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Polygonal Structure Model of Ring Inhibitory Neural Network

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Abstract

The ring inhibitory neural network is a typical model to generate oscillation with a simple structure. Neurons of the ring inhibitory neural network are connected in one direction inhibitory. In this study, we propose the ring inhibitory neural network with polygonal structure. This model is composed by some ring inhibitory neural networks as conventional model. We confirm that polygonal models which based on the conventional triquetrous / pentagonal model composed by three and five neurons can oscillate. In these simulations, polygonal structure models show various oscillations by changing symmetry and circulation conditions.

1. Introduction

The neural network is calculation models which can replicate some functions of human brain. This is applied to data processing, data mining, and so on. In addition, the oscillation model of neural network is applied to pattern recognition and walking rhythm outbreak circuit[1]-[4]. The ring inhibitory neural network is a one of typical oscillation model with a simple structure and it is composed by connected neurons in one direction. Weights between each neuron are defined as nagative because neurons composing the model are inhibitory[5].

In this study, we propose the polygonal ring inhibitory neural network. The structure of polygonal model is constructed by combined conventional models under some condition. Futuremore, we explain about oscillation and synchronization phenomena of polygonal structure models.

2. Oscillation System

2.1 Ring Inhibitory Neural Network

Neural networks with limit cycle are classified in two model. One is the network model which has self-connection, and the another one does not have self-connection like ring inhibitory neural networks. We apply the ring inhibitory neural network which is comprised of connecting neurons in one direction. The ring inhibitory neural network is composed with connecting neuron to one direction. Figure 1 shows the network model of the ring inhibitory neural network. The dynamical system of this model is described as Eqs. (1) and (2). Moreover, the output function is defined to arctangent as Eq. (2).

$$\tau \frac{du_i}{dt} = -u_i + W_{ji} f(u_j) \tag{1}$$

$$f(u) = tan^{-1} u \tag{2}$$

Where τ is the time constant. Neurons of the ring inhibitory neural network are connected unidirectionally with the weight W_{ji} of negative value. The oscillation phase of this model is same as the number of neuron m. Oscillation is caused if the weight W_{ji} is set to negative value. In addition, this model can oscillate if the number of neuron is odd. We explain about this term to focus on equilibrium point. The equilibrium point u_0 has to fulfill Eq. (3). The state of N_1 is substituted sequentially as follows, when the number of neuron is m.

$$-u_i + W_{ji} f(u_j) = 0 \tag{3}$$

$$u_1 = W_{m1} f(W_{mm-1} f(W_{m-1m-2} \cdots W_{21} f(u_1) \cdots)) \tag{4}$$

Equation (4) is monotone decreasing function if the number of neuron m is odd. In case the function f is monotonic decrease, systems have only one equilibrium point.

Figure 2 shows dynamics of the ring inhibitory neural network which is comprised by three neurons. This model oscillates three-phase waveform. From Eqs. (1) and (2), the equilibrium point u_0 of the ring inhibitory neural network is the

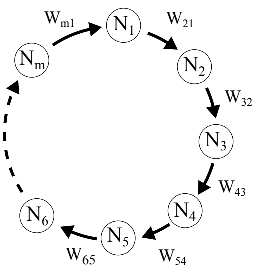


Figure 1: Ring inhibitory neural network.

origin. In addition, stability of the equilibrium point depends on the weight W_{ji}. We define jacobian matrix of the right side of Eq. (1) as matrix A. The eigenvalue λ of matrix A at the equilibrium point is calculated from the following Eq. (5).

$$det|A - \lambda E| = 0 \tag{5}$$

The equilibrium point is unstable when the ring inhibitory neural network shows limit cycle. This point is stable if real parts of all eigenvalues are negative. Therefore, the model causes hop bifurcation, and it shows limit cycle if the weight is set to $W_{ji} < -2.0$.

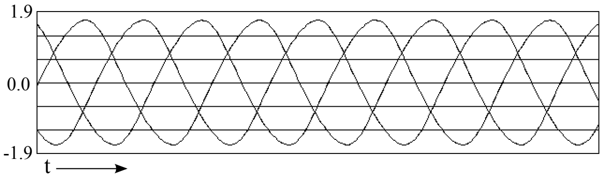


Figure 2: Dynamics of triquetrous ring inhibitory neural network ($m = 3$, $W_{ji} = -3.0$, $\tau = 1.0$).

2.2 Polygonal Structure Model

In this section, we explain about the polygonal structure model. In this study, we propose to extend the polygonal structure from the ring inhibitory neural network. The polygonal models are comprised by connecting some ring inhibitory neural networks under certain condition. We can confirm synchronization of neurons and characteristic oscillation by applying polygonal structure. The dynamical system of this model is described in Eq. (6). Weights W_{ji} between coupled neurons are unified negative value each. Whereas, the weight W_{ji} between disconnected neurons is defined as zero.

