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Investigation of Oscillatory Networks Including Two Types of Resistors

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Abstract

In this study, we investigate synchronization phenomena observed in coupled oscillators by using two types of resistors. Negative and normal resistors are used as coupling resistors in our circuit model. By using computer simulations and circuit experiments, we obtain interesting synchronization. Furthermore, we extend four coupled oscillators and compare with previous study.

1. Introduction

There are a lot of synchronization phenomena in this world. This is one of the nonlinear phenomena that we can often observe by natural beings which does collective actions. For example, firefly luminescence, cry of birds and frogs, applause of many people, swing of the pendulum and so on. Synchronization phenomena have a feature that the set of small power can produce very big power by synchronizing at a time. Therefore studies of synchronization phenomena have been widely reported not only engineering but also the physical and biological fields [1]-[6]. Investigation of coupled oscillators attracts attention from many researchers because coupled oscillatory network produce interesting phase synchronization such as the phase propagation wave, clustering and complex patterns.

In our investigations, we use several van der Pol oscillators (see Fig. 1). Van der Pol oscillators have been coupled in various form and investigated about their synchronization phenomena [7], [8]. For example, as shown in Fig. 2(a), the circuit model which coupled three van der Pol oscillators as a ring topology have been found the three phase synchronization (phase shift 120 degrees) by the interaction of each oscillator [9]. However, the three-phase synchronization was always observed stably in that system. In our previous study, we have investigated several kinds of interesting synchronization phenomena in coupled oscillatory system which has stronger frustrations as shown Fig. 2(b) [10]. By computer simulation, we observed that the phase difference between adjacent oscillators changed and the synchronization was destroyed after the adjacent oscillators synchronize with anti-phase (phase shift 180 degrees). In other words, this circuit model has the feature such as repeated anti-phase synchronous and asynchronous.

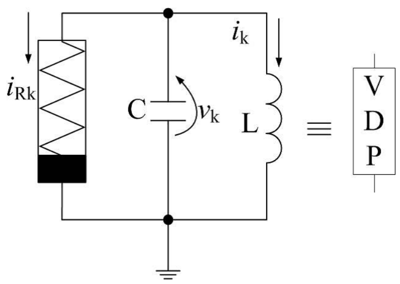

Figure 1: Van der Pol oscillator.

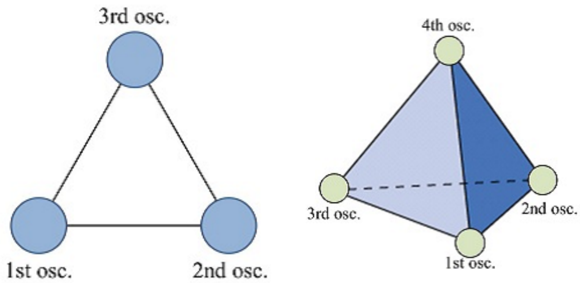

Figure 2: Coupled van der Pol oscillators. (a) Three coupled oscillators. (b) Four coupled oscillators.

In this study, we investigate synchronization phenomena of coupled oscillators using different types of resistors partially. We focus on the amplitude of each oscillator and phase differences between adjacent oscillators by changing coupling strength. By using averaging method, we compare between simulation results and theoretical analysis. Also, we increase number of oscillators and demonstrate the circuit experiment of this oscillatory system.

2. Circuit Model

Figure 3 shows the circuit model. First and second oscil-

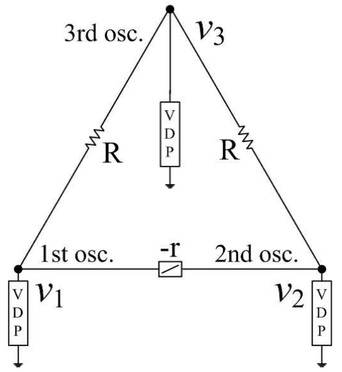

Figure 3: Conceptual circuit model for three coupled oscillators.

lators are coupled by negative resistor. And third and other oscillators are coupled by normal resistor. In the case of the negative resistor coupling, the adjacent oscillators are synchronized at anti-phase. While, we use the normal resistor as the coupling resistor, the adjacent oscillators are synchronized at in-phase.

In the computer simulations, we assume that the $v_k - i_{Rk}$ characteristics of nonlinear resistor in each oscillator is given by the following third order polynomial equation.

