2013 International Workshop on Nonlinear Circuits,
Communications and Signal Processing
NCSP'13, Israd of Hawaii, Hawaii, USA. March 4-7, 2013

Synchronization Phenomena of Two Simple RC Chaotic Circuits Coupled by a Capacitor

Ayana Shimada[1], Yoko Uwate[1], Yoshifumi Nishio[1] and Jingmin Xin[2]

[1] Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Phone: +81-88-656-7470
E-mail: {ayana,uwate,nishio}@ee.tokushima-u.ac.jp

[2] Xi'an Jiaotong University
28 Xianning Xilu, Xi'an, Shaanxi, China
Phone: +86-29-8266-8802
Email: jxin@mail.xjtu.edu.cn

Abstract

In this study, we investigate two coupled chaotic oscillators composed of RC circuits. We carry out computer calculation and circuit experiment. We could observe in-phase synchronization and anti-phase synchronization by computer calculation. Moreover, when we changed the value of coupling strength between oscillators, we observed the change of synchronization state. In addition, we observed in-phase and anti-phase attractor.

1. Introduction

Recently, many researchers have shown their interests in chaotic systems [1][2][3]. In particular, chaos synchronization has attracted many researchers' attentions and their mechanism has been gradually made clear. This is because, chaotic circuit systems are good models to explain higher dimensional nonlinear phenomena in the field of natural science. Further, chaotic behavior has been expected in various research fields. Hence, chaos has been investigated not only in engineering but also in various fields such as medicine, sociology and economics. In the field of electrical and electronic engineering, researchers have investigated the phenomena in large scale coupling chaotic circuits. Moreover, they proposed the new chaos-generating circuits and many applications using chaos as chaos communications, chaos cryptosystem, chaos neural networks and so on. In order to realize chaotic engineering systems, it is important to investigate simple coupled chaos-generating circuits.

In our previous study, a simple chaotic oscillator using two RC circuits was proposed [4]. When we change the parameter, we can observe not only periodic attractors but also chaotic attractors in this simple oscillator. Further, we investigated the chaotic behavior when the number of the coupled RC circuits are increased [5]. We investigated the attractors and the Poincaré maps when we changed the value of parameter. In addition, we found the cross correlation characteristics between neighboring oscillators. In these previous studies, we found that it is very interesting to investigate implementable chaotic circuit compose of RC circuit.

In this study, we investigate two coupled chaotic oscillators composed of RC circuits. Two chaotic oscillators coupled by one capacitor. We carry out computer calculation and circuit experiment about the cases of $V_{S1} = V_{S2}$ and $V_{S1} = -V_{S2}$.

2. Circuit Model

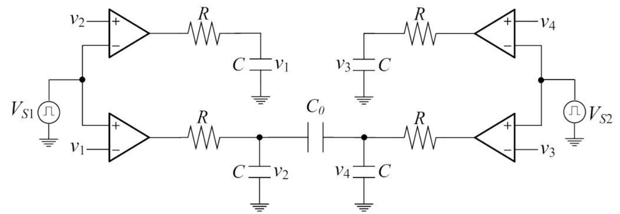

Figure 1: Circuit model

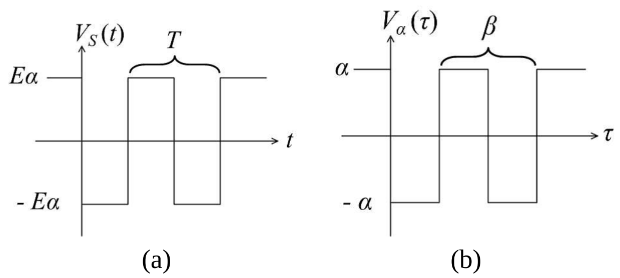

Figure 2: Rectangular voltage waveform

Figure 1 shows the circuit model. In this figure two chaotic oscillators, which was proposed in [4], are coupled via one capacitor C_0. Two independent rectangular voltage sources are connected to two comparators of each oscillator. Hence, the whole circuit consists of two rectangular voltage sources, four comparators, four resistors, four capacitors and one variable capacitor. Figure 2(a) shows the rectangular voltage waveform $V_S(t)$. $E\alpha$ is the amplitude of the rectangular voltage and T is the period of the waveform. E is the output voltage of the comparators, namely the DC supply voltage of the operational amplifiers. The circuit equations are described as follows:

$$RC\frac{dv_1}{dt} = \begin{cases} -v_1 + E & (v_2 > V_{S1}) \\ -v_1 - E & (v_2 < V_{S1}) \end{cases}$$

$$RC\frac{dv_2}{dt} = \begin{cases} -\frac{C+C_0}{C+2C_0}v_2 - \frac{C_0}{C+2C_0}v_4 - E \\ \qquad\qquad (v_1 > V_{S1}) \\ -\frac{C+C_0}{C+2C_0}v_2 - \frac{C_0}{C+2C_0}v_4 + E \\ \qquad\qquad (v_1 < V_{S1}) \end{cases}$$

$$RC\frac{dv_3}{dt} = \begin{cases} -v_3 + E & (v_4 > V_{S2}) \\ -v_3 - E & (v_4 < V_{S2}) \end{cases}$$

$$RC\frac{dv_4}{dt} = \begin{cases} -\frac{C_0}{C+2C_0}v_2 - \frac{C+C_0}{C+2C_0}v_4 - E \\ \qquad\qquad (v_3 > V_{S2}) \\ -\frac{C_0}{C+2C_0}v_2 - \frac{C+C_0}{C+2C_0}v_4 + E \\ \qquad\qquad (v_3 < V_{S2}) \end{cases}$$

By using the following variables and the parameters,

$$v_n = Ex_n,\ \gamma = \frac{C_0}{C+2C_0},\ t = RC\tau,\ T = RC\beta$$

$$(n = 1,\ 2,\ 3,\ 4),$$

we obtain the normalized circuit equations. Because the circuit equations are linear in each region, the rigorous solution of the circuit equations can be derived as follows:

$$x_1 = \begin{cases} (x_{10} - 1)e^{-\tau} + 1 & (x_2 > V_\alpha) \\ (x_{10} + 1)e^{-\tau} - 1 & (x_2 < V_\alpha) \end{cases}$$

$$x_2 = \begin{cases} \left(\frac{x_{20}+x_{40}}{2} + 1\right)e^{-\tau} + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-\gamma\tau} - 1 \\ \qquad\qquad (x_1 > V_\alpha) \\ \left(\frac{x_{20}+x_{40}}{2} - 1\right)e^{-\tau} + \left(\frac{x_{20}-x_{40}}{2}\right)e^{-\gamma\tau} + 1 \\ \qquad\qquad (x_1 < V_\alpha) \end{cases}$$

$$x_3 = \begin{cases} (x_{30} - 1)e^{-\tau} + 1 & (x_4 > V_\alpha) \\ (x_{30} + 1)e^{-\tau} - 1 & (x_4 < V_\alpha) \end{cases}$$

$$x_4 = \begin{cases} \left(\frac{x_{20}+x_{40}}{2} + 1\right)e^{-\tau} - \left(\frac{x_{20}-x_{40}}{2}\right)e^{-\gamma\tau} - 1 \\ \qquad\qquad (x_3 > V_\alpha) \\ \left(\frac{x_{20}+x_{40}}{2} - 1\right)e^{-\tau} - \left(\frac{x_{20}-x_{40}}{2}\right)e^{-\gamma\tau} + 1 \\ \qquad\qquad (x_3 < V_\alpha) \end{cases}$$

where V_α corresponds to V_S and is shown in Fig. 2(b). x_{10}, x_{20}, x_{30} and x_{40} are initial values.

3. Simulation Results

We show the results of computer calculations. Moreover, Fig. 4 shows the phase difference between the left oscillator and the right oscillator in Fig. 1.

3.1 Case of $V_{S1} = V_{S2}$

Left attractors of Fig. 3 shows the left oscillator (x_1-x_2) and center attractors shows the right oscillator (x_3-x_4) in Fig. 1. Right attractors of Fig. 3 shows the phase difference between left oscillator and right oscillator (x_1-x_3) in Fig. 1. In this computer calculation, we use fixed parameters $\alpha = 0.060$ and $\beta = 4.0$. We observe the in-phase synchronization state in Fig. 3(1c) and when Fig. 3(5c), we observe the asynchronous state. We find that when the value of the coupling strength becomes small, the strength of synchronization state of each oscillator becomes weak in Fig. 3.