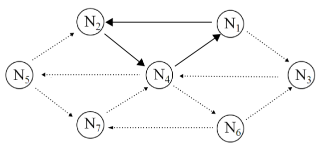
$$\tau \frac{du_i}{dt} = -u_i + \sum_{j=1}^{m} W_{ji} f(u_j) \tag{6}$$

In addition, we define some condition of connections. Connections of neuron are set to one direction which does not disturb circulation. This connection method obeys the base model like a triquetrous and pentagonal conventional network. In other words, the output of neuron circulate through three neurons, for example N_1, N_2 and N_4 in Fig. 3(a). Network models of polygonal structure based on triquetrous / pentagonal ring neural networks are shown on Figs. 4 and 5 each.

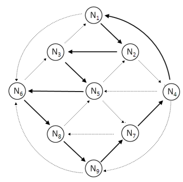
Next, we focus on stability and the equilibrium point of the polygonal models based on triquetrous network. It is important that we analyze the equilibrium point to understand the characteristic of the behavior of the system. From Eqs. (2) and (6), the equilibrium point u_0 of the polygonal model is the origin. This model can oscillate if each neuron causes limit cycle around the origin. Therefore, the equilibrium point must be unstable. The eigenvalue λ is found from Eq. (5) if jacobian matrix of the right side of Eq. (6) is matrix A. The oscillation conditon of $m = 7$ asymmetry polygonal model is $W_{ji} < -1.1$. In the same way, the oscillation condition of $m = 6$ symmetry polygonal model is $W_{ji} < -1.0$.

Moreover, Fig. 4(a) shows the non-oscillation network model which removed some combination from $m = 9$ polygonal model. This model can not oscillate in spite of having a unstable condition. Such phenomenon is confirmed in many non-oscillation models. Now, we define the loop number L. The loop number L means the number of neurons of the path through the neurons most in a single stroke. For example, the loop number of the $m = 7$ polygonal model is $L = 3$ by binding of solid lines like a Fig. 3(a). While the $m = 9$ polygonal model has $L = 9$ due to pass nine neurons when trace solid lines like a Fig. 3(b). Moreover, the loop number is related to the oscillation condition. The polygonal model can not oscillate except some models if the loop number L is even for example the non-oscillation model in Fig. 4(d). While models which have odd loop numbers like Figs. 4(a), 4(b), and 4(c) can oscillate.

Futuremore, the polygonal model can be split into three categories, asymmetry models of partial, the one of entire circulation, and symmetry models. The network is determined as the symmetry model if the number of input and output of all neurons is the same. The models of Figs. 4(d) and 5(b) are classified into the symmetry model. The asymmetry model under the neuronal number $m > L$ is defined asymmetry of partial circulation. The models of Figs. 4(a), 5(a) and 5(c) are classified into the asymmetry model of partial circulation. On the other hand, the asymmetry model of entire circulation is the case of $m = L$. The model of Fig. 4(a) is classified into asymmetry model of entire circulation. The oscillation of polygonal model varies among these models. From this, the loop number L is similar as the number of neuron in the conventional models.



(a) $m = 7$ model ($L = 3$).

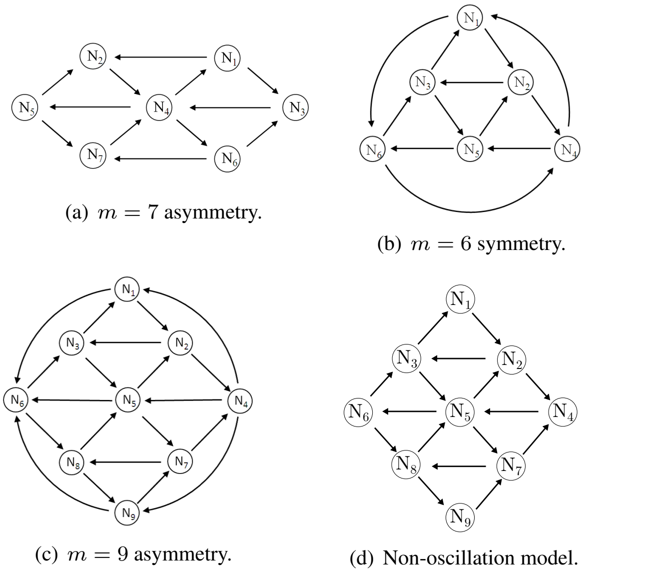


(b) $m = 9$ model ($L = 9$).

Figure 3: Polygonal model and circulation route.

