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Synchronization of Switching Phenomena Observed in Discrete and Continuous Coupled Chaotic Systems

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Abstract— **Synchronization of switching phenomena can be observed in a few coupled chaotic systems. Synchronization of self-switching phenomena have reported by Sekiya et al. Synchronization of self-switching phenomena is generated when chaotic elements are asynchronized. In our past study, we have observed similar phenomena on coupled chaotic circuits with Nishio circuit. Although coupling topology and coupled elements are different each other, synchronization of switching phenomena have observed in each system. In this study, we have investigate the phenomena on coupled chaotic maps. This study shows that the phenomena can be observed in discrete and continuous system. Synchronization of self-switching phenomena have generality and may be observed in many natural system.**

I. Introduction

There are many of nonlinear phenomena in the natural world. For example, chaos synchronization, clustering phenomenon, and so on. It is important that grasping the mechanism of those phenomena to clarify nonlinear phenomena exist in the natural world. Especially, synchronization phenomena observed in coupled chaotic systems have been attracted attentions since these system might be regarded as the models of natural system and give contribution to clarify the high-order complex nonlinear phenomena.

On the other hand, some chaotic elements have coexisting attractors. Double-scroll attractor can be observed by changing the parameters for those chaotic elements. In chaotic oscillators, double-scroll means the behavior that the solution switches two attractors without external force. This phenomenon is called "self-switching phenomenon". Moreover, "synchronization of self-switching phenomena" have reported by Sekiya et al[1]. The phenomena mean that the switching of attractor is occur synchronously in each subcircuit though the system keeps asynchronous state. It is very interesting that there are simultaneity though the system behaves disorderly. We have observed similar phenomena in simple coupled chaotic circuit[2]. The study shows that the phenomena have generality and might be observed high-order coupled chaotic system. It should be noted that the coupled chaotic system, the chaotic element of which are included in each study, has double-scroll attractor. In this study, we investigate synchronization of switching phenomena on coupled chaotic map. The investigated system is composed with a cubic function. This chaotic element have symmetrical structure and the element which is used in previous study also have same characteristic. Namely, it is considered that a cubic function belongs to double-scroll family. Although coupled chaotic map is discrete system, synchronization of self-switching phenomena have observed. This study indicates that the phenomena have broad generality, and gives contribution to clarify the mechanism of synchronization of swithcing phenomena.

II. Coupled Chaotic Systems

Coupled chaotic system shown in Fig. 1 is investigated in this study. Chaotic elements are shown in Fig. 2 (a) and (b). Figure 2 (a) is a simple chaotic oscillators proposed by Nishio et al. Equations for each system are described as follows. In chaotic element as shown Fig. 2, by changing parameters and

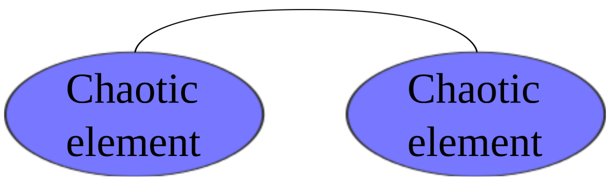
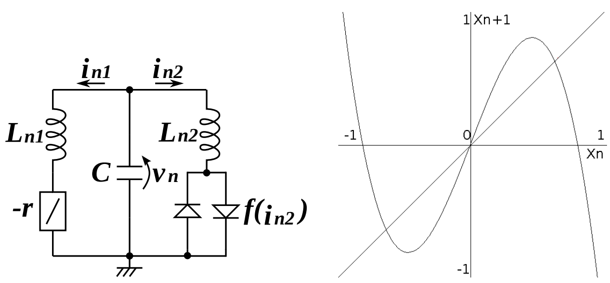


Fig. 1. System model.