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \qquad (g_1,\ g_3 > 0), \qquad (k = 1,\ 2,\ 3). \tag{1}$$

The normalized circuit equations are expressed as:

[First oscillator]

$$\begin{cases} \dfrac{dx_1}{d\tau} = \varepsilon(1 - \dfrac{1}{3}{x_1}^2)x_1 - y_1 \quad +\gamma_1(x_1 - x_2) \\ \qquad\qquad\qquad\qquad\qquad\quad -\gamma_2(x_1 - x_3) \\ \dfrac{dy_1}{d\tau} = x_1. \end{cases} \tag{2}$$

[Second oscillator]

$$\begin{cases} \dfrac{dx_2}{d\tau} = \varepsilon(1 - \dfrac{1}{3}{x_k}^2)x_2 - y_2 \quad +\gamma_1(x_2 - x_1) \\ \qquad\qquad\qquad\qquad\qquad\quad -\gamma_2(x_2 - x_3) \\ \dfrac{dy_2}{d\tau} = x_2. \end{cases} \tag{3}$$

[Third oscillator]

$$\begin{cases} \dfrac{dx_3}{d\tau} = \varepsilon(1 - \dfrac{1}{3}{x_3}^2)x_3 \quad -y_3 \\ \qquad\qquad\qquad\qquad -\gamma_2(2x_3 - x_1 - x_2) \\ \dfrac{dy_3}{d\tau} = x_3. \end{cases} \tag{4}$$

We use the following normalizations:

$$t = \sqrt{LC}\tau\ ,\ v_k = \sqrt{\frac{g_1}{3g_3}}\,x_k\ ,\ i_k = \sqrt{\frac{g_1 C}{3g_3 L}}\,y_k,$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}}\ ,\ \gamma_1 = \frac{1}{r}\sqrt{\frac{L}{C}}\ ,\ \gamma_2 = \frac{1}{R}\sqrt{\frac{L}{C}},$$

where ε is the nonlinearity, γ_1 is the coupling strength of negative resistor r and γ_2 is the coupling strength of normal resistor R. In the computer simulations, we calculate the phase differences between adjacent oscillators to observe synchronization states.

3. Synchronization Phenomena

We calculate Eqs. (2)-(4) using the fourth-order Runge-Kutta method with the step size $h = 0.001$. We show the simulation result of the synchronization phenomena in Figs. 4 and 5. In Fig. 4, we show the attractor of each oscillator. We set the parameters as follows ε = 0.02, γ_1 = 0.05 and γ_2 = 0.05. Obvious difference of the amplitude of each oscillator appears in this parameters. Namely, oscillation death is observed in third oscillator. Phase differences between first and second oscillators shows anti-phase synchronization, because third oscillator stops oscillating in order to match consistency.

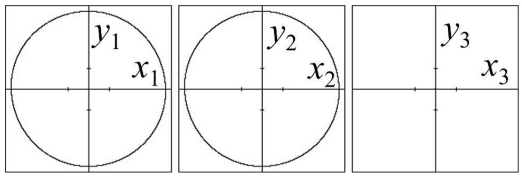

Figure 4: Attractor between adjacent oscillators (horizontal axis:voltage, vertical axis:electric current.

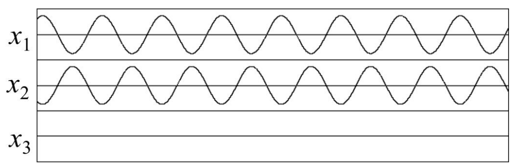

Figure 5: The time waveform of the each oscillator.

In this oscillatory system, the nonlinearity and coupling strength influence synchronization phenomena. Here we fix the parameter $\varepsilon = 0.02$. In case γ_1 is larger than or equal to γ_2, first and second oscillator synchronize in anti-phase (see Fig. 6(a)). The other way, in the case of γ_2 bigger, all of the phase differences between adjacent oscillators converge in-phase synchronization because effect of normal resistor increases. Figure 6(b) shows in-phase synchronization using the following parameters $\gamma_1 = 0.01$ and $\gamma_2 = 0.05$.

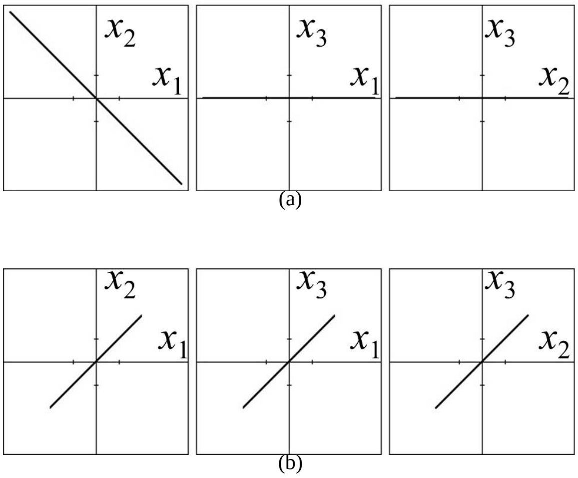

Figure 6: Phase differences between adjacent oscillators. (a) Anti-phase synchronization. (b) In-phase synchronization.

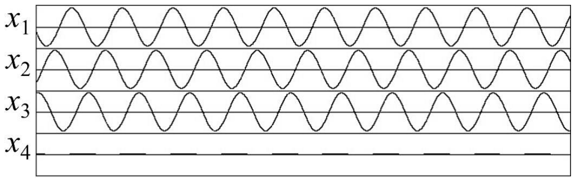

Figure 8: The time waveform of four coupled oscillators.

