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Clustering Patterns Depending on Density of Chaotic Circuits in Networks

Yuji Takamaru†, Yoko Uwate†, Thomas Ott‡ and Yoshifumi Nishio†

† Dept. of Electrical and Electronic Engineering, Tokushima University,
2-1 Minami-Josanjima, Tokushima, 770-8506, Japan
phone:+81-88-656-7470, fax: +81-88-656-7471
Email:{takamaru, uwate, nishio}@ee.tokushima-u.ac.jp
‡ Institute of Applied Simulation, Zurich University of Applied Sciences,
Einsiedlerstrasse 31a, 8820 Waedenswil, Switzerland
phone:+41-058-934-56-84
Email:thomas.ott@zhaw.ch

Abstract

In this study, we investigate clustering patterns generated in coupled chaotic circuits networks. In these networks, the coupling strength is reflected the distance information and each chaotic circuit is connected to all chaotic circuits. We consider the relationship between coupling strength and phase difference by changing the scaling parameter of coupling strength. Furthermore, we confirm the various phase synchronization patterns when we change the number of chaotic circuits.

1. Introduction

Recently, our lives deal with large amount of information. Our society is called advanced information network society. Because of this, many informations are gradually processed day by day. Therefore, the ideas of clustering algorithms were proposed and applied to information processing. Clustering algorithms have widespread applications in different fields, such as business data mining, image processing and analysis of biological data. There are variety of different clustering algorithms along with the many applications. Many algorithms were proposed to utilize synchronization phenomena, for instance in Coupled Map Lattics (CML), for clustering [1]-[3]. Previously, many of these studies were using discrete time model for clustering, however analysis of using a continuous time model has not almost studied. Therefore, we focus on research of clustering phenomena using electronic circuits in continuous time model.

On the other hand, synchronization phenomena is one of typical phenomena when we analyze coupled chaotic circuits. This phenomenon widely can be observed and studied in the field of natural and technical sciences. In order to understand synchronization phenomena in detail, we analyze electronic circuits. Coupled chaotic circuit is composed of an electronic circuits and one of suitable model to analyze the synchronization phenomena. Moreover, we can see various phenomena not only synchronization phenomena. However, all phenomena are not really investigated. Therefore, we consider that our study is new approach to investigate the synchronization phenomena and clustering phenomena in coupled chaotic circuits.

In a previous study, we investigated the relationship between clustering and density of coupled chaotic circuits in 2-dimensional place [4]-[6]. For this investigation, the coupling strength reflected the distance information and we changed the number of circuits in cluster. We showed that clustering phenomena affected other cluster when density in the chaotic circuits was high. We also observed that networks of coupled chaotic circuits could split into different synchronized groups.

In this study, we investigate clustering patterns generated in coupled chaotic circuits networks. In these networks, chaotic circuits are connected to all chaotic circuits. For this investigation, we change the variable parameter of coupling strength and density of chaotic circuits. From the results, the network can be observed 3 clustering patterns. We consider the relationship between scaling parameter and phase difference by changing the scaling parameter of coupling strength. Additionally, we confirm the various phase synchronization patterns when we change the density of chaotic circuits.

2. Circuit Model

Figure 1 shows the model of the chaotic circuit, investigated in [7]-[9].

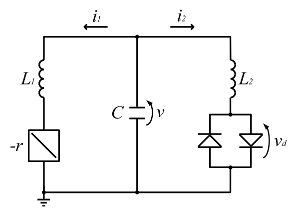


Figure 1: Chaotic circuit.

