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Investigation of Associative Memory by Small World Hopfield Neural Network with Characteristics of Social Network

Tomoya Shima, Chihiro Ikuta, Yoko Uwate and Yoshifumi Nishio

Tokushima University
2-1 Minami-Josanjima Tokushima 770-8506, JAPAN
Phone: +81-88-656-7470, Fax: +81-88-656-7471
E-mail: {s-tomoya, ikuta, uwate, nishio}@ee.tokushima-u.ac.jp

Abstract

In the Hopfield Neural Network (HNN), each neuron is connected to every other neuron. Thereby, the HNN causes high cost to generate the network in terms of implementation. Small World Hopfield Neural Network (SWHNN) improves ability for transmission by introducing the shortcut connections into the sparse regular network. However the storage ability of the SWHNN decreases for associative memory because the SWHNN has sparse connectivity. In this study, we propose the SWHNN with characteristic of Local Bridge (SWHNN-LB) using “local bridge”. And we explore performance of the SWHNN-LB using associative memory.

1. Introduction

Many researchers have developed artificial neural network models. One of the network models is Hopfield Neural Network (HNN) [1], and the HNN intends to act as associative memory in this study. The HNN has full and symmetric connectivity and causes high cost to generate network in terms of implementation. A few years ago, Bohland et al. showed that Small World HNN (SWHNN) [2] could be effective associative memory. The SWHNN is combined the HNN with the small world network which has sparse connectivity and is one of social network [4] which is structure which shows social relation, e.g. aerial line, infection with a virus, World Wide Web and so on. The effective associative memory of the SWHNN comes from two properties of the small world network [3]. The small world network has high propagation efficiency by two properties. First property results from introduction of regular network, and second property results from a few long range path which are called “shortcut”. However, the storage capability of the SWHNN is less than the conventional HNN by sparse connectivity. We believe that this problem can be solved if we apply the SWHNN to an important characteristic in the real social network like the small world network. In this study, we focus on “local bridge” which is found by Granovetter [6] and is one of characteristics in social network. Example of the local bridge is shown in Fig. 1.

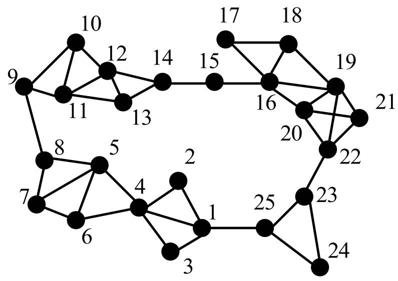

Figure 1: Example of the local bridge.

In Fig. 1, the shortest route from 1 to 25 is 1-25. One of second shortest route from 1 to 25 is 1-4-5-8-9-11-13-14-15-16-20-22-23-25. The 1-25 route is predominantly shorter than other routes. In this case, the 1-25 route is called the local bridge. Similarly, it is found that 8-9, 14-15, 15-16 and 22-23 route are local bridge. In the social network, the local bridge affects the entire network in terms of propagation of information. Because the local bridge creates more and shorter routes. Thereby, the local bridge has greater influence on network if the second shortest route is longer. We consider that the local bridge is similar to the shortcut of the small world network. If the shortcut has a similar property to the local bridge, influence on network differs by shortcuts.

In this study, we propose the SWHNN with characteristics of Local Bridge (SWHNN-LB). In our proposed method, we define “significance” of every shortcut based on theory of the local bridge. Here, the significance is the second shortest route length of two nodes which are connected by shortcut. The determination method of the significance is based on the property of the local bridge. This property is that the local bridge has greater influence on network if the second shortest route is longer. After that, we adjust the weight parameter of shortcuts based on the significance and investigate influence on network by different shortcuts of significance. We compare our proposed method to the conventional SWHNN by exploring the network performance.

2. Small World Hopfield Neural Network Model [2]

In this study, we consider an one-dimensional ring model of the small world network. We start with a n-node ring regular lattice. Each unit in a n-node ring regular lattice is connected to its k nearest neighbors by edges. Here, the number of k is very small compared to the number of n. Then, each unit on the network is randomly rewired to other units with probability p.