(a) (b)

Fig. 2. Chaotic elements. (a) Chaotic circuit. (b) Cubic function.

variables as follows,

$$t = \sqrt{L_1 C}\tau, \quad i_{n1} = V_{th}\sqrt{\frac{C}{L_1}}x_n, \quad i_{n2} = y_n,$$
$$v_n = V_{th}z_n, \quad \text{"}\cdot\text{"} = \frac{d}{d\tau}, \quad \alpha = g\sqrt{\frac{C}{L_1}}, \; v_{dn} = V_{th}v'_{dn}$$
$$\beta = V_{th}\frac{\sqrt{L_1 C}}{L_2}, \; \gamma = \frac{1}{V_{th}}\sqrt{\frac{C}{L_1}} \; \text{ and } \; \delta = G\sqrt{\frac{L_1}{C}} \qquad (1)$$

the system equation of Fig. 2 (a) is described as follows:

$$\begin{cases} \dot{x}_n = \alpha x_n + z_n \\ \dot{y}_n = \beta\{z_n - v'_{dn}\} \\ \dot{z}_n = -x_n - \gamma y_n - \delta\left(N z_n - \sum_{k=1}^{N} z_k\right) \end{cases} \qquad (2)$$

We approximate i-v characteristic of bi-directionally coupled diodes as following function.

$$v'_{dn} = (2u(y_{n2}) - 1) \qquad (3)$$

Chaotic element as shown Fig.2 (b) is written as

$$f(x) = -ax + ax^3. \qquad (4)$$

Then, system equation is described as follows:

$$x_{n+1}(i) = (1-\epsilon)f(x_n(i)) + \frac{\varepsilon}{N}\sum_{j=1}^{N} f(x_n(j)). \qquad (5)$$

Where N and $u()$ denote the number of chaotic elements and step function respectively.

III. Coexisting attractors

Figure 3 shows the bifurcation diagram and coexisting attractors of chaotic element as shown Fig. 2 (a) and (b) respectively. Each bifurcation diagram is described with different initial values. Period-doubling bifurcation have begun at different point of the vertical because these chaotic element are the double-scroll family. Coexisting attractors of each element are shown in Fig. 3 (2). We distinguished each coexisting attractors from bifurcation diagram as follows.

In chaotic element (a),

(1) The Poincaré section is defined as $z_n = 1.0$ and $\dot{y}_n < 0$.

(2) When the solution hits the Poincaré section, and $x_n \leq 0.675$, the color is set as red. In the case of $y_n > 0.675$, the color is set as blue.

In chaotic element (b),

(1) When $x_n(i) \leq 0$, the color is set as red. In the case of $x_n(i) > 0$, the color is set as blue.

These definitions are important to investigate switching of attractors and as such, are being applied to the simulations of this study.

TABLE I
Attractor-state definition.

Attractor-state	State
{Red, Red}	A
{Red, Blue}	B

IV. In case of N = 2.

Figure 4 shows simulation results of each system model. These figures show time waveforms Fig. 4(1) and lissajous figure and scatter diagram Fig. 4(2). In the time waveforms, there red and blue waves that are show attractor state. Namely, the change of color means switching phenomena have occurred. Green line also shows a difference of attractor state. Synchronization of switching phenomena occurred in Fig. 4. In each simulation, the amplitude of $z_1 - z_2$ or $x_n(1) - x_n(2)$ is observed that denote each system keeps asynchronous state. Although, asynchronous state is kept at each system, switching phenomena are synchronized. We call such phenomena "synchronization of switching phenomena". These phenomena are collapsing according to decrease the coupling strength. In circuit experiment, similar phenomena also can be observed as shown Fig. 5. It should be noted that anti-phase switching is observed in both systems.

