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Detailed Investigation of Synchronizations in Three Cross-Coupled Chaotic Circuits

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Abstract

Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields. In this study, we investigate synchronization phenomena observed in three simple chaotic circuits cross-coupled by three inductors. Interesting synchronization phenomena can be confirmed by computer simulations and circuit experiments.

1. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled chaotic circuits are extensively carried out in various fields [1]- [5]. We consider that it is very important to investigate the phenomena related with chaos synchronization to realize future engineering application utilizing chaos. In our previous study, we have investigated the cross-coupled chaotic circuits and reported various interesting synchronization phenomena [6]- [9].

In this study, we consider three Shinriki-Mori chaotic circuits [9] cross-coupled via three inductors. The circuit is a special version of the ring considered in [7], but we have not investigated the phenomena in detail because we did not notice the phase differences of small oscillations until we noticed it in 2009 [8]. By computer simulations and circuit experiments, we investigate the synchronization phenomena in detail, in particular, three-phase synchronizations of small oscillations are confirmed to be generated.

2. Circuit Model

Figure 1 shows the circuit model. In the circuit, three Shinriki-Mori chaotic circuits are cross-coupled via inductors L_2.

First, we approximate the $v - i$ characteristics of the nonlinear resistors consisting of the diodes by the following 3-

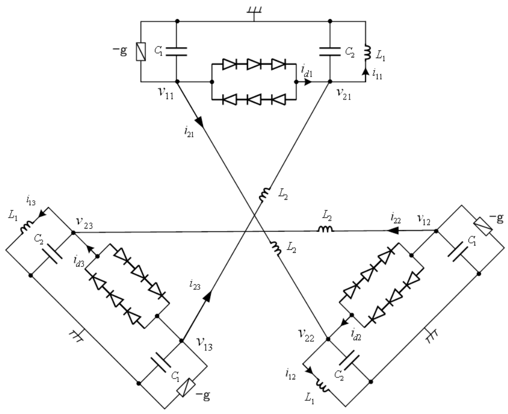


Figure 1: Circuit model.

segment piecewise-linear functions.

$$i_{dn} = \begin{cases} G(v_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|v_{1n} - v_{2n}| \leq V) \\ G(v_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V) \end{cases} \quad (1)$$

$$(n = 1, 2, 3)$$

The circuit equations are described as follows.

$$\begin{cases} L_1 \dfrac{di_{1n}}{dt} = v_{2n} \\ C_1 \dfrac{dv_{1n}}{dt} = gv_{1n} - i_{2n} - i_{dn} \\ C_2 \dfrac{dv_{2n}}{dt} = i_{dn} + i_{2,n+1} - i_{1n} \\ L_2 \dfrac{di_{2n}}{dt} = v_{1n} - v_{2,n-1}, \end{cases} \quad (2)$$

$$(n = 1, 2, 3)$$

where $v_{20} = v_{23}$ and $v_{24} = v_{21}$. By using the following parameters

$$\begin{cases} \alpha = \dfrac{C_2}{C_1},\ \beta = \sqrt{\dfrac{L_1}{C_2}}G,\ \gamma = \sqrt{\dfrac{L_1}{C_2}}g,\ \delta = \dfrac{L_1}{L_2}, \\ t = \sqrt{L_1 C_2}\tau, \end{cases} \tag{3}$$

and variables

$$\begin{cases} i_{11} = \sqrt{\dfrac{C_2}{L_1}}Vx_1,\ i_{12} = \sqrt{\dfrac{C_2}{L_1}}Vx_2,\ i_{13} = \sqrt{\dfrac{C_2}{L_1}}Vx_3, \\ v_{11} = Vy_1,\ v_{12} = Vy_2,\ v_{13} = Vy_3, \\ v_{21} = Vz_1,\ v_{22} = Vz_2,\ v_{23} = Vz_3, \\ i_{21} = \sqrt{\dfrac{C_2}{L_1}}V\omega_1,\ i_{22} = \sqrt{\dfrac{C_2}{L_1}}V\omega_2,\ i_{23} = \sqrt{\dfrac{C_2}{L_1}}V\omega_3, \end{cases} \tag{4}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x_n} = z_n \\ \dot{y_n} = \alpha(\gamma y_n - \omega_n - \beta f(y_n - z_n)) \\ \dot{z_n} = \beta f(y_n - z_n) + \omega_{n+1} - x_n \\ \dot{\omega_n} = \delta(y_n - z_{n+1}) \end{cases} \tag{5}$$

