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# Chaotic Circuit Using a Colpitts Oscillator and a Resonator

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**Abstract**

In this study, a novel chaotic circuit is proposed. The circuit consists of a Colpitts oscillator, a resonator and a diode. By using a idealized diode model, exact solutions are derived. Chaotic attractors are shown in circuit experiments and computer calculations of exact solutions.

## 1. Introduction

Chaos can be observed various field of the natural world. Therefore, many researchers pay attention to this phenomenon and related phenomena. In a field of electric engineering, chaotic phenomena have been investigated using a chaotic circuit. Elements of an electric circuit can be obtained in very low cost and the specific is very high. Additionally, a chaotic circuit has a simple structure. Thus, it is easy to generate chaotic phenomena on an electric circuit.

However, designing a novel chaotic circuit is difficult except experts like [1]–[3] . A designing method is proposed in [4]. This method is that coupling an oscillator and a resonator with diodes as shown in Fig. 1. The advantage is that the method is simple. Therefore, this method is very useful method for many researchers. On the other hand, this advantage is that there is no rule for setting the parameters. Breaking the disadvantage is essential to make this method perfected.

In this study, a novel chaotic circuit applied this method is proposed. The circuit consists of a Colpitts oscillator, a resonator and a diode. Developing many sample circuits is an important task for making this method perfected.

## 2. Circuit Model

Figure 2 is a proposed circuit in this study. A Colpitts oscillator connects to a resonator via a diode. In this model, all elements except a diode are modeled as linear elements. Diode model is shown in Fig. 3. Its $v$-$i$ characteristics is shown in Fig. 4. This model is an idealized model. Using this model,

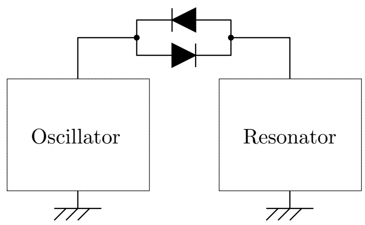


Figure 1: Designing method for a chaotic circuit.

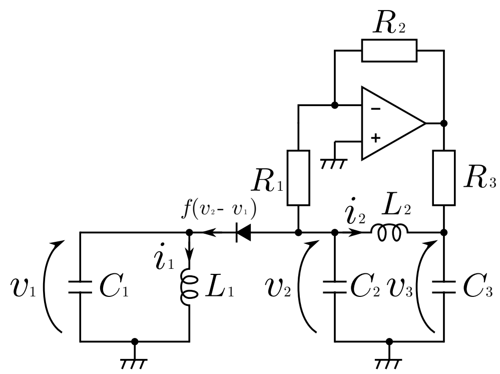


Figure 2: Proposed system.

the analysis becomes easily. In order to investigate validity of this model, a circuit equation is derived.

$$\begin{cases} (C_1 + C_2)\dfrac{dv_1}{dt} & = & -\dfrac{1}{R_1}(v_1 + V_{th}) - i_1 - i_2, \\ C_3\dfrac{dv_3}{dt} & = & -\dfrac{1}{R_3}\left\{v_3 + \dfrac{R_2}{R_1}(v_1 + V_{th})\right\} + i_2, \\ L_1\dfrac{di_1}{dt} & = & v_1, \\ L_2\dfrac{di_2}{dt} & = & v_1 - v_3 + V_{th}, \end{cases} \tag{1}$$

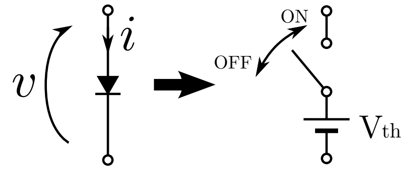


Figure 3: Idealized diode model.

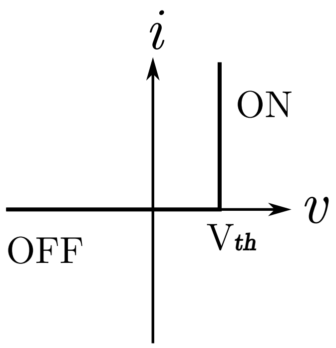


Figure 4: $v$-$i$ characteristic of the idealized diode model as shown in Fig. 3.

$$\begin{cases} C_1 \dfrac{dv_1}{dt} &= -i_1, \\ L_1 \dfrac{di_1}{dt} &= v_1, \end{cases} \tag{2}$$

$$\begin{cases} C_2 \dfrac{dv_2}{dt} &= \dfrac{C_1}{C_2}\left(-\dfrac{1}{R_1}v_2 - i_2\right), \\ C_3 \dfrac{dv_3}{dt} &= -\dfrac{1}{R_3}\left(v_3 + \dfrac{R_2}{R_1}v_2\right) + i_2, \\ L_2 \dfrac{di_2}{dt} &= v_2 - v_3. \end{cases} \tag{3}$$

