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Synchronization of an Array of Chaotic Circuits Coupled by Different Coupling Strength

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Abstract

This paper presents synchronization of coupled chaotic circuits as an array topology when coupling strength has different value depending on distance between the chaotic circuits. By using computer simulations, we observe interesting synchronization phenomena. Namely, several types of synchronization are observed by depending on arrangement method of chaotic circuits.

1. Introduction

Synchronization is one of the basic phenomena and it can be observed everywhere in our living life [1]-[5]. Therefore, studies on synchronization phenomena have investigated in the various fields. In particular, we are interested in synchronization phenomena observed in electronic circuits. We consider that there are many real physical phenomena exhibiting by coupled chaotic circuits, and it is important to investigate synchronization phenomena observed from coupled chaotic circuits for future engineering applications.

Recently, we have investigated synchronization phenomena observed from complex networks of the coupled chaotic circuits [6], [7]. The chaotic circuits are placed on 2-dimensional space and the coupled parameters depend on the distance between the circuits. We have observed interesting clustering phenomena from this proposed system by computer simulations and circuit experimental results. In order to understand the obtained results in detail, we need to consider more simple circuit system.

In this study, we investigate synchronization phenomena in coupled chaotic circuits as a ladder topology (1-dimensional space). In this circuit system, the chaotic circuits are arranged in one dimensional space irregularly. The value of coupling strength depends on the distance between the circuits. By using computer simulations, we confirm that the coupled chaotic circuits are synchronized at in-phase or asynchronous states. The chaotic circuits are synchronized at in-phase when the arrangement of chaotic circuits is symmetry, however the chaotic circuits are not synchronized when the chaotic circuits are arranged in asymmetry. The obtained synchronization states depend on the coupling strength and the circuit parameters.

2. Circuit model

Figure 1 shows the model of the used chaotic circuit called Nishio-Inaba circuit, investigated in [8]-[10].

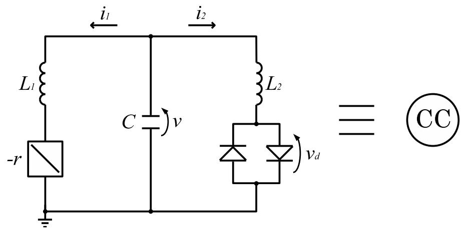


Figure 1: Nishio-Inaba chaotic circuit.

The circuit consists of a negative resistance, a nonlinear resistance consisting of two diodes, a capacitor and two inductors. The following equation describes the approximated $I - V$ characteristic of the nonlinear resistance, where the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \quad (1)$$

By using the variables and the parameters as follows;

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1C}}{L_2}Vy;\ v = Vz;$$
$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1C}}{L_2} = \delta;$$
$$t = \sqrt{L_1C}\tau;\ \ "\cdot" = \frac{d}{d\tau}$$

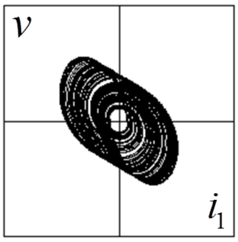
We obtain the normalized circuit equations.

$$\begin{aligned} \dot{x} &= \alpha x + z \\ \dot{y} &= z - f(y) \\ \dot{z} &= -x - \beta y \end{aligned} \tag{2}$$

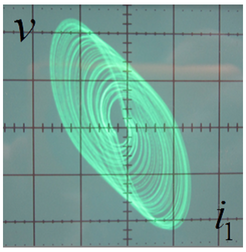
where $f(y)$ is described as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \tag{3}$$

Figure 2 shows the chaotic attractor generated by the chaotic circuit, obtained from a computer simulation (Fig. 2 (a)) and measured in a circuit experiment (Fig. 2 (b)). For the computer simulation, we set the parameters as $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. For the circuit experiment, the parameters are fixed with $L_1 = 500[mH]$, $L_2 = 200[mH]$, $C = 0.015[\mu F]$, and $r_d = 1.46[M\Omega]$.



(a) Computer simulation.



(b) Circuit experiment.

Figure 2: Chaotic attractor.

