2013 International Workshop on Nonlinear Circuits,
Communications and Signal Processing
NCSP'13, Isrand of Hawaii, Hawaii, USA. March 4-7, 2013

Synchronization of Switching Phenomena in a Coupled Chaotic System with Inaba Circuit

Takuya Nishimoto†, Yasuteru Hosokawa† and Yoshifumi Nishio‡

†Shikoku University
Furukawa, Ohjin, Tokushima, Japan
Email: s2137034@edu.shikoku-u.ac.jp

‡Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email: nishio@ee.tokushima-u.ac.jp

Abstract

Synchronization of switching phenomena can be observed in a few coupled chaotic systems. In our past study, we have observed synchronization of switching phenomena in coupled chaotic circuits with Nishio circuit. However, these phenomena have reported only two cases. In this study, we investigate the phenomena in other chaotic system with Inaba circuit.

1. Introduction

Many kinds of complex phenomena can be observed on the large-scale coupled chaotic circuits. Investigations of these phenomena are very important works in order to declare nonlinear phenomena in the natural world. An electric circuit is suitable for models of large-scale coupled nonlinear systems because of the following reasons. Getting electric parts is easy and inexpensive, short experiment time, high repeatability of experiments and it is a real physical system. Therefore, there are many studies of large-scale coupled circuit systems. In these studies, synchronization phenomena are attracted attentions.

On the other hand, some chaotic circuits have coexisting attractors. In these circuits, switching phenomena of attractors can be observed. Normally, in the case of a synchronization state, the switching of attractors is also synchronized. And in the case of asynchronous states, the switching of attractors is also asynchronized. However, synchronization of self-switching phenomena on full-coupled chaotic oscillators is reported by [1]. Additionally, we have observed similar phenomena in some chaotic systems with Nishio circuit [2][3]. We investigated full-coupled, ladder-coupled, star-coupled and ring-coupled chaotic system with Nishio circuit is investigated.

In this study, we investigate these phenomena in full-coupled chaotic systems with Inaba circuit.

2. System Model

Figure 1 shows a circuit model used in this study. This circuit consists of a linear negative resister, two inductors, a capacitor and a bi-directionally coupled diodes. Figure 2 shows i-v characteristic of the diodes that are idealized model. Figure 3 shows a coupled chaotic system with Inaba circuit.

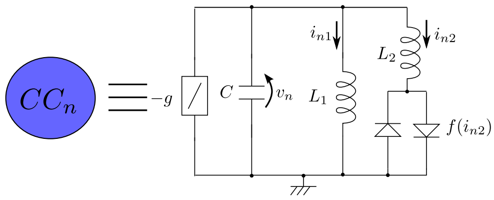

Figure 1: A circuit model

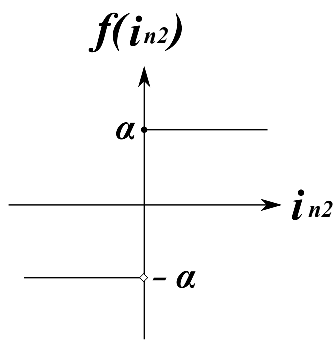

Figure 2: System model

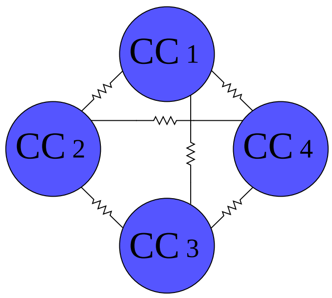

Figure 3: System model($N = 4$)

Full coupled chaotic system is investigated in this study. The system equation is described as follows:

$$\begin{cases} C\dfrac{dv_n}{dt} &= gv_n - (i_{n1} + i_{n2}) - G\left(Nv_n - \displaystyle\sum_{k=1}^{N} v_k\right), \\ L_1\dfrac{di_{n1}}{dt} &= v_n, \\ L_2\dfrac{di_{n2}}{dt} &= v_n - f(i_{n2}). \\ & (n = 1, 2, \cdots, N) \end{cases} \tag{1}$$