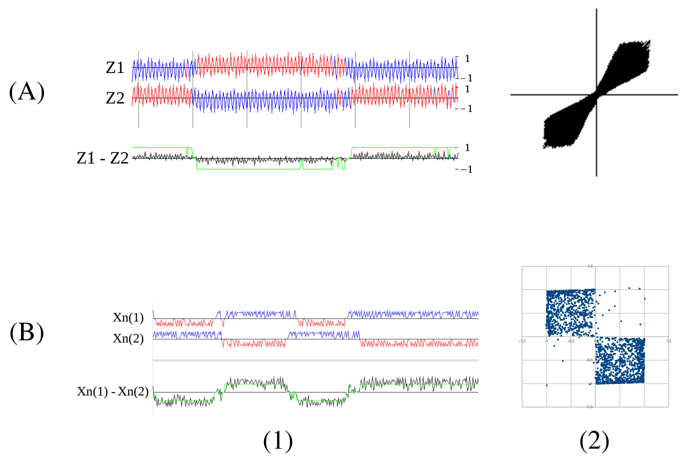
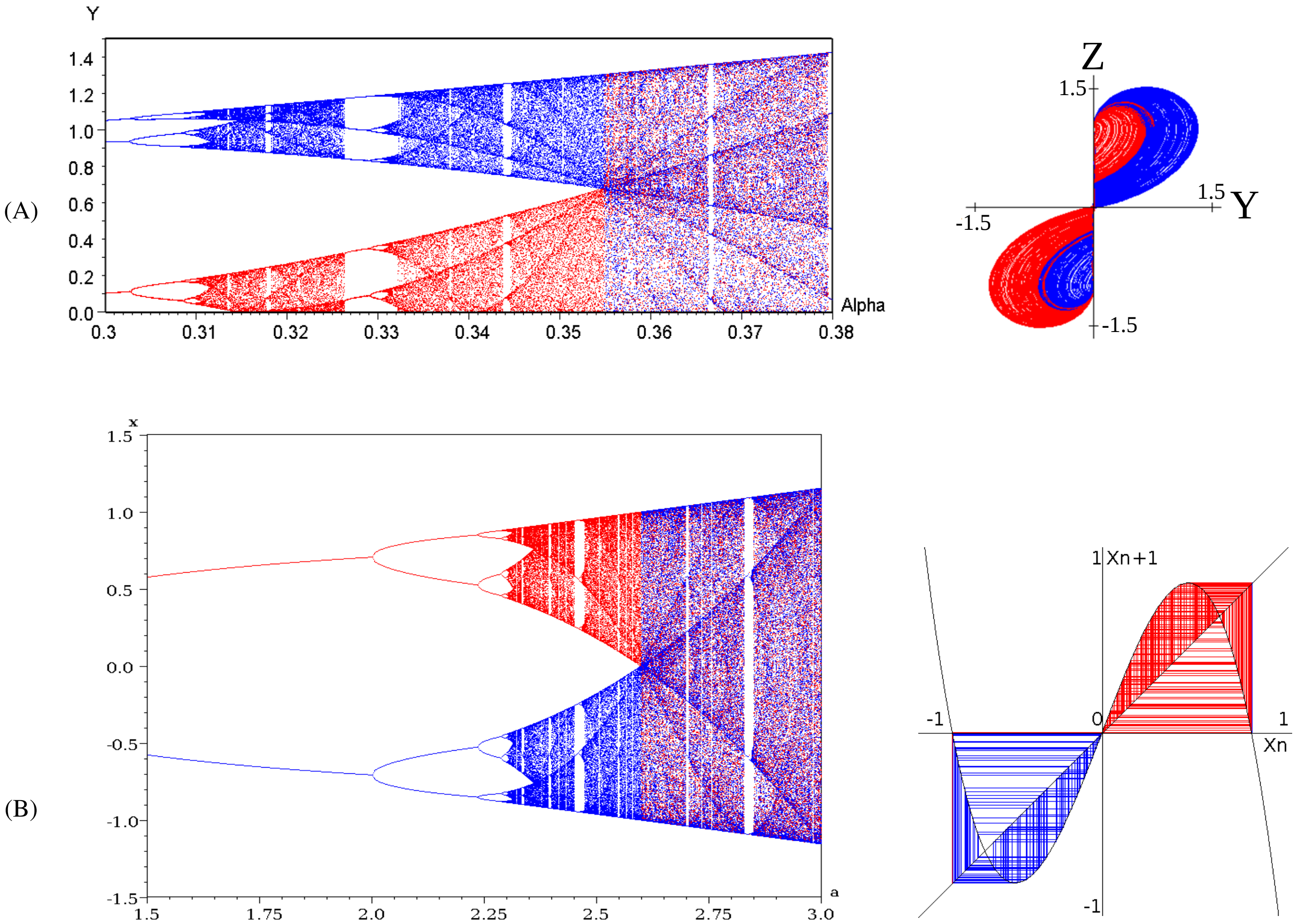


Fig. 4. Computer simulation result. (A) Chaotic element as shown Fig. 2(a). $N = 2$, $\alpha = 0.40$, $\beta = 3.0$ and $\delta = 0.2$. (1) Time waveforms of z_i and $z_1 - z_2$. (2) z_1 vs z_2. (B) Chaotic element (b). $N = 2$, $a = 2.8$ and $\varepsilon = 0.875$.(1) Time waveforms of $x_n(i)$. (2) Scatter diagram of $x_n(1)$ vs $x_n(2)$.