The following equations show circuit equations when each chaotic circuit is coupled globally with each other.

$$\begin{aligned}
\frac{dx_i}{d\tau} &= \alpha x_i + z_i \\
\frac{dy_i}{d\tau} &= z_i + f(y) \\
\frac{dz_i}{d\tau} &= -x_i - \beta y_i - \sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\
(i, j &= 1, 2, \cdots, N)
\end{aligned} \tag{1}$$

For the computer simulation, we set the parameters as α = 0.460, β = 3.0 and δ = 470. The characteristic of the function $f(y)$ can be described 3-segment piecewise-linear function. The value of γ_{ij} reflects the distance between the circuits in an inverse way, described by the following equation:

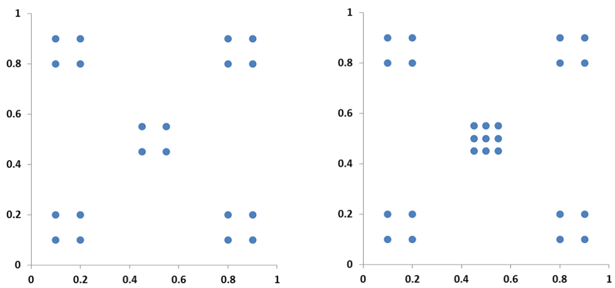
$$\gamma_{ij} = \frac{g}{(length_{ij})^2}. \tag{2}$$

$length_{ij}$ denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit. The parameter g is a scaling parameter that determines the coupling strengths.

3. Clustering Phenomena

3.1 Clustering Phenomena

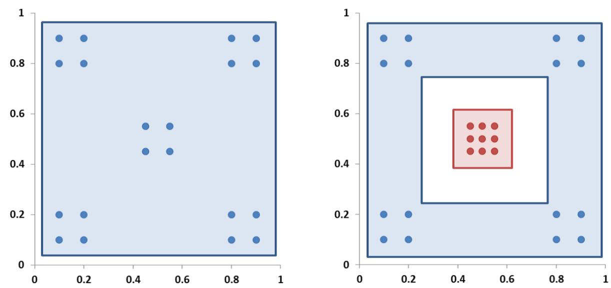
In this section, we investigate clustering phenomena when we configure network of coupled chaotic circuits in 2-dimensional place. In our previous study [6], we researched the relationship between clustering and density of coupled chaotic circuits when we changed density of chaotic circuits. Arrangements of chaotic circuits are shown in Fig. 2. Figure 2 (a) is composed same number of chaotic circuits, however Fig. 2 (b) is composed high density of chaotic circuits in inside and some low density chaotic circuits groups. In these networks, we replace chaotic circuits with a simple model like small circle.



(a) Same number of groups. (b) Different number of groups.

Figure 2: Arrangements of chaotic circuits.

Simulation results in these networks are shown in Fig. 3. From simulation results, all chaotic circuits are synchronized in one cluster from the result of Fig. 3 (a), however we can see 2 clusters from chaos synchronization between high density group and same low density groups from the result of Fig. 3 (b).



(a) One cluster. (b) Two clusters.

Figure 3: The clustering results.

From these results, clustering phenomena are related density of coupled chaotic circuits.

3.2 Investigation of Clustering Phenomena

Next, we investigate the clustering result corresponding to Fig. 3 (b) in detail. This network can be divided 2 clusters between high density and low density from chaos synchronization. We consider the state of this network when we change the parameter g determined by Eq. (2). Furthermore, we calculate the phase difference between chaotic circuits using computer simulation.

For this simulation, the iteration is set to τ_p = 10,000 for calculating the result more precisely. Figure 4 shows the phase difference between two chaotic circuits when we change the value of parameter g.

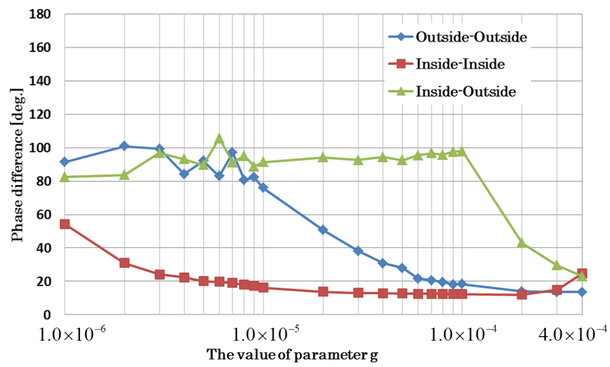


Figure 4: The relationship between g and phase difference.

We define state of synchronization patterns from the average of phase difference when we calculate τ_p = 10,000. Synchronized state can be defined if the average of phase difference below 40°. Similarly, we define asynchronous state that the average of phase difference is between 70° and 110°.