By changing parameters and variables as follows,

$$\begin{aligned} & v_n = x_n,\ \ i_{n1} = \sqrt{\frac{C}{L_1}}y_n,\ \ i_{n2} = \sqrt{\frac{C}{L_1}}z_n, \\ & t = \sqrt{L_1C}\tau,\ \ \text{"}\cdot\text{"} = \frac{d}{d\tau},\ \ b = g\sqrt{\frac{L_1}{C}}, \\ & n = \frac{L_2}{L_1}\ \text{ and }\ \delta = G\sqrt{\frac{L_1}{C}}. \end{aligned} \tag{2}$$

the normalized system equation is described as follows: when $z_n > 0$,

$$\begin{cases} \dot{x}_n &= bx_n - y_n - z_n - \delta\left(Nz_n - \displaystyle\sum_{k=1}^{N} z_k\right), \\ \dot{y}_n &= x_n, \\ \dot{z}_n &= \dfrac{1}{n}x_n - \dfrac{\alpha}{n}. \end{cases} \tag{3}$$

when $z_n = 0$,

$$\begin{cases} \dot{x}_n &= bx_n - y_n - z_n - \delta\left(Nz_n - \displaystyle\sum_{k=1}^{N} z_k\right), \\ \dot{y}_n &= x_n.S \end{cases} \tag{4}$$

when $z_n < 0$,

$$\begin{cases} \dot{x}_n &= bx_n - y_n - z_n - \delta\left(Nz_n - \displaystyle\sum_{k=1}^{N} z_k\right), \\ \dot{y}_n &= x_n, \\ \dot{z}_n &= \dfrac{1}{n}x_n + \dfrac{\alpha}{n}. \end{cases} \tag{5}$$

where N is a number of circuits. x_n, y_n and z_n are corresponding to v_n, i_{n1} and i_{n2}, respectively.

3. Computer Simulations

This section shows numerical experiments of the system model.

3.1 Coexisting Attractors

Figure 5 shows a bifurcation diagram. The points are plotted when a solution hits the poincaré map that is defined where $z = 0.7$ and $\dot{y} < 0$. From this bifurcation diagram, the attractors is classified by colors according to following definition. When the solution hits the poincaré map and $y \leq 0.32$, the color is set as blue. The other case, namely, When the solution hits the poincaré map and $y > 0.32$, the color is set as red. This definition is important to investigate switching of attractors and as such, is being applyed to the simulations of this study.

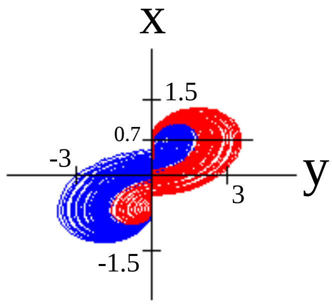

Figure 4: Coexisting attractors. $\alpha = 0.7$, $b = 1.49, n = 0.0495$ and $V_0 = 1$.

3.2 Case of $N = 2$

Figure 6 shows simulation result of $N = 2$. These figures

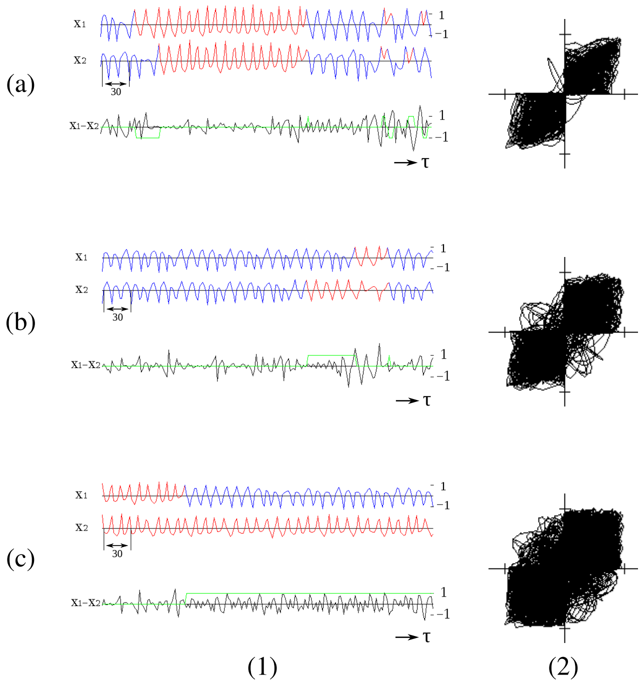

Figure 6: Computer simulation results. $N = 2$, (a) $\delta = 0.30$. (b) $\delta = 0.15$. (c) $\delta = 0.09$. (1)Time waveforms of x_i and difference. (2) z_1 vs z_2.