V. Mechanism of Synchronization of Switching Phenomena

In this section, we investigate the mechanism of synchronization of switching phenomena. Figure 6 shows the distribution of $x_n(1)$. Note that the requirement of switching is when the element satisfy $|x_n(i)| > 1$. Blue pins show the distribution of $x_n(1)$ in case of state A, red pins show in case of state B. Table. I shows state definition of attractor. Red pins are sifted to center of the distribution compare with blue pins. Namely, in case of state B, switching phenomena



(1) (2)

Fig. 3. Computer simulation results. (A) Chaotic element as shown in Fig. 2(a). (1)Bifurcation diagram. $\beta = 3.0$. Horizontal: α. (2) Coexisting attractors. $\alpha = 0.4$ and $\beta = 3.0$. (B) Chaotic element as shown Fig. 2(b).(1) Bifurcation diagram. Horizontal: a. (2) Coexisting attractors. $a = 2.8$.

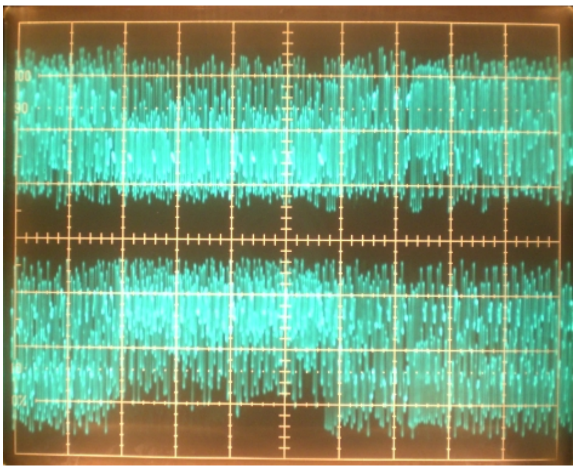


Fig. 5. Circuit experiment of coupled chaotic circuit of Fig. 2 (a). Vertical:v_n[2V/dvi], Horizontal:time[10ms/dvi]. $N = 2$, $L_1 = 642.8$[mH], $L_2 = 211.1$[mH], $C = 7.06$[nF], $G = 0.089$[mS].

hardly occur than state A. Figure 8 shows the probability of switching. We measured the probability of switching in $1 < n < 100000$. Both state are decreased according to increasing the coupling strength. However, state A have a certain probability of switching independent of ϵ. On the contrary, the probability is decreased substantially according to increasing the coupling strength. Namely, state B is stable compare with state A for this system. Such behavior can be explained as follows. In case of state A, each element has positive value. Therefore probability of switching is increased since the absolute value of $x_n(1)$ become large. In case of state B, an element takes positive value while the other takes negative value. For such occasion, the absolute value of $x_n(1)$ become small and probability of switching is decreased. It is considered that the mechanism of switching phenomena in this system. Figure 7 shows the probability of switching per 1τ in the system of Fig. 2 (b). We measured the probability in $2000 < \tau < 2000000$. The probability is also decreased by increasing the coupling strength. We assume that there are similar mechanisms in the system.

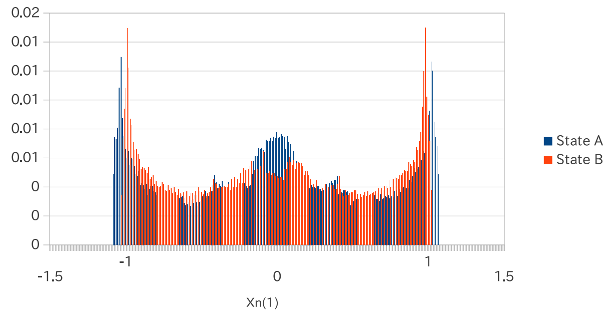


Fig. 6. Distribution of the value. $a = 2.8$, $\varepsilon = 0.875$.