Next, we show the relationship between the coupling strength and the degree of synchronization. We use fixed parameters $\alpha = 0.060$ and $\beta = 4.0$. Figures 3(a) to (c) show the degree of synchronization by changing coupling strength γ. Vertical axis is degree of synchronization D_{syn} defined as follows;

$$D_{syn} = \max\{x_1 - x_3\} - \min\{x_1 - x_3\}.$$

When D_{syn} is small, our model is close to the synchronization state. When the value of γ is small, we find that each oscillator do not become synchronization completely. However, the value of $\gamma = 0.80$, we observe the in-phase synchronization state. There is clearly a difference when one compares $\gamma = 0.05$ and $\gamma = 0.80$.

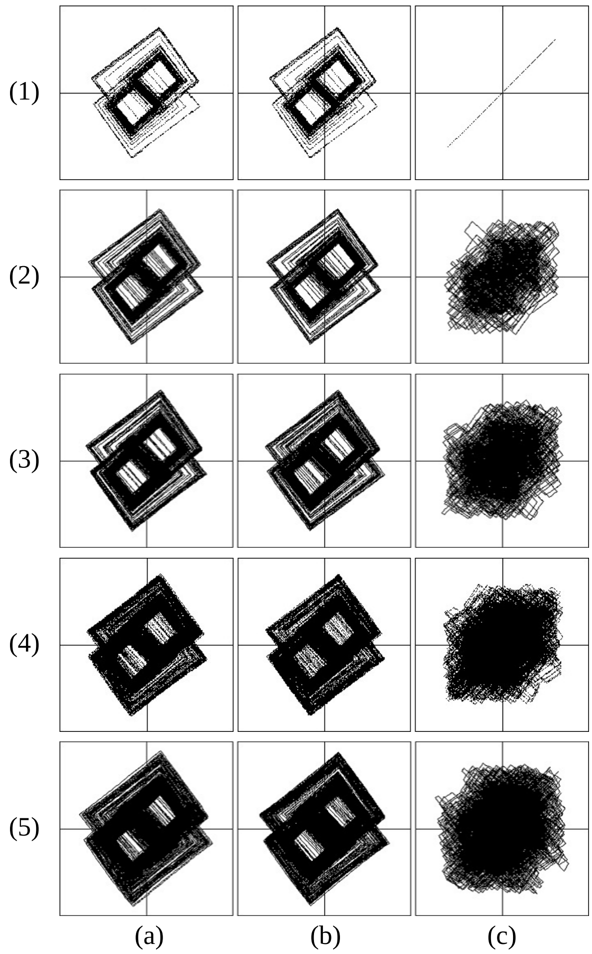

Figure 3: Attractor and phase difference in $V_{S1} = V_{S2}$ (1) $\gamma = 0.80$, (2) $\gamma = 0.75$, (3) $\gamma = 0.50$, (4) $\gamma = 0.45$ and (5) $\gamma = 0.01$

3.2 Case of $V_{S1} = -V_{S2}$

Left attractors of Fig. 5 shows the left oscillator (x_1-x_2) and center attractors shows the right oscillator (x_3-x_4) in Fig. 1. Right attractors of Fig. 5 shows the phase difference between left oscillator and right oscillator (x_1-x_3) in Fig. 1. In this computer calculation, we use fixed parameters $\alpha = 0.060$ and $\beta = 4.0$. From Figs. 5(1c) and (4c), we observe the anti-phase synchronization. Moreover, we observe the synchronization and asynchronous state regardless of the value of coupling strength.

4. Circuit Experiment

Figures 6 and 7 show the circuit experimental results. Figs. 6(a) and (b) show the attractor and time waveform by difference of the value of C_0 in the case of $V_{S1} = V_{S2}$. Figs. 7(a) and (b) show the attractor and time waveform by difference of the value of C_0 in the case of $V_{S1} = -V_{S2}$. From all of the time waveform, we find that chaos appears. From Figs. 6(a) and (b), when the value of each rectangular voltage source is $V_{S1} = V_{S2}$, each oscillator becomes in-phase synchronization. Compare Fig. 6(a) with (b), we observe in-phase synchronization clearly in Fig. 6(a). By weakening the coupling strength, we can observe the state, such as asynchronous. Figure 7 shows the attractor and time waveform about the case of $V_{S1} = -V_{S2}$. We can not observe the anti-phase synchronization clearly the case of $V_{S1} = -V_{S2}$. However, we can observe the attractor of anti-phase. Moreover, we obtain the near results in Figs. 5(2c), (3c) and (5c).