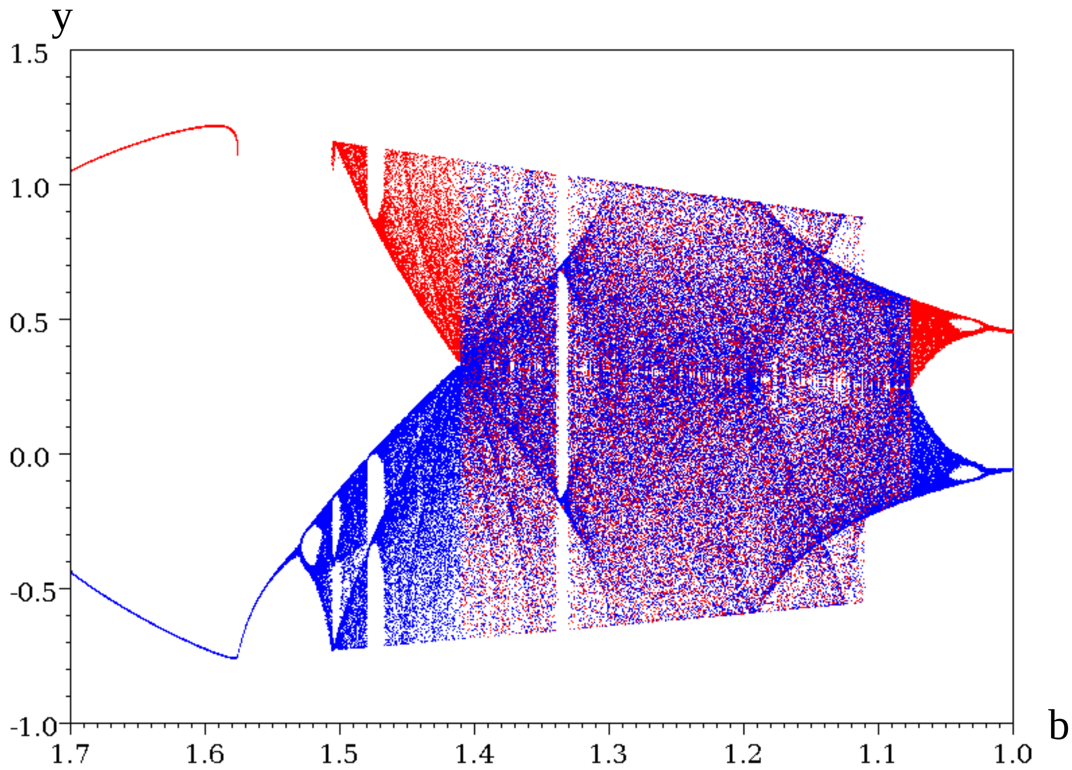

Figure 5: Bifurcation diagram. $\alpha = 0.7, V_0 = 1$ and $n = 0.0495$.

show time waveforms(1) and lissajous figure(2). In the time

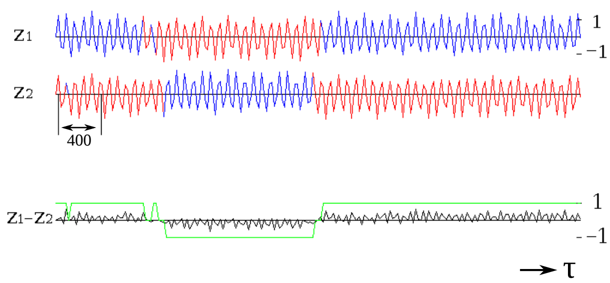

Figure 7: Synchronization of switching phenomena in Nishio circuit.

waveforms, there are red and blue waves show that the attractor states. Green lines also show a difference of attractor state. Namely, if each circuit takes different attractor state, the amplitude of this green line is observed. Synchronization of switching phenomena occured in Fig. 6 (a). However this system keeps asynchronous state at all because the wave of $x_1 - x_2$ is observed. Though asynchronous state is kept at this system, switching phenomena are occured in both circuit at almost same time. This phenomenon is very interesting at the point that there is behavior which has simultaneity in asynchronous behavior. Fig. 6 (b) and (c) show simulation results which can be observed when decrease the coupling strength from Fig. 6 (a). In this case, synchronization of switching phenomena cannot be observed and lissajous figure gradually changed. This phenomenon is collapsing according to decrease the parameter value of δ.

In our past study, similar phenomena is observed with Nishio circuit. However, there is a point of difference between Nishio circuit and Inaba circuit. The synchronization of switching phenomena in Nishio circuit is anti-phase attractor switching as shown in Fig. 7. In contrast, the phenomena in Inaba circuit is in-phase attractor switching. It is a interesting that though the structure of both circuit have a similar structure, the switching of these circuits are different.