The small world network is quantified properties of the network structure by average route length L and clustering coefficient C. Here, the average route length L is measured the separation between two nodes in the network, while the clustering coefficient C is measured the degree of cliquishness connectivity in the network. Figure 2 shows L-C characteristic.

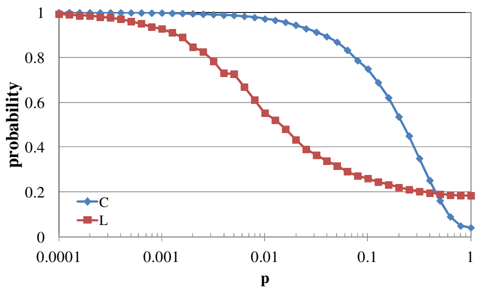

Figure 2: L-C characteristic of SW, when $n = 200$ and $k = 16$.

The definition of the small world network model by Watts is $L \sim L_{random}$ and $C \gg C_{random}$. From Fig 2, the definition of the small world network model is satisfied, when $0.01 \leq p \leq 0.1$. These properties result from introduction of a few long range path. It is known that a few long range path are "shortcut". Here, the shortcut connections are longer than edges which connects each unit to its k nearest neighbors in this study. The SWHNN is combined the small world network like Fig. 2 with the HNN theory.

3. Small World Hopfield Neural Network with Local Bridge Model

We consider that the performance of the SWHNN improves by introducing the local bridge. In this study, we focus on the local bridge which is one of characteristic of the social network and important connection in terms of propagation of information in the real network. The introduction method of the local bridge is that the significance is defined for each shortcut. Here, the significance is the second shortest route length of two nodes which are connected by shortcut. The determination method of the significance is based on the property of the local bridge. This property is that the local bridge has greater influence on network if the second shortest route is longer. Example of the significance is shown in Fig 3. In this study, we define the significance of every shortcut on the SWHNN. The method of adjustment is multiplied the weight parameters of each shortcut by α which is constant. The SWHNN-LB is similarly used the HNN theory.

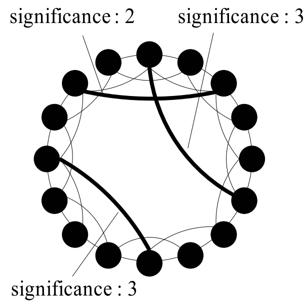

Figure 3: Example of the SWHNN-LB ($n = 16$ and $k = 4$).

4. HNN Working as Associative Memory

The HNN based on the model of biological neurons was proposed by J.J. Hopfield in 1982 [1]. Every neurons in the HNN is connected to each other neuron in the network with no self-connection.

The HNN is used as the associative memory by exploiting the property that the network has multiple stable states. Namely, if the parameters of the network can be decided in such a way that the patterns to be stored become stable states of the network, the network produces a stored pattern that is similar to an input pattern. The process of the associative memory by the conventional HNN is described as follows.

[Step1 (HNN)]: If we apply the conventional HNN for the associative memory, one pixel in an image corresponds with one neuron.

[Step2 (HNN)]: The HNN stores images by determining the weight parameter. The weight parameter is given by

$$w_{ij} = \begin{cases} \frac{1}{P}\sum_{p=1}^{P} x_i^{(p)} x_j^{(p)} & (i \neq j) \\ 0 & (i = j), \end{cases} \quad (1)$$

where P is the number of patterns, w is the weight parameter and x is the pattern of memory. It turns out that the weight parameter is symmetrical by Eq. (1).

[Step3 (HNN)]: The neurons is initialized by an unknown image. Namely, the network is given the input. The state of each neuron is determined by Eq. (2).

$$u_i(t) = \sum_{j=1}^{n} w_{ij}(t)x_j(t), \quad (2)$$

where u is the internal state of the neuron and x is the input or output. The value of u is binary.