By substituting the normalized variables and the parameters,

$$\begin{aligned} &x_n = \frac{v_n}{V_{th}} \quad \text{for} \quad n = 1, 2, 3, \\ &x_n = \sqrt{\frac{L_1}{C_1}}\frac{i_{n-3}}{V_{th}} \quad \text{for} \quad n = 4, 5, \\ &\frac{d}{dt} = \text{"}\cdot\text{"}, \quad \tau = \frac{1}{\sqrt{L_1 C_1}}t, \quad \alpha = \frac{1}{R_1}\sqrt{\frac{L_1}{C_1}}, \\ &\beta = \frac{R_2}{R_3}, \quad \gamma = \frac{1}{R_2}\sqrt{\frac{L_1}{C_1}}, \quad \delta = \frac{L_1}{L_2} \quad \text{and} \\ &\varepsilon = \frac{C_1}{C_2} = \frac{C_1}{C_3}. \end{aligned} \tag{4}$$

Equations (1), (2) and (3) are normalized as

$$\begin{cases} \dot{x}_1 &= \dfrac{\varepsilon}{\varepsilon+1}(-\alpha x_1 - x_4 - x_5 - \alpha), \\ \dot{x}_3 &= \varepsilon(-\alpha\beta x_1 - \beta x_3 + x_5 - \alpha\beta), \\ \dot{x}_4 &= x_1, \\ \dot{x}_5 &= \delta(x_1 - x_3 + 1), \end{cases} \tag{5}$$

$$\begin{cases} \dot{x}_1 &= x_4, \\ \dot{x}_4 &= x_1, \end{cases} \tag{6}$$

$$\begin{cases} \dot{x}_2 &= \varepsilon(-\alpha x_2 - x_5), \\ \dot{x}_3 &= \varepsilon(-\gamma x_3 - \alpha\beta x_2 + x_5), \\ \dot{x}_5 &= \delta(x_2 - x_3), \end{cases} \tag{7}$$

We define two piecewise-linear regions as follows.

$$\begin{aligned} &\mathbf{R_1} \equiv \{(x_1, x_3, x_4, x_5) | \dot{x}_1 + i_1 > 0\} \\ &\mathbf{R_2} \equiv \{(x_1, x_2, x_3, x_4, x_5) | \dot{x}_2 - x_1 - 1 < 0\} \end{aligned} \tag{8}$$

These regions are corresponding to the two state of the diode. Note that the circuit equations in $\mathbf{R_2}$ are completely decoupled into two and three dimensional equations. In order to derive exact solutions, eigenvalues in two regions are derived. The characteristic equation of the circuit equations in each linear region is given as follows.
In $\mathbf{R_1}$:

$$\begin{aligned} &(\varepsilon+1)m^4 + \varepsilon(\alpha+\beta+\beta\varepsilon)m^3 + \varepsilon(\alpha\beta\varepsilon + 2\delta \\ &+\delta\varepsilon + 1)m^2 + \varepsilon^2(\alpha\beta\delta + \alpha\delta + \beta\delta + \beta)m + \delta\varepsilon^2 \\ &= 0. \end{aligned} \tag{9}$$

In $\mathbf{R_2}$: For the resonator,

$$m^2 + 1 = 0. \tag{10}$$

For the Colpitts Oscillator,

$$\begin{aligned} &m^3 + \varepsilon(\alpha+\gamma)m^2 + \varepsilon(\alpha\gamma\varepsilon + 2\delta)m + \delta\varepsilon^2(\alpha + \alpha\beta \\ &+\gamma) = 0. \end{aligned} \tag{11}$$

The eigenvalues in each region can be calculated from the characteristic equations ( 9 ) - ( 11)

From circuit experiment and computer simulation using Eq. (5) - (7), The eigenvalues are calculated as follows:

$$\begin{aligned} &\mathbf{R_1} : \lambda_1, \lambda_2, \sigma \pm j\omega \\ &\mathbf{R_2}(\text{Resonator}) : \pm j \\ &\mathbf{R_2}(\text{Colpitts Osc.}) : \lambda, \sigma_1 \pm j\omega_1 \end{aligned} \tag{12}$$

Further, the equilibrium points in $\mathbf{R_1}$ is described as follows:

$$\mathbf{B} = [\quad -\alpha - \beta - \alpha\beta \quad 0 \quad 0 \quad 0 \quad ]^T. \tag{13}$$

These values are calculated by making the right-hand side of Eq. (5) to be equal to zero. The equilibrium point in $\mathbf{R_2}$ is the origin. Then, we can describe the exact solutions in each linear region as follows.
In $\mathbf{R_1}$:

$$\mathbf{x}(\tau) - \mathbf{B} = \mathbf{F}(\tau) \cdot \mathbf{F}^{-1}(0) \cdot (\mathbf{x}(0) - \mathbf{B}). \tag{14}$$