3.3 Case of $N = 3$

Figure 8 shows the simulation result case of $N = 3$. We can confirm a synchronization of switching phenomena. The behavior of $N = 3$ is nearly synchronous state compare with case of $N = 2$. Because the amplitude of difference wave become smaller than $N = 2$. Moreover, frequency of the switching is decreased. This tendency also can be observed in the system with Nishio circuit. The behavior of the phenom-

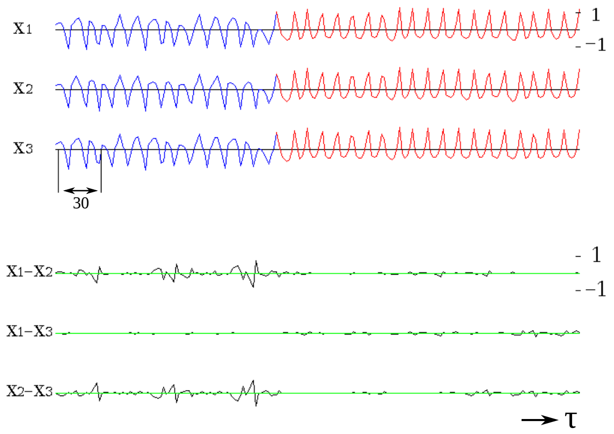

Figure 8: A computer simulation result for $N = 3. \alpha = 0.7, V_0 = 1, b = 1.56, n = 0.0495$ and $\delta = 0.27$.

ena have characteristics, such as the waveform of difference becomes larger before occur the switching.

3.4 Case of $N > 3$

We also investigate case of $N > 3$. One of the simulation results is shown as Fig. 9. Frequency of switching is decreased than case of $N < 4$. Although observation of

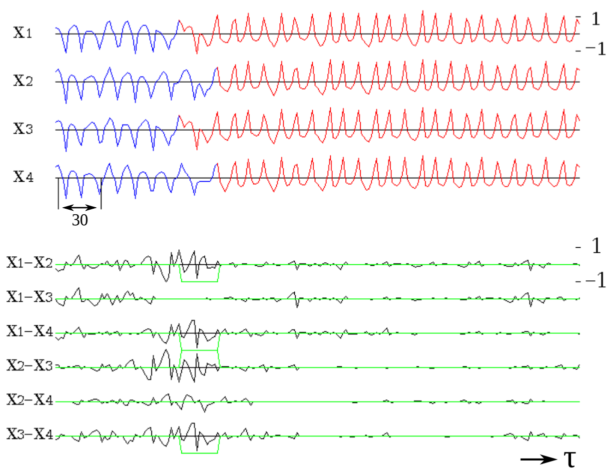

Figure 9: A computer simulation result for $N = 4. \alpha = 0.7, V_0 = 1, b = 1.54, n = 0.0495$ and $\delta = 0.21$.

the phenomena in Nishio circuit is remarkably difficult at $N > 3$, in Inaba circuit, still can be observed. In the case of increased the number of the circuit from Fig. 9, the system becomes almost synchronous state. Though, observation of synchronization of switching phenomena in Nishio circuit can be around $N = 3$, the phenomena in Inaba circuit can be observed around $N = 5$. In the case of $N > 5$, the system becomes almost synchronous state and synchronization of switching phenomena cannot be observed.

4. Conclusions

In this study, synchronization of switching phenomena is investigated with Inaba circuit. As a result, In-phase synchronization of switching phenomena is observed. This result is different from the synchronization of switching phenomena in the system with Nishio circuit. The switching of the system with Inaba circuit is in-phase and the phenomena can be observed around $N = 5$. Moreover, there are tendencies as follows. In case of the number of the circuit is increased, frequency of switching is decreased and gradually become synchronous state. The waveform of difference becomes bigger before occur the switching.

References

[1] H. Sekiya, S. Mori and I. Sasase, “Synchronization of Self-Switching Phenomena on Full-Coupled Chaotic Oscillators,” *IEICE Trans.*, vol. J83-A, no. 11, pp. 1264–1275, 2000.

[2] T. Nishimoto, Y. Hosokawa and Y. Nishio, “Anti-phase Synchronization of Switching Phenomena in Globally Coupled System of Chaotic Circuits,” *IEICE Technical Report*, no. NLP-48 & CAS2010-32, pp. 1–4, 2010.

[3] T. Nishimoto, Y. Hosokawa and Y. Nishio, “In-Phase and Anti-Phase Synchronization of Switching Phenomena in Coupled Chaotic Circuits” *Proc. of IEEE Workshop on Nonlinear Circuit Networks (NCN’10)* , pp. 15–18, 2010.

[4] Y. Nishio, N. Inaba, S. Mori and T. Saito, “Rigorous Analyses of Windows in a Symmetric Circuit,” *IEEE Trans. Circuit and Systems*, vol. 37, no. 4, pp. 473–487, 1990.