[Step4 (HNN)]: Neuron's output is determined by a linear function. The equation of the determining Neuron's output is given as follows.

$$x_i(t+1) = \begin{cases} 1, & (u_i(t) \geq 0) \\ -1, & (u_i(t) < 0). \end{cases} \quad (3)$$

[Step5 (HNN)]: The HNN recalls a similar image to the input image from the stored images by repeating Step4 to Step5.

5. Simulation Results

In this study, we apply three networks which are the SWHNN and the SWHNN-LB to the associative memory. And, we compare recall rate of each network when k is changed. In the computer simulations, we consider each network with 200 and 300 nodes. We prepare 3 stored binary patterns at random and 1 binary pattern to be input, which differs from one stored pattern by 100%. Example of stored patterns is shown in Fig. 4. It is practically reasonable that a pattern within 1% error is considered as being recalled. We randomly compose 25 each network. And, we define that the recall rate is the average of success rate of recall for each network.

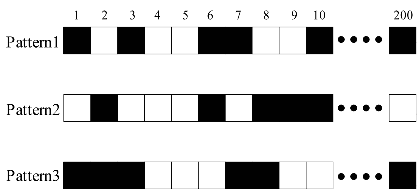

Figure 4: Example of stored patterns.

5.1 Network with 200 Nodes

In the case of each network with 200 nodes, the simulation results are shown in Fig. 5. Figure 5 shows the recall rate when k is changed. The network (a) is the SWHNN. The network (b), (c) or (d) are the SWHNN-LB for adjusting shortcuts of the significance 2, 3 or 4 respectively. The SWHNN-LB for the significance 2 and 3 show better recall rate than the SWHNN. In particular, the SWHNN-LB for the significance 3 shows better recall rate than the SWHNN totally.

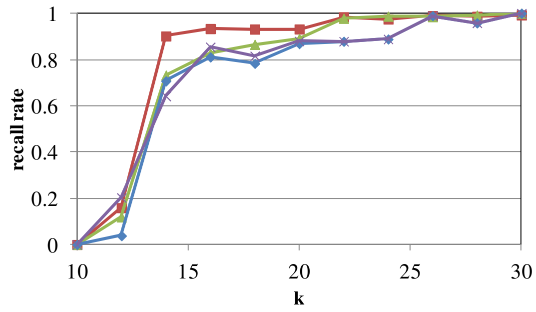

Figure 5: Recall rate of each network depending on k, $p = 0.1$. (a) The SWHNN. (b) The SWHNN-LB for the significance $= 2$ and $\alpha = 3.9$. (c) The SWHNN-LB for the significance $= 3$ and $\alpha = 3.1$. (d) The SWHNN-LB for the significance $= 4$ and $\alpha = 3.1$.

On the other hand, the performance of the SWHNN-LB for the significance 4 is as same as the SWHNN. From Fig. 5, it is found that the performance of the SWHNN-LB differs by the significance. However, we expected that the SWHNN-LB shows better performance if the significance becomes bigger. In order to examine this assumption, we explore existing probability of shortcuts of each significance in the SWHNN-LB. Figure 6 shows the existing probability.

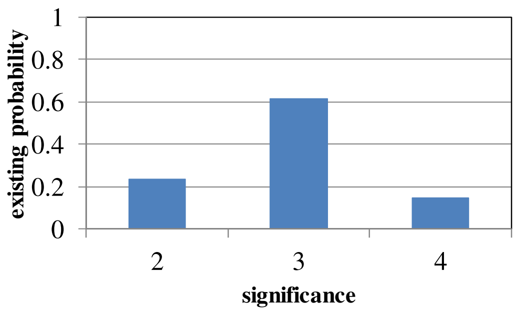

Figure 6: The existing probability the SWHNN depending on the significance ($k = 14$).

From Figs. 5 and 6, it is found that the shortcuts for the significance 4 can not affect the network because the number of them is too small or nothing if k is more than 16. On the other hand, the shortcuts for the significance 4 affect the network if k is 14. Thereby we assume that the SWHNN-LB depends on a number of the shortcuts of each significance. Furthermore, we explore the SWHNN-LB performance if we adjust the same number of the shortcuts of each significance. Figure 7 shows the SWHNN-LB performance if we adjust 10 randomly-selected shortcuts of each significance in the SWHNN-LB. From Fig. 7, it is found that the SWHNN-LB shows better performance if the significance becomes bigger.