3. Simulation Result

In this study, we confirm oscillation by the Runge-Kutta method. The weight W_{ji} is set to $W_{ji} = -3.0$. The time constant τ is set to $\tau = 2.0$.



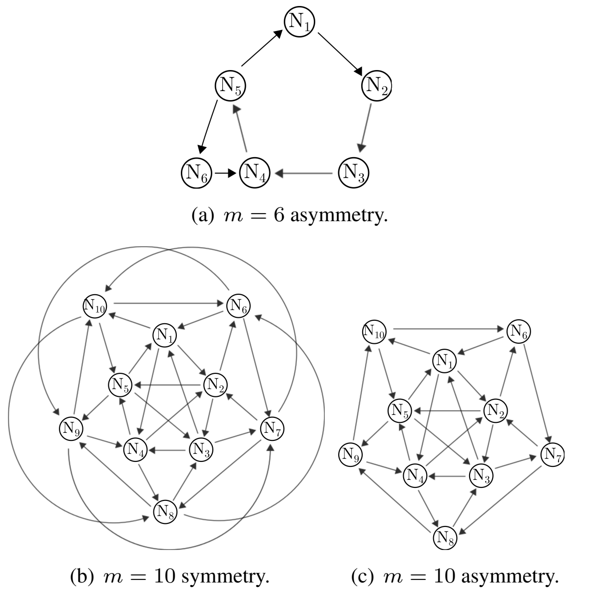
(a) $m = 7$ asymmetry.

(b) $m = 6$ symmetry.

(c) $m = 9$ asymmetry.

(d) Non-oscillation model.

Figure 4: Polygonal model based on $m = 3$ (triquetrous model).



(a) $m = 6$ asymmetry.

(b) $m = 10$ symmetry.

(c) $m = 10$ asymmetry.

Figure 5: Polygonal model based on m = 5 (pentagonal model).

3.1 Polygonal Model based on m = 3

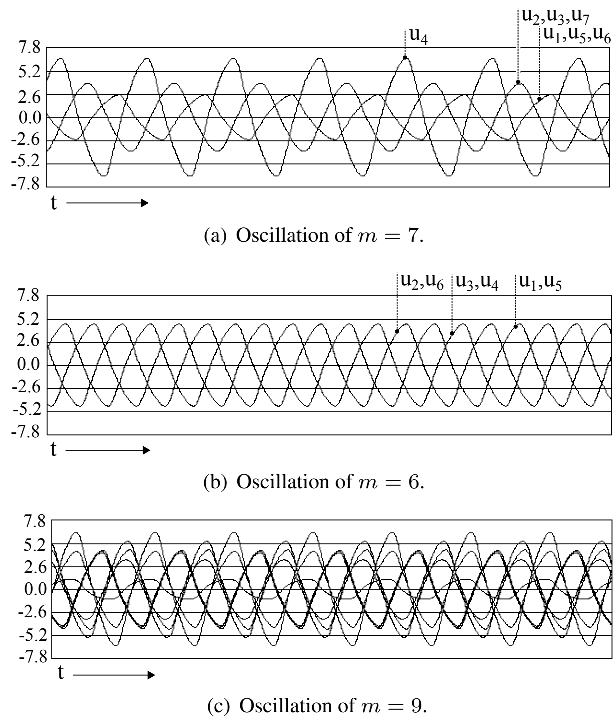
In this section, we explain about simulation results of the polygonal structure model based on conventional model which was composed by three neurons.

From the $m = 7$ polygonal model of Fig. 4(a), this model is classified into an asymmetry model of partial circulation. This model has the loop number $L = 3$ and oscillates the waveform like Fig. 6(a). The $m = 7$ polygonal asymmetry model of partial circulation oscillates three-phase waveform, and each wave pattern has the different amplitude. In addition to this, neurons which have the same number of input and output synchronize like u_1, u_5 and u_6. This becomes clear by calculating the circuit equation.

From the $m = 6$ polygonal model of Fig. 4(b), this model is classified into a symmetry model. The loop number L of this model is $L = 6$. The waveform which is oscillated by this model is shown on Fig. 6(a). This model oscillate three-phase waveform, and each wave pattern has the same amplitude only in this case. As a different from the asymmetry model, only this model can oscillate in spite of having an even loop number. Neurons at the symmetry position synchronize like u_1 and u_5. This becomes clear by calculating a circuit equation.

From the $m = 9$ polygonal model of Fig. 4(c), this model is classified into an asymmetry model of entire circulation. Figure 6(c) shows the oscillation of this model. The asymmetry model of entire circulation oscillates nine-phase waveform, and each wave pattern has the different amplitude. As different features with the $m = 7$ and $m = 6$ models, this oscillates L-phase wave without some models if the loop number is L only in this case.