Table 1: State of networks

The value of g	In - In	In - Out	Out - Out
$2.0 \times 10^{-6} \leq g \leq 3.0 \times 10^{-5}$	Syn.	Not-syn.	Not-syn.
$4.0 \times 10^{-5} \leq g \leq 2.0 \times 10^{-4}$	Syn.	Not-syn.	Syn.
$3.0 \times 10^{-4} \leq g \leq 4.0 \times 10^{-4}$	Syn.	Syn.	Syn.

In the region between $g = 2.0 \times 10^{-6}$ and $g = 3.0 \times 10^{-5}$ inside chaotic circuits group is synchronized one cluster, however other chaotic circuits that composed outside groups are not synchronized. In the region between $g = 4.0 \times 10^{-5}$ and $g = 2.0 \times 10^{-4}$ inside chaotic circuits group is synchronized one cluster, also outside chaotic circuits groups are synchronized one cluster. Therefore, clustering phenomena can be observed in this region. Finally, all chaotic circuits are synchronized in one cluster if the region between $g = 3.0 \times 10^{-4}$ and $g = 4.0 \times 10^{-4}$. Thus, this network can be observed 3 clustering patterns from the average of phase difference.

4. Relationship between Phase Difference and Density

In this section, we calculate the phase difference between inside chaotic circuits and inside chaotic circuits, and between inside chaotic circuits and outside chaotic circuits when we change density of chaotic circuits in inside shown in Fig. 3. Moreover, we calculate range of phase difference between maximum and minimum. Here, we change the number of inside chaotic circuits in inside from 1 to 9.

First, we calculate the phase difference inside chaotic circuits shown in Tab. 2. Table 2 shows the phase difference, maximum value and minimum value. Figure 5 shows the result of relationship between phase difference and density of chaotic circuits. In this result, the average of each phase difference is below 40°. Thus chaotic circuits in inside are synchronized regardless of the density.

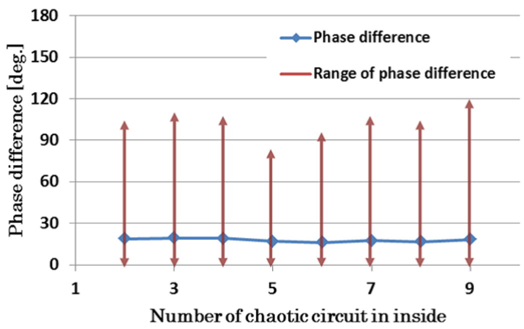


Figure 5: Phase difference and range of phase difference (in-in).

Next, we calculate the phase difference between inside chaotic circuits and outside chaotic circuits shown in Tab. 3. Table 3 shows the phase difference, maximum value and minimum value. Figure 6 shows the result of relationship between phase difference and density of chaotic circuits. From

Table 2: The phase difference (in-in)

Density of circuits	Ave.	Max.	Min.
2	18.894°	102.898°	0.001°
3	19.502°	109.258°	0.001°
4	19.029°	107.431°	0.002°
5	17.067°	86.363°	0.001°
6	16.272°	94.816°	0.002°
7	17.554°	106.801°	0.002°
8	16.678°	101.588°	0.001°
9	18.375°	118.286°	0.003°

this result, chaotic circuits between inside and outside are synchronized when the density in inside is between 1 and 5, however state of other density are not clear. Therefore, we use the frequency distribution to reveal the synchronized state.

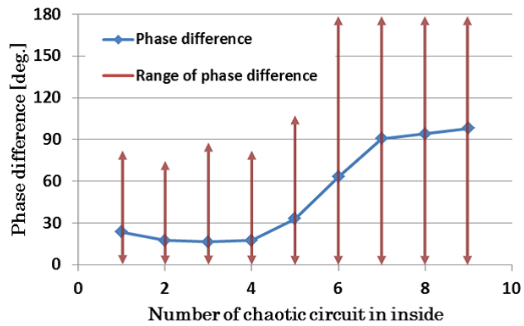
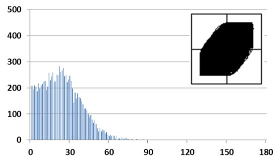


Figure 6: Phase difference and range of phase difference (in-out).