$$(n = 1, 2, 3)$$

where $z_{20} = z_{23}$ and $z_{24} = z_{21}$. The nonlinear function $f(\cdot)$ corresponds to the $v - i$ characteristics of the nonlinear resistors consisting of the diodes and are assumed to be described by the following 3-segment piecewise-linear functions:

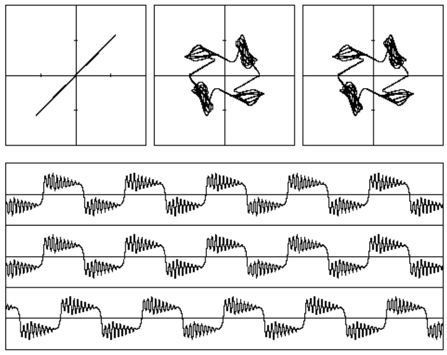
$$f(y_n - z_n) = \begin{cases} y_n - z_n - 1 & (y_n - z_n > V) \\ 0 & (|y_n - z_n| \leq V) \\ y_n - z_n + 1 & (y_n - z_n < -V) \end{cases} \tag{6}$$

$$(n = 1, 2, 3)$$

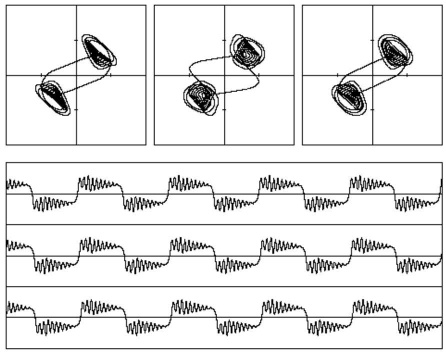
3. Synchronization Phenomenon

The coupled circuits generate various synchronization states. Two examples of steady states are shown in Fig. 2. These two synchronization states are obtained for the same parameter set of $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.15$ and $\delta = 0.004$ however with different initial conditions. In Fig. 2(a), two circuits are completely synchronized in in-phase and the other circuit exhibits some phase difference (time waveform shows that the switching from plus to minus or from minus to plus delays 90 degrees from the other circuits. On the other hand, in Fig. 2(b), all three circuits seem to be synchronized in in-phase from the time waveforms, however, the attractors do not show in-phase synchronizations.

In order to investigate how many steady states coexist and why the shape of attractors in Fig. 2(b) does not show in-phase, we investigate the time waveforms of the voltages



(a)



(b)

Figure 2: Typical examples of observed synchronization phenomena for $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.15$ and $\delta = 0.004$. Upper figures show the attractor on $v_{11} - v_{12}$ phase plane, on $v_{11} - v_{13}$ phase plane and on $v_{12} - v_{13}$ phase plane from the left. Lower figure shows the time waveforms of v_{11}. v_{12} and v_{13}.

more carefully, because we realized in 2009 that small oscillations could synchronize when two circuits are cross-coupled [13]. Figure 3 shows closer looks of the time waveforms of the three voltages. Figures 3(a) and 3(b) correspond to the steady states in Figs. 2(a) and 2(b), respectively. Because v_{11} and v_{12} are completely synchronized in in-phase in Fig. 2(a), we can see only two waveforms in Fig. 3(a). From this figure we can notice that the small oscillation of v_{13} is synchronized in anti-phase with the small oscillations of v_{11} and v_{12}. Although all three waveforms looked like to be synchronized in in-phase in Fig. 2(b), the closer look in Fig. 3(b) revealed that the small oscillations of these three voltages are synchronized clearly in 3-phase. This is why the attractors in Fig. 2(b) are not on the in-phase synchronization space completely. Also we can understand why the shapes of the attractors show some phase differences in the first and the third quadrants of the phase space in Fig. 2(b). This fact encouraged us to search many more coexisting steady states, because we can expect a large number of steady states with local 3-phase of small oscillations. As we expected, we could