Next, we consider the following equations when each chaotic circuits is coupled globally.

$$\begin{aligned} \frac{dx_i}{d\tau} &= \alpha x_i + z_i \\ \frac{dy_i}{d\tau} &= z_i + f(y) \\ \frac{dz_i}{d\tau} &= -x_i - \beta y_i - \sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ & (i, j = 1, 2, \cdots, N) \end{aligned} \tag{4}$$

where i in the equation represents the circuit itself, and j indicates the coupling with other circuits. The parameter γ_{ij} represents the coupling strength between the circuits. The value of γ_{ij} reflects the distance between the circuits in an inverse way, described by the following equation:

$$\gamma_{ij} = \frac{g}{(length_{ij})^2}. \tag{5}$$

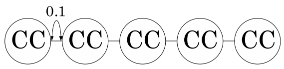
$length_{ij}$ denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit. The parameter g is a weighting or scaling parameter that determines the coupling strengths. For our study, we set the parameter to $g = 1.0 \times 10^{-5}$.

3. Basic Synchronization phenomena

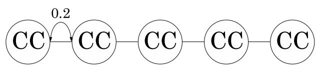
First, we investigate the case of basic synchronization phenomena. We arrange some chaotic circuits in one-dimensional an array at regular intervals. In this simulation, all chaotic circuits are connected to each other by resistors and the coupling strength between two circuits is determined by Eq. (6). Furthermore, the value of coupling strength depends on the distance between the circuits.

The arrangement of chaotic circuits is shown in Fig. 3. In this figure, four chaotic circuits are coupled as the array topoloby. The distance between each chaotic circuits are different in each pattern. This distance for each pattern are defined by $d = 0.1$, $d = 0.2$ and $d = 0.3$.

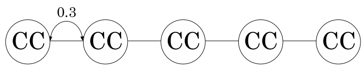
Figure 4 shows the computer simulation results of the phase differences between the adjacent chaotic circuits. From these results, we confirm that the chaotic circuits are synchronized at in-phase when the distance parameter is set to $d = 0.10$. By increasing the value of d, the chaos synchronization is broken and asynchronous can be obtained when the distance parameter is set to $d = 0.30$. For simulations, we fix the distance parameter with $d = 0.10$ for synchronization.



(a) $d = 0.10$.



(b) $d = 0.20$.

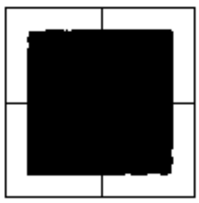


(c) $d = 0.30$.

Figure 3: Arrangement of chaotic circuits.

(a) $d = 0.10$.

(b) $d = 0.20$.



(c) $d = 0.30$.

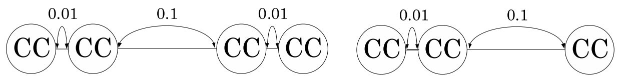
Figure 4: Phase difference between the chaotic circuits.

4. Synchronization phenomena

In this section, several types of the circuit arrangement are considered as follows.

4.1 Synchronization phenomena I

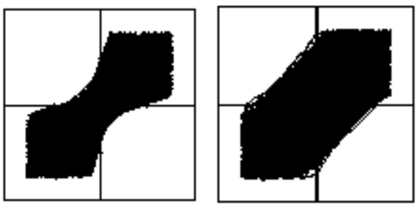
Here, we investigate synchronization phenomena when the chaotic circuits are coupled at irregular intervals. The two types arrangements of chaotic circuits are considered symmetric or asymmetric model shown in Fig 5. In this figures, $d = 0.01$ is the most nearest distance between the chaotic circuits. The arrangement of Fig. 5 (a) shows the symmetric arrangement across the coupling $d = 0.10$, while Fig. 5 (b) shows the asymmetric arrangement across the coupling $d = 0.10$.



(a) Symmetric arrangement. (b) Asymmetric arrangement.

Figure 5: Arrangement of chaotic circuits.

