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Synchronization Phenomena in Chaotic Circuit with Stochastically Coupling

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Abstract

In this study, we investigate the synchronization phenomena observed from two chaotic circuits with stochastically coupling. First, we research the basic synchronization phenomena. Second, we confirm on synchronization ratio of two chaotic circuit when the coupling probability is changed. Finally, we focus on synchronization ratio of chaotic circuit when the five coupling probability is changed. Then we compare the difference in synchronization ratio between two and five coupled chaotic circuits.

1. Introduction

Synchronization phenomena is one of the basic nonlinear phenomena and it can be observed everywhere in our living life [1]-[5]. For example, vibration of the cells of the human and begins to croak at the same time of the frog. Recently, in particular, synchronization phenomena in coupled chaotic circuits are attracted many researchers attentions. There are many real physical phenomena exhibiting by coupled chaotic circuits, and it is important to investigate synchronization phenomena observed from coupled chaotic circuits for future engineering applications. We consider that there are many real physical phenomena exhibiting by coupled chaotic circuits, and it is important to investigate synchronization phenomena observed from coupled chaotic circuits for future engineering applications.

In this study, we investigate the relationships between network connection and synchronization phenomena. First, we focus on synchronization ratio of two chaotic circuits when the coupling probability is changed. For this simulation, we change the bifurcation parameters in chaotic circuit and observe a change of synchronization ratio. Finally we focus on synchronization ratio of five chaotic circuits when the coupling probability is changed. Then, we compare difference of synchronization with two chaotic circuits and five chaotic circuits.

2. Circuit Model

2.1 Nishio-Inaba Chaotic Circuit

Figure 1 shows the model of the used chaotic circuit called Nishio-Inaba circuit, investigated in [6]-[7].
The approximated I-V characteristic of the nonlinear resistance is given in Fig 2.

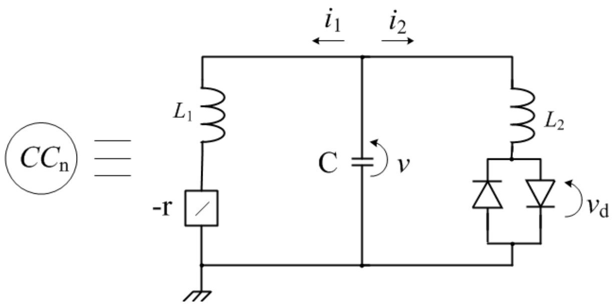


Figure 1: Nishio-Inaba Chaotic Circuit

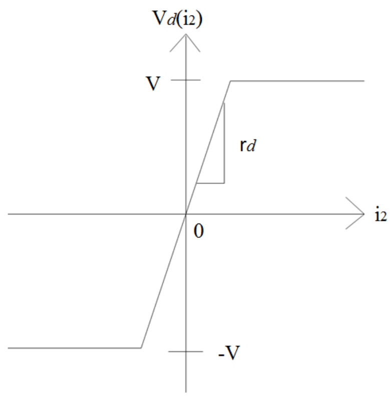


Figure 2: The I-V characteristic of the nonlinear resistance in Fig. 1.

The circuit consists of a negative resistance, a nonlinear resistance consisting of two diodes, a capacitor and two inductors. The following equation describes the approximated I-V characteristic of the nonlinear resistance , where the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) \;=\; \frac{r_d}{2}(|i_2 + \frac{V}{r_d}| - |i_2 - \frac{V}{r_d}|). \quad (1)$$

The circuit dynamics is then described by the following piecewise-linear third-order ordinary differential equation:

$$\begin{aligned} L_1\frac{di_1}{dt} &= v + ri_1 \\ L_1\frac{di_1}{dt} &= v - v_d(i_2) \\ L_1\frac{di_1}{dt} &= v - v_d(i_2). \end{aligned} \quad (2)$$