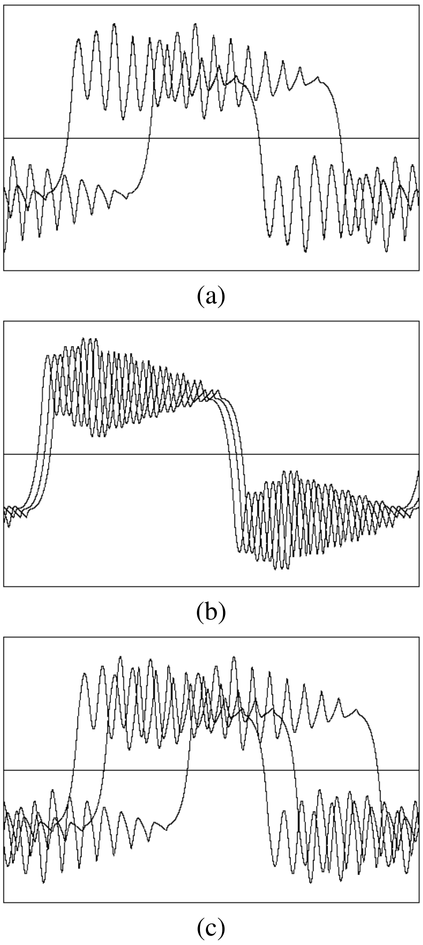


Figure 3: Magnified time waveforms of three voltages obtained from three circuits. (a) and (b) correspond to Fig. 2(a) and (b), respectively.

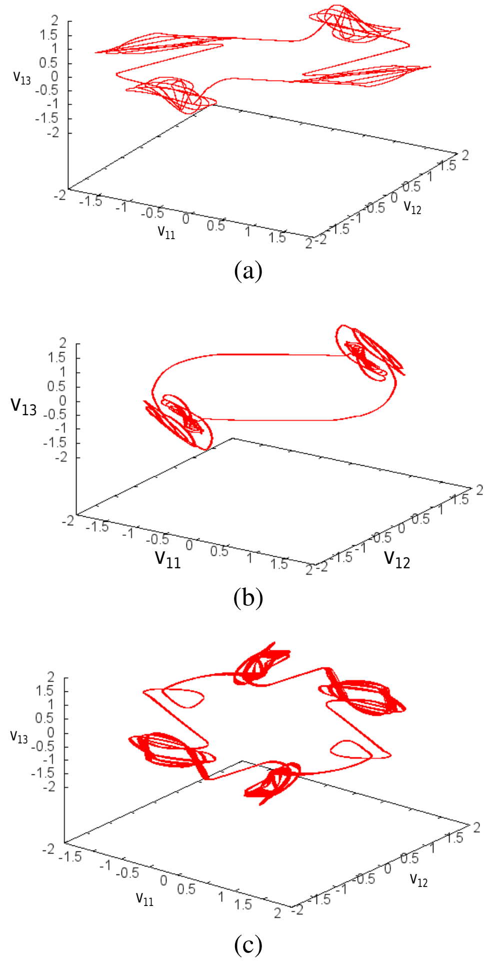


Figure 4: Attractors in 3-dimensional phase space ($v_{11} - v_{12} - v_{13}$). (a), (b) and (c) correspond to Fig. 3(a), (b) and (c), respectively.

find many different steady states and one example is shown in Fig. 3(c). Combinations of the timing shift of the switching from minus to plus or from plus to minus with local 3-phase of small oscillations make it possible to generate a large number of steady states.