In $\mathbf{R_2}$:
For the resonator,

$$\mathbf{x_a}(\tau) = \mathbf{G_a}(\tau) \cdot \mathbf{G_a}^{-1}(0) \cdot (\mathbf{x_a}(0)). \tag{15}$$

For the Colpitts osc.,

$$\mathbf{x_b}(\tau) = \mathbf{G_b}(\tau) \cdot \mathbf{G_b}^{-1}(0) \cdot (\mathbf{x_b}(0)). \tag{16}$$

In order to derive the Poincaré map, Let us define the following subspace

$$\mathbf{S} = \mathbf{S_1} \cap \mathbf{S_2}. \tag{17}$$

where

$$\begin{aligned} \mathbf{S_1} &: \dot{x}_1 = -x_4 = \frac{\varepsilon(-\alpha x_1 - x_4 - x_5 - \alpha)}{(\varepsilon + 1)}. \\ \mathbf{S_2} &: x_2 - x_1 = 1. \end{aligned} \tag{18}$$

The subspace $\mathbf{S_1}$ corresponds to the boundary condition between $\mathbf{R_1}$ and $\mathbf{R_2}$, while the subspace $\mathbf{S_2}$ corresponds to the ON state of the diode. Namely, $\mathbf{S}$ corresponds to the transitional condition from $\mathbf{R_1}$ to $\mathbf{R_2}$. Let us consider the solution starting from an initial point on $\mathbf{S}$. The solution returns back to $\mathbf{S}$ again after two subspaces as shown in Fig. 5. Hence, the Poincaré map can be derived as follows:

$$\mathbf{T} : \mathbf{S} \to \mathbf{S}, \quad \mathbf{x} \mapsto \mathbf{T}(\mathbf{x_0}). \tag{19}$$

where $\mathbf{x_0}$ is an initial pont on $\mathbf{S}$, while $\mathbf{T}(\mathbf{x_0})$ is the pint at which the solution starting from $\mathbf{x_0}$ hits $\mathbf{S}$ again. The Poincaré map can be represented as a composit map of the submaps $\mathbf{T_1}$ and $\mathbf{T_2}$. Namely, the Poincaré map can be obtained as

$$\mathbf{T} = \mathbf{T_1} \circ \mathbf{T_2}. \tag{20}$$

## 3. Circuit Experiments and Computer Calculations

Figure 6 shows the circuit experimental results and computer calculated results. In the circuit experiments, parameters are set as $R_1 = 1$ [kΩ], $R_3 = 200$ [Ω], $C_1 = 0.10$ [$\mu$F], $C_2 = C_3 = 0.022$ [$\mu$F], $L_1 = 200$[mH] and $L_1 = 30$[mH]. In the computer calculations, parameters are set as $\alpha = 1.41$, $\gamma = 4.71$, $\delta = 6.67$ and $\varepsilon = 4.5$. $R_2$ and $\beta$ are set as control parameters. A periodic orbit is observed in Fig. 6 (1) and (5). Chaotic attractors are observed in Fig. 6 (2), (3) and

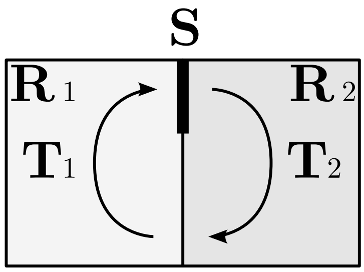


Figure 5: Route map.

(4). These results shows this model does not lose important features of the original circuit related with the generation of chaos. Additionally, the nonlinear element is only the diode. It is mean that the Colpitts oscillator plays a role of expanding and the diode play a role of folding. These are known as the essence of generating chaos.

## 4. Conclusion

In this study, the novel chaotic circuit has been proposed. By applying a idealized diode model, exact solutions of each regions are derived. Additionally, the Poincaré map is defined. In circuit experiments and computer calculations, chaotic phenomena are observed.

In our future works, calculating the largest Lyapunov exponents, the circuit analysis using exact solutions and so on will be carried out.

## References

[1] L. O. Chua, M. Komuro and T. Matsumoto, "The double scroll family," *IEEE Trans. Circuits Syst.*, vol. 33, no. 11, pp. 1073–1118, 1986.

[2] N. Inaba, S. Mori and T. Saito, "Chaotic Phenomena in 3rd Order Autonomous Circuits with a Linear Negative Resistance and an Ideal Diode II," *Technical Report of IEICE*, Vol. 87, No. 271, pp.9-15, Nov. 1987.