Figure 6 shows the simulation results of the phase differences between the chaotic circuits across the coupling $d = 0.10$ and $d = 0.01$ as shown in Fig. 5 (a). Similarly, Fig. 7 shows the results of the phase differences between the chaotic circuits across the coupling $d = 0.10$ and $d = 0.01$ as shown in Fig. 5 (b). From these results, all chaotic circuits are synchronized when the chaotic circuits are coupled by symmetry. However, the case of that the chaotic circuits care coupled by asymmetry, asynchronous phenomena can be observed from the adjacent chaotic circuits between the coupling $d = 0.10$.



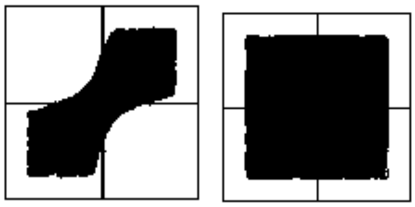
(a) $d = 0.01$. (b) $d = 0.1$.

Figure 6: Phase difference of symmetric arrangement.

4.2 Synchronization phenomena II

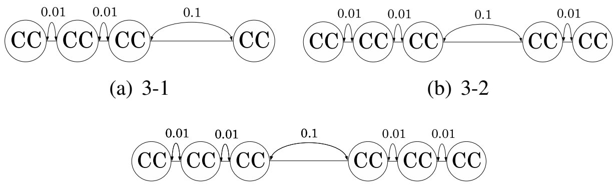
Next, the number of the chaotic circuits is changed as shown in Fig. 8. We consider three types of the arrangements as symmetry and asymmetry cases; 3-1, 3-2 and 3-3.

Figure 9 shows the simulation results of the phase differences between the chaotic circuits via the coupling $d = 0.10$. From these results, the chaotic circuits coupled by the coupling parameter $d = 0.10$ are not synchronized when the chaotic circuits are coupled by asymmetry (Fig. 9 (a)). In the case of the symmetry case, all chaotic circuits are synchronized with in-phase as shown in Fig. 9 (c).



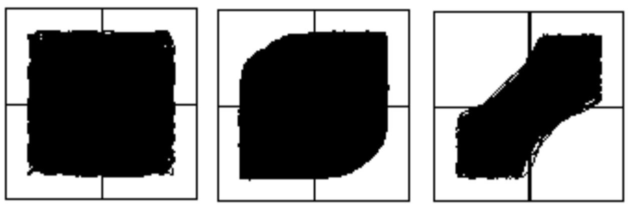
(a) $d = 0.01$. (b) $d = 0.1$.

Figure 7: Phase difference of asymmetric arrangement.



(a) 3-1 (b) 3-2 (c) 3-3

Figure 8: Arrangement of chaotic circuits.



(a) 3-1 (b) 3-2 (c) 3-3

Figure 9: Phase difference between the chaotic circuits ($d = 0.10$).

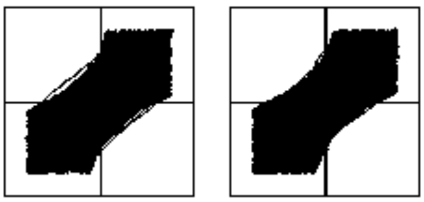
4.3 Synchronization phenomena III

Finally, we show some simulation results when the number of the chaotic circuits is increased. The results of four types of the arrangements (4-1, 4-2 4-3, and 4-4) are shown in Fig. 10. We observe chaos synchronization when the chaotic circuits are arranged as 4-3 and 4-4. We first confirm that chaos synchronization is occurred even if the chaotic circuits are coupled by asymmetry.

Figure 11 shows the results of five types of the arrangements (5-1, 5-2 5-3, 5-4 and 5-5). Similarly, we observe chaos synchronization when the chaotic circuits are arranged as 5-3, 5-4 and 5-5. The adjacent chaotic circuits between the coupling $d = 0.10$ can be synchronized when the number of the chaotic circuits is more than 3 for the one side group. Namely, more than half number of the chaotic circuits against the total number circuits in the one side group is needed to synchronize all circuits.