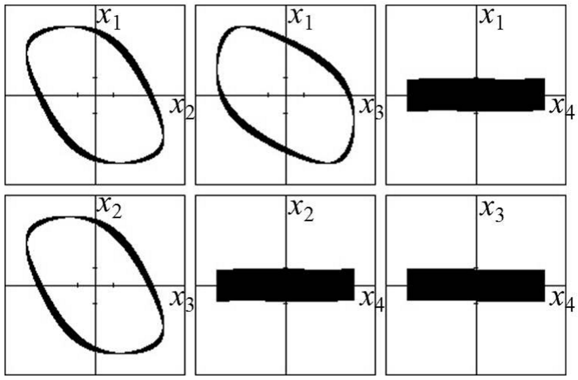

Figure 9: Lissajous figures.

4. Four Coupled van der Pol Oscillators

Next, we investigate synchronization phenomena in tetrahedral form as shown in Fig. 7. This oscillatory system exists three triangles as shown in Fig. 3. These oscillators from first to the third are coupled using same negative resistors but we couple normal resistor R between fourth oscillator and others.

In the case of γ_1 bigger than γ_2, we can observe three-phase synchronization between first, second and third oscillators because fourth oscillator occurs the oscillation death. Figure 8 shows the simulation results for $\varepsilon = 0.02, \gamma_1 = 0.05$ and $\gamma_2 = 0.01$. In the case of four coupled oscillators, we can find out synchronization phenomena that three coupled oscillators do not produce. When fourth oscillator continues to oscillate but attenuated, the synchronization phenomena about fourth oscillator are observed asynchronous. Also, first, second and third oscillator do not produce three-phase synchronization clearly because fourth oscillator effect other oscillators. Namely three-phase synchronization keeps vibrating few degrees. We confirm that this vibration width increases if amplitude of fourth oscillator is big. Figure 9 shows the asynchronous of simulation result, we set the parameters ε = 0.10, γ_1 = 0.10 and γ_2 = 0.03.

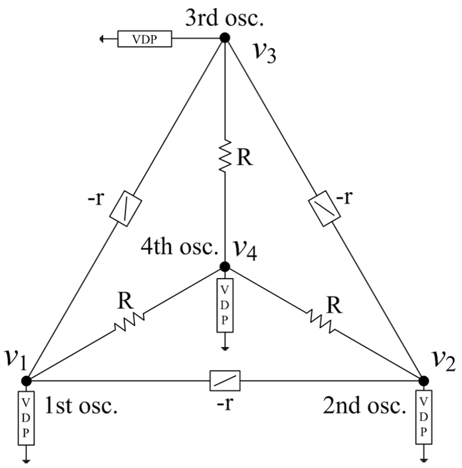

Figure 7: Conceptual circuit model for tetrahedron form.

5. Circuit Experiment

We perform the circuit experiment and compare with simulation results. All of these oscillators are used same value

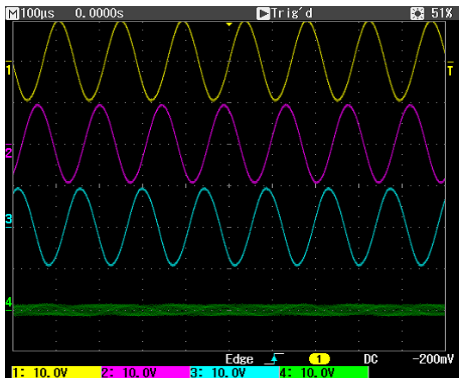

Figure 10: The time waveform of the each oscillator.

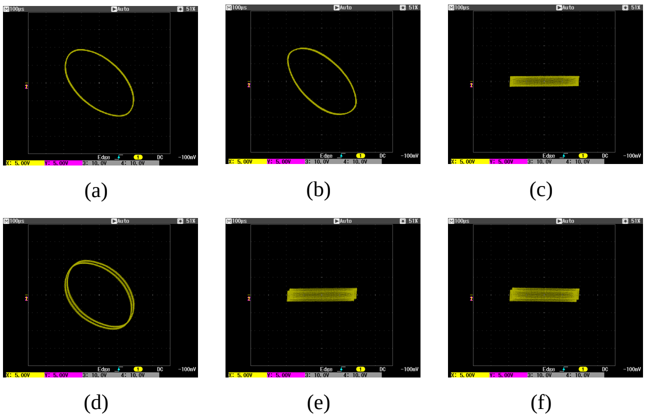

Figure 11: The phase differences between adjacent oscillators. (a) 1st-2nd. (b) 1st-3rd. (c) 1st-4th. (d) 2nd-3rd. (e) 2nd-4th. (f) 3rd-4th.

elements $C = 33[nF]$, $L = 30[mH]$. Observed synchronization phenomena hardly decide two types of resistors r and R. The experiment results in this study are found out several interesting patterns of synchronization phenomena. Figures 10 and 11 show the comparison chart of the simulation results of Fig. 9. In this circuit experiment, we use the following resistors $r = 3.3[k\Omega]$ and $R = 33[k\Omega]$.

6. Conclusion

This paper has presented the synchronization phenomena of coupled oscillators which is used two types of resistors. In this circuit model, we have been able to observe several patterns of synchronization phenomena. And the oscillator which is coupled by normal resistor R has occurred oscillation death. These synchronization phenomena have been observed by circuit experiments.

Acknowledgment

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