[3] M. Shinriki, M. Yamamoto and S. Mori, "Multimode oscillations in a modified van der Pol Oscillator Containing a Positive Nonlinear Conductance," *Proc. IEEE*, vol. 69, pp. 394–395, 1981.

[4] Y. Hosokawa, Y. Nishio and A. Ushida, "A Design Method of Chaotic Circuits Using an Oscillator and a Resonator," *Proceedings of IEEE International Symposium on Circuits and Systems (ISCAS'01)*, vol. 3, pp. 373-376, 2001.

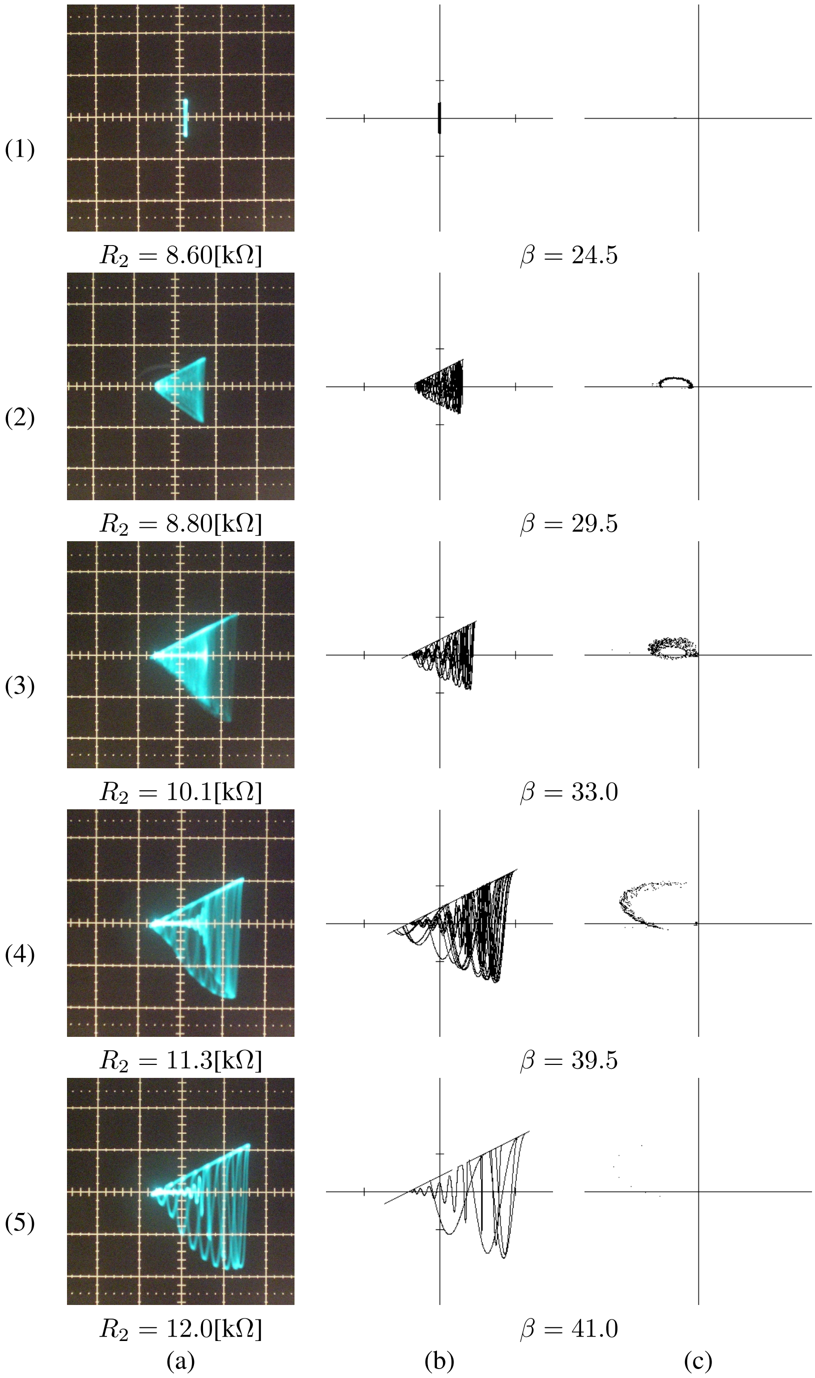


Figure 6: Circuit Experimental Results and Computer Calculation Results. (a) Circuit Experiments. Horizontal axis: $v_1$ ( 1.0 [V/div.] ). Vertical axis: $v_2$ ( 2.0 [V/div.] ). (b) Attractors. Horizontal axis: $x_1$. Vertical axis: $x_2$. Gridlines show 2.5. (c) Poincaré maps of (b). Horizontal axis: $x_1$. Vertical axis: $x_2$.