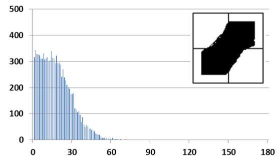
Table 3: The phase difference (in-out)

Density of circuits	Ave.	Max.	Min.
1	23.640°	85.773°	0.003°
2	17.425°	77.697°	0.009°
3	16.442°	87.398°	0.001°
4	17.500°	83.591°	0.008°
5	32.938°	112.724°	0.006°
6	63.203°	178.035°	0.135°
7	90.618°	179.972°	0.032°
8	94.031°	179.998°	0.043°
9	97.988°	179.996°	0.007°

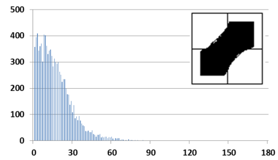
Figure 7 shows the synchronization patterns from Lissajous figures. Each pattern in these figures correspond to Fig. 6 and Tab. 3. We calculate the number of phase domain for the phase difference. The number of counts is set to $\tau_p = 10,000$. From these figures, in the small region of the average of phase difference are synchronized with small value for phase domain. Also, in the region near value of 90° are not synchronized from Lissajous figures and phase domain. However, in the region of number of 6 chaotic circuits, synchronization state is not sure. Additionally, the maximum value is 178.035° in the phase domain. In this region, the phase domain is between 0.135° and 178.035°, namely this region composed of 6 chaotic circuits is not synchronized.



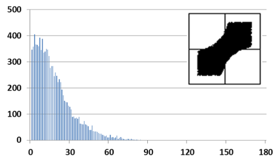
(a) 1 chaotic circuit.



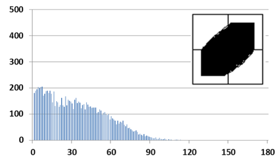
(b) 2 chaotic circuits.



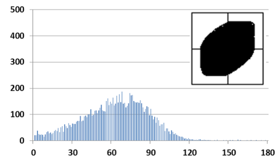
(c) 3 chaotic circuits.



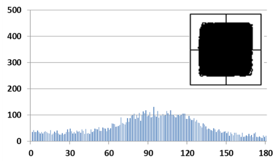
(d) 4 chaotic circuits.



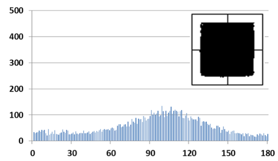
(e) 5 chaotic circuits.



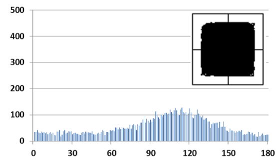
(f) 6 chaotic circuits.



(g) 7 chaotic circuits.



(h) 8 chaotic circuits.



(i) 9 chaotic circuits.

Figure 7: State of synchronization patterns (horizontal axis: phase difference, vertical axis: frequency distribution).

Therefore, we define the state of synchronization or asynchronous by using the average of phase difference and phase domain.

As we mentioned before, we define synchronized state that phase difference is below 40°. Also, we define asynchronous state that the phase difference is between 70° and 110° and the phase domain is between 0° and 180°. Thus, we define synchronization patterns by using phase difference and phase domain. Furthermore, the coupled chaotic circuits networks can be observed some clustering patterns from chaos synchronization.

5. Conclusions

In this study, we have investigated clustering patterns generated in coupled chaotic circuits networks. In these networks, the coupling strength reflected the distance information and each chaotic circuit is connected to all chaotic circuits. For this investigation, we have changed the scaling parameter of coupling strength and number of chaotic circuits composed of cluster. We have observed some clustering patterns from chaos synchronization. From computer simulation results, we have confirmed that the state of clustering patterns depend on the scaling parameter g and density of chaotic circuits networks. Furthermore, we have made clear that it is efficient for using the average of phase difference and phase domain.

In our future work, we would like to study the clustering phenomena in the case of large scale networks. Additionally, we hope to apply this clustering method for data mining, image processing and something application in our lives.

Acknowledgment

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