2}) \end{cases} \tag{2}$$

where $V_{\alpha n}$ is a parameter corresponding to V_{Sn} $(n = 1, 2)$, β is parameter corresponding to T and γ is a coupling strength. x_{10}, x_{20}, x_{30} and x_{40} are initial values.
Because these circuit equations are linear in each region, the rigorous solution of these circuit equations can be derived.

We carry out computer calculation, the value of the frequency error δ is fixed with 0.05% to remove synchronization isn't due to chaotic phenomena.

3. Simulations

First, we explain input wave. The input wave's period β is 10. The step size of τ is 0.01. We define deviation of two input waves as "gap". In this study, we investigate synchronization state when the gap is changed from 0 to 5. For example, Fig. 4 shows two input waves when gap is 2.50.

Second, we explain how to judge in-phase synchronization and anti-phase synchronization. To judge in-phase synchronization and anti-phase synchronization quantitatively, we use correlation coefficient in this study. Correlation coefficient is described as follows:

$$c = \frac{\frac{1}{k}\sum_{i=0}^{k}(x_1(i)-\overline{x_1})(x_2(i)-\overline{x_2})}{\sqrt{\frac{1}{k}\sum_{i=0}^{k}(x_1(i)-\overline{x_2})^2}\sqrt{\frac{1}{k}\sum_{i=0}^{k}(x_2(i)-\overline{x_2})^2}} \tag{3}$$

where x_1 and x_2 are state variables and k is number of trials. If c is near 1.0, synchronization state is in-phase. In contrast, if c is near -1.0, synchronization state is anti-phase.

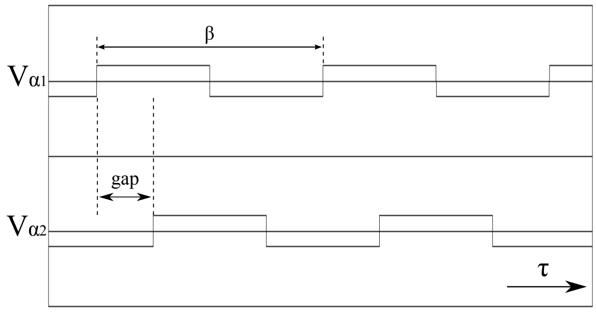


Figure 4: Two input wave (gap $= 2.50$).

4. Simulation Results

We show the results of the computer calculations and use fixed parameters $\alpha = 0.06$ and $\beta = 10$.

First, we show the result of circuit model 1 when the coupling strength are 0.1 and 1.0 and circuit model 2 when the coupling strength is 0.5.

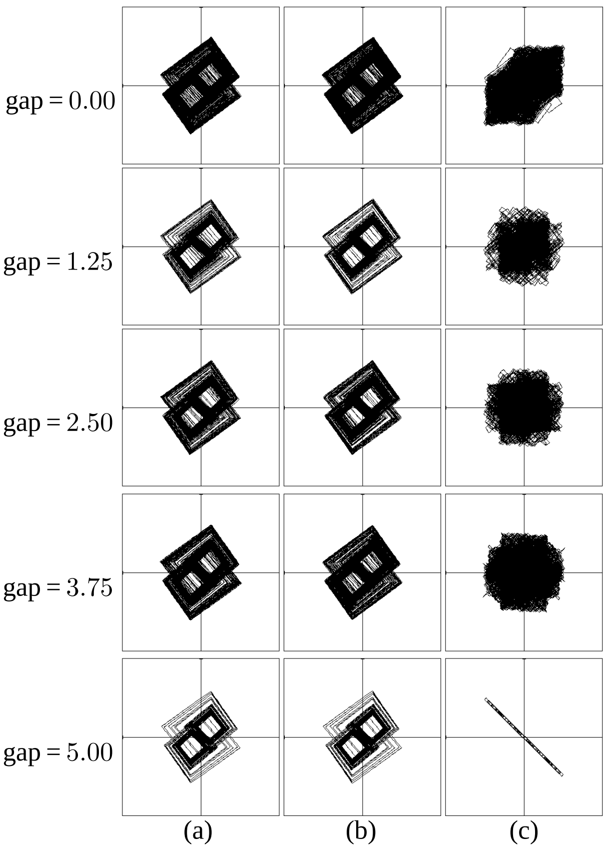


Figure 5: Chaotic attractor of circuit model 1 obtained from computer calculation ($\gamma = 0.1$). (a) $x_1 - x_2$, (b) $x_3 - x_4$, and (c) $x_1 - x_3$.