Equation (2) can be normalized by changing the variables according to

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\;\; i_2 = \frac{\sqrt{L_1C}}{L_2}Vy;\;\; v = Vz;$$

$$\alpha = r\sqrt{\frac{C}{L_1}};\;\; \beta = \frac{L_1}{L_2}\;\; \delta = r_d\frac{\sqrt{L_1C}}{L_2};.\;\; t = \sqrt{L_1C}\tau.$$

yielding

$$\begin{aligned} \dot{x} &= \alpha x_i + z_i \\ \dot{y} &= z - f(y) \\ \dot{z} &= -x_i - \beta y_i - \beta y \end{aligned} \quad (3)$$

where $f(y)$ is described as follows:

$$f(y) \;=\; \frac{\delta}{2}(|y + \frac{1}{\delta}| - |y - \frac{1}{\delta}|). \quad (4)$$

Next, we consider the following equations when each chaotic circuits is coupled globally.

$$\begin{aligned} \dot{x} &= \alpha x_i + z_i \\ \dot{y} &= z_i - f(y) \\ \dot{z} &= -x_i - \beta y_i - \gamma_{ij}\sum_{j=1}^{n}(z_i - z_j) \\ & \quad (i, j = 1, 2, \cdots, n) \end{aligned} \quad (5)$$

where i in the equation represents the circuit itself, and j indicates the coupling with other circuits. The parameter γ_{ij} represents the coupling strength between the circuits.

2.2 Propose Model

Figure 3 shows the circuit model used in this study. Each circuit is connected by a resistor. Also, coupled state (coupling or uncoupling) is determined by stochastically. We define the uncoupling probability ($= p$) in chaotic circuit. For example, the two chaotic circuits are coupled, if the uncoupling probability is $p = 0$%. Whereas, the two chaotic circuits are uncoupling, if the uncoupling probability is $p = 100$%.

In this study, we investigate the ratio of synchronization. In order to analyze synchronization ratio, we define the synchronization as following equation,

$$|z_M - z_N| < 0.1. \quad (6)$$

where M is the number of circuits and N is circuit which connected circuit M.

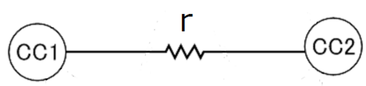


Figure 3: Circuit model ($n = 2$).

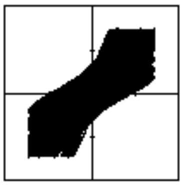
3. Synchronization Phenomena

3.1 Basic Phenomena

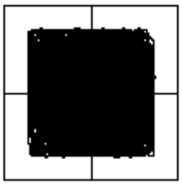
In this section, we compare the synchronization phenomena, in the case of $p = 0\%$ and $p = 100\%$. First, we set parameter as α= 0.46 , β= 3.0, γ= 470 and δ. In this study, we focus on the parameter δ. Figures 4 (a) and 4 (b) show the simulation result of the phase difference. From these results, we confirm synchronization phenomena state, in the case of $p = 0\%$ and $p = 100\%$.

3.2 Synchronization Phenomena with Stochastically Coupling ($n = 2$)

In this section, we focus on changing synchronization ratio with range from $p = 0\%$ to $p = 100\%$. For this investigating, we change the bifurcation parameters in chaotic circuit and observe a change of synchronization ratio. First, we set the parameters as α, β= 3.0, γ= 470, and δ= 0.64. Figure 5 shows the chaotic attractors in the case of α= 0.37, 0.4 and 0.46. Figure 6 shows the simulation results of the synchronization ratio by changing the bifurcation parameters of the coupled chaotic circuits with stochastically coupling. The horizontal axis is an indicator of uncoupling probability. The vertical axis is an



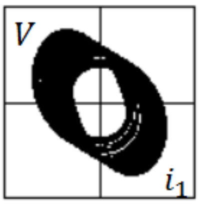
(a) coupling ($p = 0\%$).



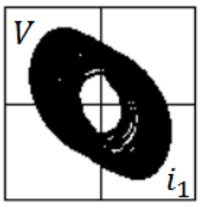
(b) un-coupling ($p = 100\%$).

Figure 4: Simulation results of phase difference.