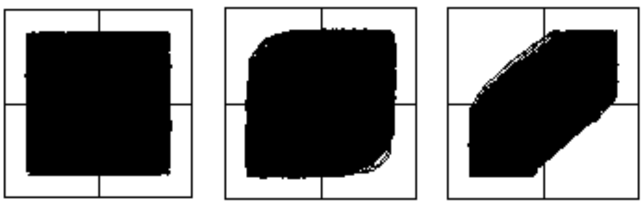
From these results, we confirm that the synchronization phenomena changes due to the arrangement of the chaotic circuits when the number of chaotic circuits has large number. Although all chaotic circuits are synchronized with in-phase when the arrangement of chaotic circuits is set to symmetry.

(a) 4-1 (b) 4-2

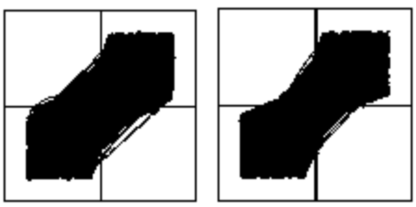


(c) 4-3 (d) 4-4

Figure 10: Phase difference between the chaotic circuits ($d = 0.10$).



(a) 5-1 (b) 5-2 (c) 5-3



(d) 5-4 (e) 5-5

Figure 11: Phase difference between the chaotic circuits ($d = 0.10$).

5. Conclusion

In this study, we have investigated the synchronization phenomena of coupled chaotic circuits as the array topology. The arrangements for chaotic circuits are set to symmetric or asymmetric types via the coupling resistors. By carrying out computer simulations, we confirm that the chaotic circuits are synchronized at in-phase when the chaotic circuits are coupled symmetry. Also, chaos synchronization could be observed if the number of chaotic circuits has more than half against the total number circuits in one side group.

In our future works, we would like to apply this proposed system for large-scale networks.

Acknowledgment

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References

[1] L.L. Bonilla, C.J. Perez Vicente and R. Spigler, “Time-periodic phases in populations of nonlinearly coupled oscillators with bimodal frequency distributions,” *Physica D: Nonlinear Phenomena,* vol. 113, no. 1, pp. 79-97, Feb. 1998.

[2] J.A. Sherratt, “Invading wave fronts and their oscillatory wakes are linked by a modulated traveling phase resetting wave,” *Physica D: Nonlinear Phenomena,* vol. 117, no. 1-4, pp. 145-166, Jul. 1998.

[3] G. Abramson, V.M. Kenkre and A.R. Bishop, “Analytic solutions for nonlinear waves in coupled reacting systems,” *Physica A: Statistical Mechanics and its Applications,* vol. 305, no. 3-4, pp. 427-436, Mar. 2002.

[4] I. Belykh, M. Hasler, M. Lauret and H. Nijmeijer, “Synchronization and graph topology,” *Int. J. Bifurcation and Chaos,* vol. 15, no. 11, pp. 3423-3433, Nov. 2005.

[5] C.M. Gray, “Synchronous oscillations in neural systems: mechanisms and functions,” *J. Computational Neuroscience,* vol. 1, pp. 11-38, 1994.

[6] Y. Takamaru, H. Kataoka, Y. Uwate and Y. Nishio, “Clustering phenomena in complex networks of chaotic circuits”, *Proc. of ISCAS’12,* pp. 914-917, Mar. 2012.

[7] Y. Takamaru, Y. Uwate, T. Ott and Y. Nishio, “Clustering phenomena of coupled chaotic circuits for large scale networks”, *Proc. of NDES’12,* pp. 70-73, Jul. 2012.

[8] Y. Nishio, N. Inaba, S. Mori and T. Saito, “Rigorous analyses of windows in a symmetric circuit,” *IEEE Transactions on Circuits and Systems,* vol. 37, no. 4, pp. 473-487, Apr. 1990.

[9] C. Bonatto and J. A. C. Gallas, “Periodicity hub and nested spirals in the phase diagram of a simple resistive circuit,” *Phys. Rev. Lett.,* 101, 054101, Aug. 2008.

[10] R. Stoop, P. Benner and Y Uwate, “Real-world existence and origins of the spiral organization of shrimp-shaped domains,” *Phys. Rev. Lett.,* 105, 074102, Aug. 2010.