The amount of the timing shift of the switching influences the shape of the attractors. Figure 4 shows the attractors in 3-dimensional phase space. Attractors in Figs. 4(a), (b) and (c) correspond to the time waveforms in Figs. 3(a), (b) and (c), respectively. We can see that the sojourn time of the solution in the plus or the minus region decides the shape of the attractors.

4. Parameter Dependency

Figure 5 shows the parameter dependency of the observed phenomena. As increasing the coupling parameter δ, the sojourn time in the plus or the minus regions becomes shorter.

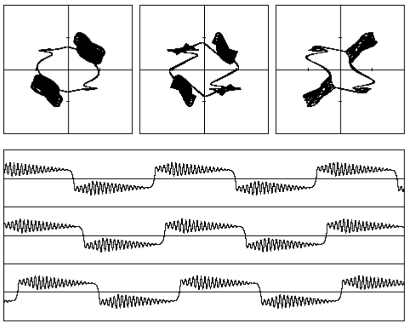
5. Circuit Experiments

Finally, circuit experimental results are shown in Fig. 6. We can say that this interesting phenomenon can be observed from both computer calculations and circuit experiments.

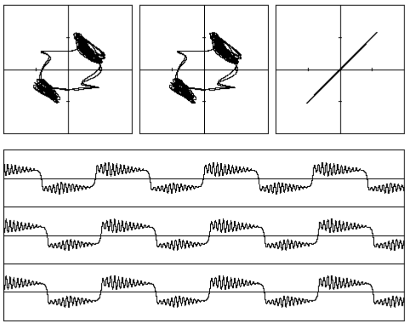
6. Conclusions

In this study, we have investigated the synchronization phenomena observed from three simple chaotic circuits cross-coupled by inductors. The detailed investigation of time waveforms clarified several interesting phenomena.

Investigating the coexistence of the states and statistical analysis of the observed phenomena are our important future work as well as more detailed explanation of the mechanism of the generations.



(a)



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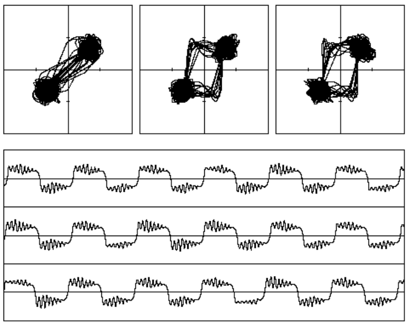

(c)

Figure 5: Parameter dependency of synchronization. (a) δ=0.002. (b) δ=0.003. (c) δ=0.005.

Acknowledgment

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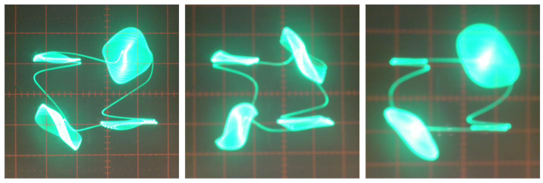
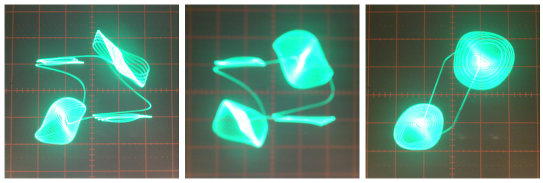
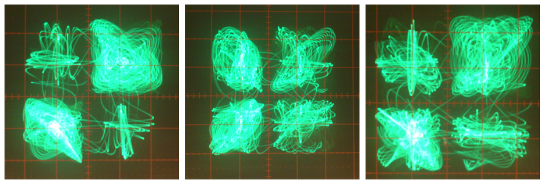


(a)



(b)



(c)

Figure 6: Circuit experimental results. From the left: $v_{11} - v_{12}$, $v_{11} - v_{13}$ and $v_{12} - v_{13}$. C_1=22nF, C_2=47nF, L_2=500mH, g=1.78mS. (a) L_1=0.9mH. (b) L_1=1.35mH. (c) L_1=1.8mH.

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