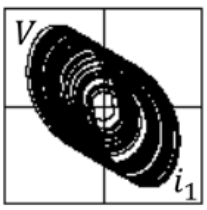
indicator of synchronization ratio. We confirm changing synchronization ratio of chaotic circuit when the coupling probability. From these results, the ratio of asynchronous increases with the uncoupling probability in each parameter α. Then, in the case of that the bifurcation parameter is higher, the synchronization ratio tends to low proportion.



(a) α=0.37.



(b) α=0.4.



(c) α=0.46.

Figure 5: Chaotic attractor.

3.3 Synchronization Phenomena with Stochastically Coupling ($n = 5$)

In this section, we investigate the synchronization phenomena with five coupled chaotic circuits ($n = 5$). Figure 7 shows the circuit model. Figure 8 shows the chaotic attractor. We set the parameter as α= 0.46, β= 3.0, γ= 470 and δ=

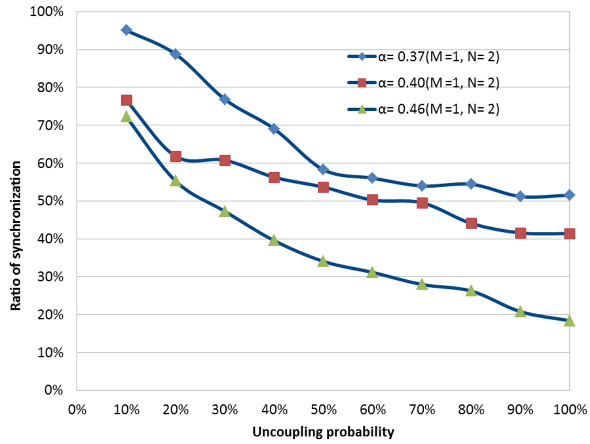


Figure 6: Simulation result ($n = 2$).

0.151. Then we compare difference in synchronization when the coupling probability is changed with two and five coupled chaotic circuits. Figure 9 shows the simulation result of the synchronization phenomena. From these results, there are similar results as far as 10% for 30%. However, by increasing the uncoupling probability, as the number of coupled chaotic circuits are larger, predisposition toward synchronization.

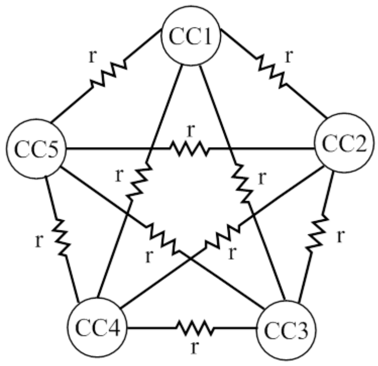


Figure 7: circuit model ($n = 5$).

4. Conclusions

In this study, we investigate the synchronization phenomena observed from chaotic circuits with stochastically coupling. We focus on synchronization in chaotic circuits with stochastically. For this investigation, we change the bifurcation parameters in chaotic circuit and observe a change in synchronization ratio. Also, we observe the difference in synchronization phenomena with two and five coupled chaotic circuits. By using the computer simulations, we have observed the interesting synchronization phenomena.

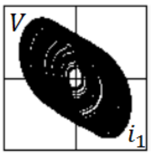


Figure 8: Chaotic attractor($n = 5$).

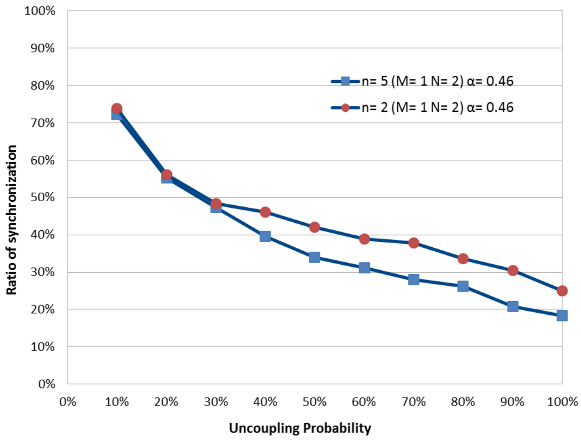


Figure 9: Simulation result.

In the future work, we would like to examine more complex networks. Additionally, we would like to investigate the mechanism of observed synchronization phenomena in detail.

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