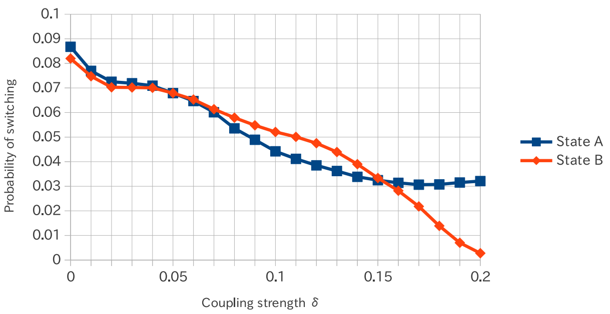


Fig. 7. Probability of switching of element as shown Fig. 2(a). $\alpha = 0.4$, $\beta = 3.0$.

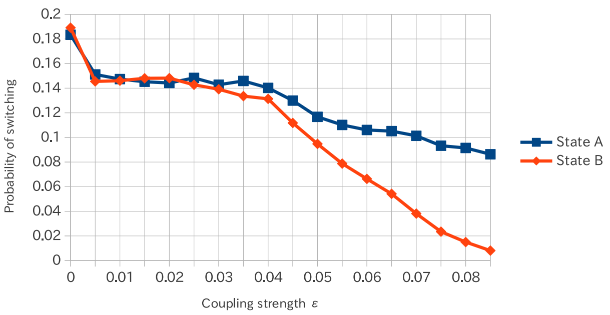


Fig. 8. Probability of switching of Fig. 2(b). $a = 2.8$.

VI. In case of N = 3

Figure 10 shows simulation results of coupled chaotic map in case of N = 3. Synchronization of switching phenomena can be observed, however, the phenomena is not occurred constantly. Additionally, the simultaneity of switching is slightly collapsed. Figure 9 shows simulation result of coupled chaotic circuit. In this case, the phenomena are not also occurred constantly. Anti-phase switching is observed in both systems. In case of N = 3, the systems obey the mechanism explained in section V. In the system proposed by Sekiya et al, synchronization of switching phenomena is observed with in-phase

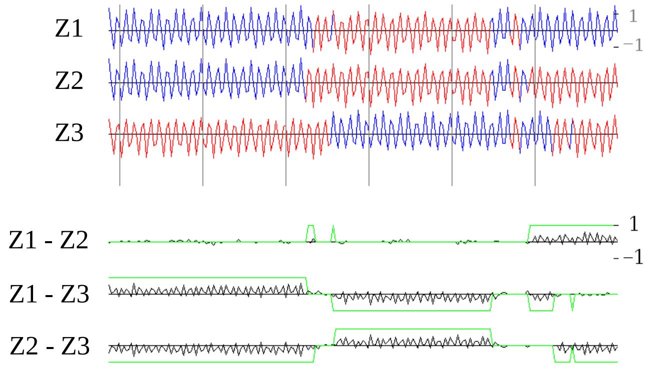


Fig. 9. Computer simulation resul of chaotic element as shown Fig. 2(a). $N = 3$, $\alpha = 0.4$, $\beta = 3.0$ and $\delta = 0.17$.

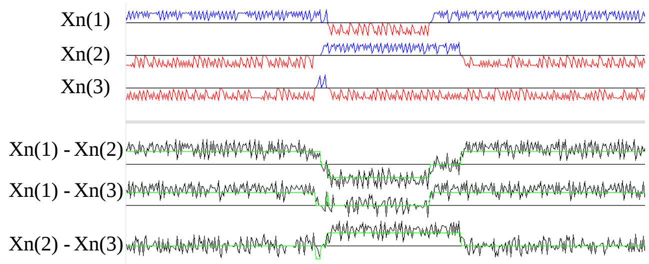


Fig. 10. Computer simulation resul of chaotic element as shown Fig. 2(b). $N = 3$, $a = 2.8$, $\delta = 0.105$.

switching.

VII. Conclusion

In this study, synchronization of switching phenomena have investigated in continuous and discrete systems. As a result, anti-phase synchronization of switching phenomena have observed in each system. Additionally, mechanism of the phenomena have declared in discrete system. This mechanism give contribution to clarify the phenomena observed in continuous system. These results denote that synchronization of switching phenomena may be observed in other natural system generally.

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