Figures 5 to 7 shows attractor and the phase difference. Left attractors show the left oscillator ($x_1 - x_2$) and center attractors show the right oscillator ($x_3 - x_4$). Right attractors show the phase difference between left and right oscillators ($x_1 - x_3$).

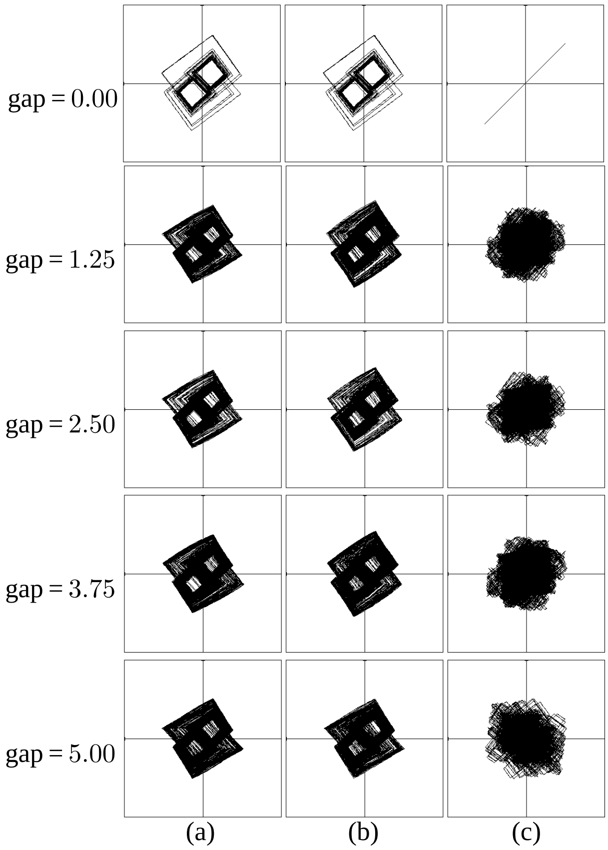


Figure 6: Chaotic attractor of circuit model 1 obtained from computer calculation ($\gamma = 1.0$). (a) $x_1 - x_2$, (b) $x_3 - x_4$, and (c) $x_1 - x_3$.

We observe the in-phase synchronization state in Figs. 6(c) and 7(c). These results are caused by two rectangular voltages are in-phase relations. Furthermore, we can observe the anti-phase synchronization state in Fig. 5(c) and 7(c). Moreover, we observe in-phase and anti-phase synchronization in same parameter when two oscillators are coupled by capacitor. However, we can not observe in-phase and anti-phase synchronization in same parameter when two oscillators are coupled by resister.

Next, Fig. 8 shows the relation between ”gap” and correlation coefficient between the two oscillators in Figs. 1 and 2. In this computer simulation, we define $k = 100000$. And, we calculate average of solution to derive correlation values.

We obtain correlation values is small when ”gap” is large from Fig. 8. This result is caused by two rectangular voltages are gradually anti-phase relations. Moreover, it becomes synchronization state when the value of coupling strength is large. And, we obtain when the value of coupling strength is small, we observe anti-phase synchronization. There is