The model of Fig. 4(d) can not oscillate, because this model is asymmetric and the loop number is $L = 6$. From this result, we consider that the asymmetric model of even L can not oscillate.



(a) Oscillation of $m = 7$.

(b) Oscillation of $m = 6$.

(c) Oscillation of $m = 9$.

Figure 6: Oscillation of the polygonal model based on $m = 3$ triquetrous network ($W_{ji} = -3.0$, $\tau = 2.0$).

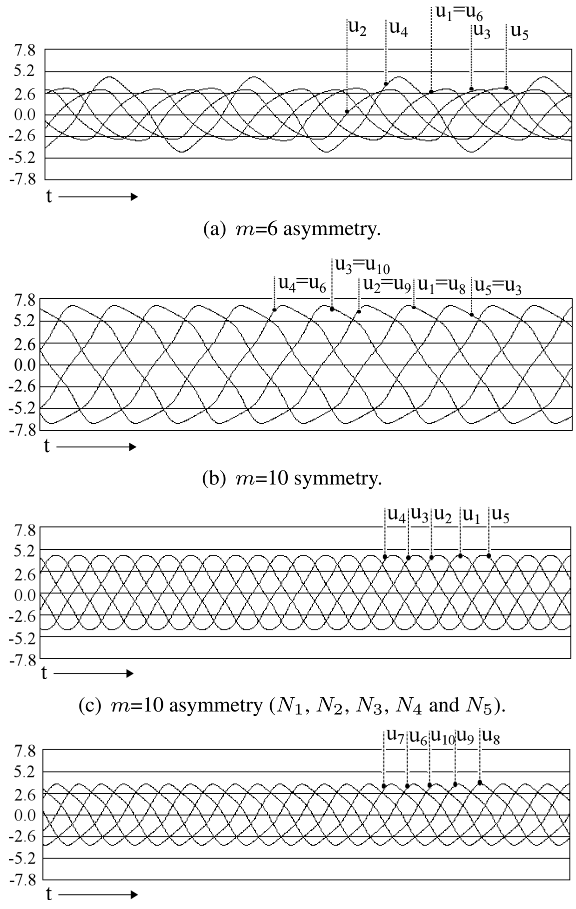
3.2 Polygonal Model based on m = 5

In this section, we explain about simulation results of the polygonal model based on $m = 5$ (pentagonal model). The $m = 5$ conventional model oscillates five-phase wave, and neurons of this model have unified amplitude.

The network model in Fig. 5(a) shows the combination of the triquetrous and pentagonal models. This model fulfills condition of asymmetry model of partial circulation. Figure 7(a) shows an oscillation of this model. The $m = 10$ polygonal model oscillates five-phase wave, and wave patterns have the different amplitude each. Neurons which have the same number of input and output synchronize as u_1 and u_6. This becomes clear by calculating a circuit equation. The feature of this model follows an asymmetry model of partial circulation.

The network model in Fig. 5(b) is the $m = 10$ symmetric model. This model fulfills condition of the symmetry model, because values of input and output of all neurons are the same. The oscillation of this model shows in Fig. 7(b). The $m = 10$ model oscillate five-phase wave, and wave patterns have the same amplitude each like the symmetry model of Fig. 4(b). In addition, neurons located on symmetry oscillate same wave like N_1 and N_8. These models have same characteristic as the polygonal models based on $m = 3$ network model.

Figure 5(c) shows the network model of $m = 10$ asymmetry. The oscillation of $m = 10$ asymmetric model is shown in Figs. 7(c) and 7(d). Two kinds of five-phase wave are oscillated simultaneously. The input and output value of N_1 to N_5 are three and the one of N_6 to N_{10} neurons are two. While the loop number L is $L \neq 5$, the waves oscillated by this model show five-phase.



(a) m=6 asymmetry.

(b) m=10 symmetry.

(c) m=10 asymmetry (N_1, N_2, N_3, N_4 and N_5).

(d) m=10 asymmetry (N_6, N_7, N_8, N_9 and N_{10}).

Figure 7: Oscillation of the polygonal model based on m=5 pentagonal network (W_{ji}=-3.0, τ=2.0).

4. Conclusion

In this study, we have proposed the structure model based on the ring inhibitory neural network and studied synchronization and oscillation phenomena. The polygonal structure models can be split into three categories and oscillate different waveform in each model. Synchronization phenomena are confirmed by the simulation. The polygonal model can not oscillate if the loop number L is even in the asymmetry model. Here, the eigenvalue of non-oscillation model is instability. Finding cause of this matter is a future work. The multi-phase oscillation is confirmed on some polygonal models. It is expect that this phenomenon develops in large scale networks. In addition, analyzing oscillations of the polygonal model by the averaging method is desirable.

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