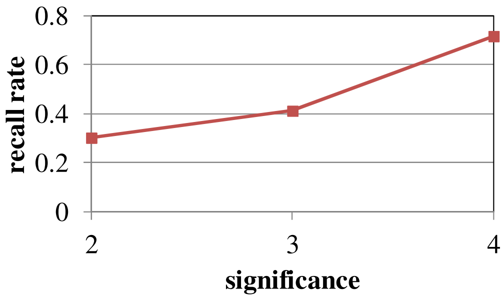

Figure 7: The SWHNN-LB performance for adjusting 10 randomly-selected shortcuts of each significance.

5.2 Network with 300 Nodes

Figure 8 shows the simulation result for 300 nodes. If node of the SWHNN-LB increases, all networks show the better recall rate than the SWHNN totally because the shortcuts for the significance 4 in the SWHNN-LB increase by increasing node.

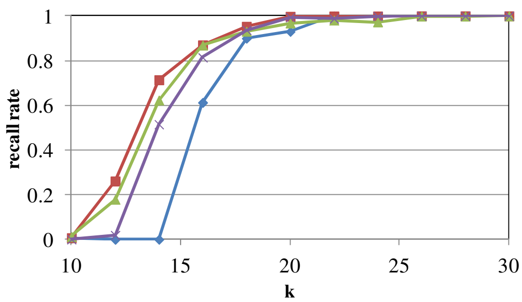

Figure 8: Recall rate of each network depending on k, $p = 0.1$. (a) The SWHNN. (b) The SWHNN-LB for the significance $= 2$ and $\alpha = 3.2$. (c) The SWHNN-LB for the significance $= 3$ and $\alpha = 2.8$. (d) The SWHNN-LB for the significance $= 4$ and $\alpha = 3.2$.

Furthermore, in the same way, we explore the existing probability of the shortcuts of each significance in the SWHNN-LB and the SWHNN-LB performance if we adjust the same number of the shortcuts of each significance. Figures 9 and 10 show the existing probability and the SWHNN performance respectively. From Fig. 10, it is found that the SWHNN-LB shows better performance if the significance becomes bigger.

6. Conclusion

In this study, we have proposed the SWHNN-LB using the local bridge which is the characteristic in the social network. And we have explored performance of the SWHNN-LB using the associative memory. The computer simulation results showed that the SWHNN-LB provided the better re-

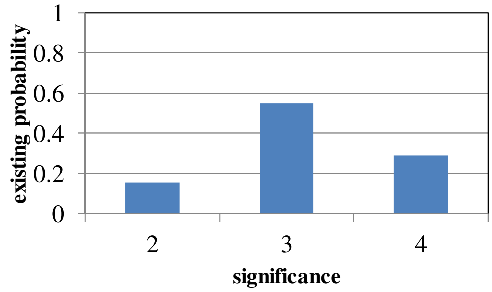

Figure 9: The existing probability the SWHNN depending on the significance ($k = 14$).

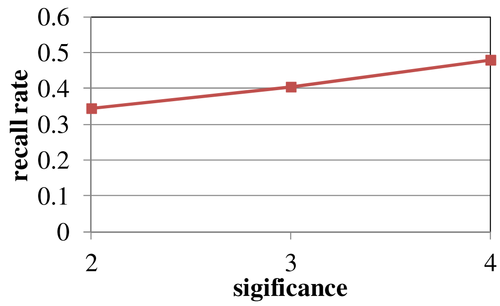

Figure 10: The SWHNN performance for adjusting 10 randomly-selected shortcuts of each significance.

call performance than the SWHNN. Also, it was found that the SWHNN-LB shows better performance if the significance becomes bigger. Therefore, we showed that the part of characteristic of social network affects the propagation ability of the artificial network.

Acknowledgment

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