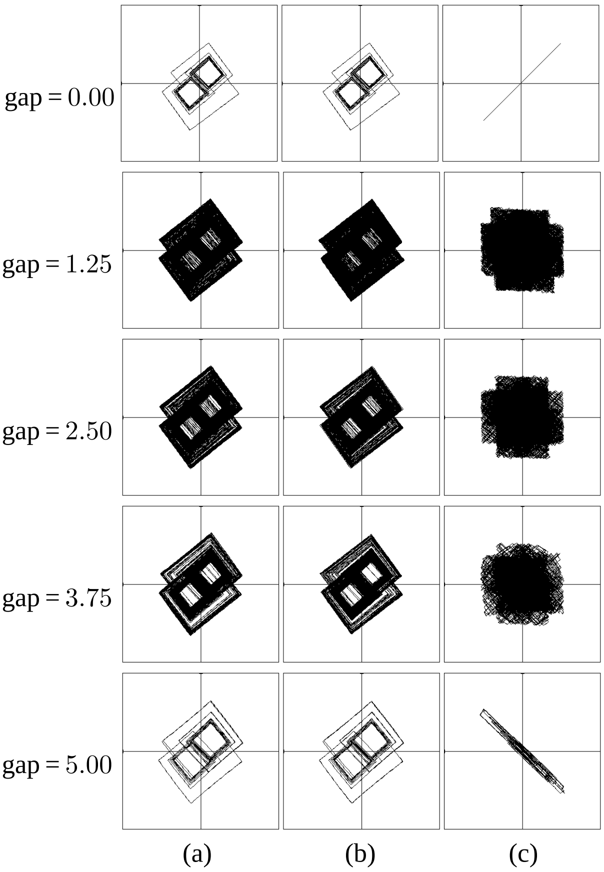
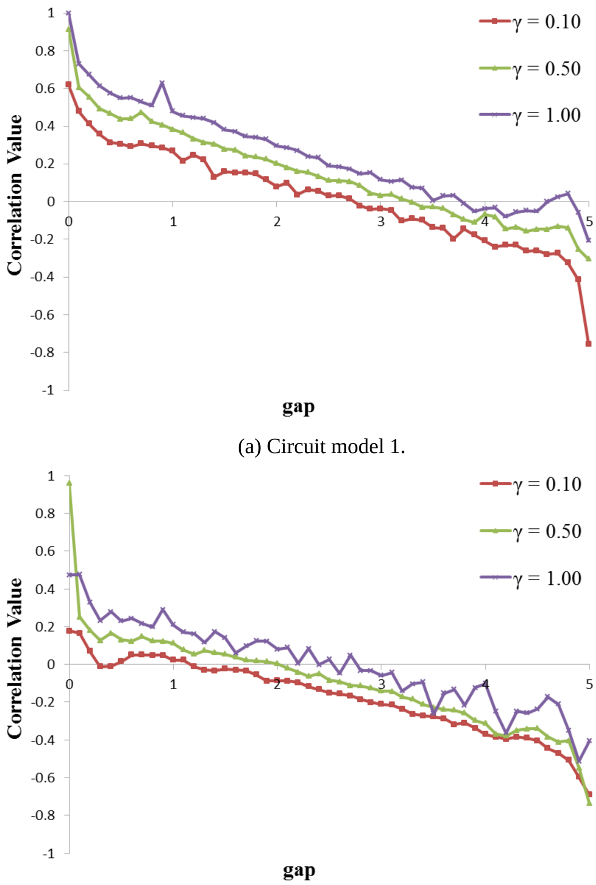


Figure 7: Chaotic attractor of circuit model 2 obtained from computer calculation ($\gamma = 0.5$). (a) $x_1 - x_2$, (b) $x_3 - x_4$, and (c) $x_1 - x_3$.



(a) Circuit model 1.

(b) Circuit model 2.

Figure 8: Relation between "gap" and correlation values.

clearly a difference when one compares $\gamma = 0.1$ and $\gamma = 1.0$ in Fig. 8(a). In Fig 8(a), when γ is large, we observe in-phase synchronization. However, we can not observe anti-phase synchronization when γ is large. By contrast, when γ is small, we observe anti-phase synchronization. However, we can not observe in-phase synchronization when γ is small. From these results, we find it is impossible to observe changing into in-phase synchronization to anti-phase synchronization or into anti-phase synchronization to in-phase synchronization by shifting input wave when two chaotic oscillators are coupled resistor. Furthermore, all locus are similar.

Figure 8(b) is almost similar result to Fig. 8(a) without $\gamma = 0.5$. We observe in-phase and anti-phase synchronization when $\gamma = 0.5$.

5. Conclusion

In this study, we have investigated synchronization phenomena observed in two coupled chaotic oscillators composed with shifting input wave. Two chaotic oscillators were coupled by resistor or capacitor. We carried out computer calculation to investigate. We could observe in-phase and anti-phase synchronization by using correlation coefficient. When two chaotic oscillators were coupled by resistor, we could not observed in-phase and anti-phase synchronization in the same parameter. When two chaotic oscillators were coupled by capacitor, we observed in-phase and anti-phase synchronization in the same parameter.

In our future works, we investigate to change input square wave into other wave as sin wave and saw wave.

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