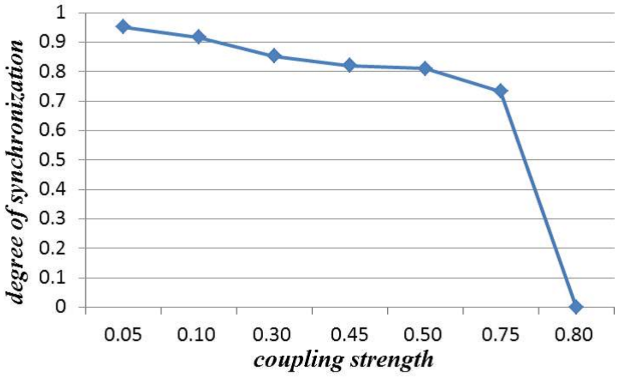

Figure 4: Relation between coupling strength and degree of synchronization

5. Conclusions

In this study, we have investigated two coupled chaotic oscillators composed of RC circuits. Two chaotic oscillators were coupled by one capacitor C_0. We carried out computer calculation and circuit experiment about the case of $V_{S1} = V_{S2}$ and $V_{S1} = -V_{S2}$. In $V_{S1} = V_{S2}$ of computer calculation, we confirmed that the relationship between coupling strength and synchronization state. We found that when the value of the coupling strength becomes small, the strength of synchronization state of each oscillator becomes weak. From the graph, we found the same relationship. In $V_{S1} = V_{S2}$, we obtained the difference phenomena from $V_{S1} = V_{S2}$. It regardless of the coupling strength, synchronization and asynchronous state are observed. In addition, we could be obtained the same result as computer calculation results by circuit experiment.

In our future work, we will make a detailed research of this proposed circuit model. Moreover, we would like to investigate lager scale circuits.

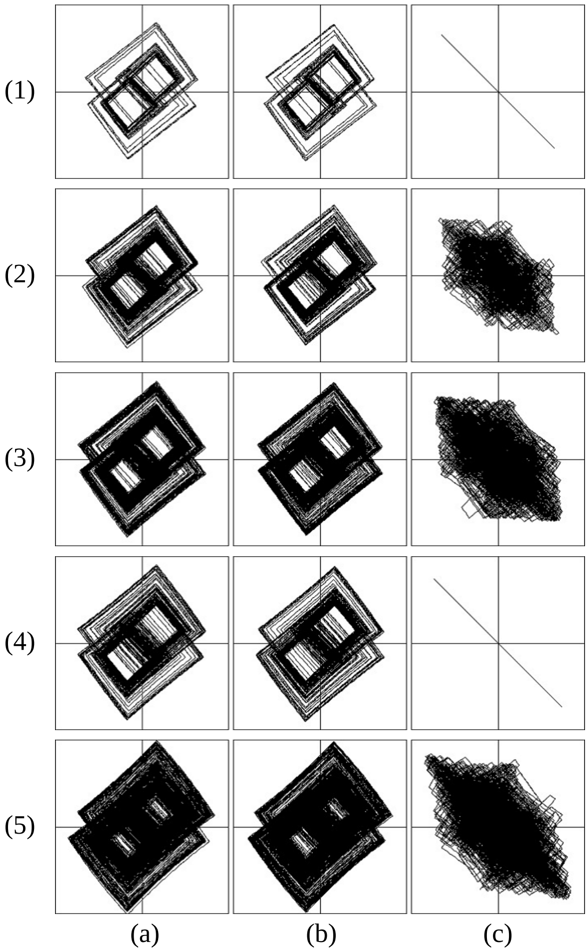

Figure 5: Attractor and phase difference in $V_{S1} = -V_{S2}$ (1) $\gamma = 0.80$, (2) $\gamma = 0.75$, (3) $\gamma = 0.50$, (4) $\gamma = 0.45$ and (5) $\gamma = 0.01$

Acknowledgment

This work was partly supported by JSPS Grant-in-Aid for Scientific Research 22500203.

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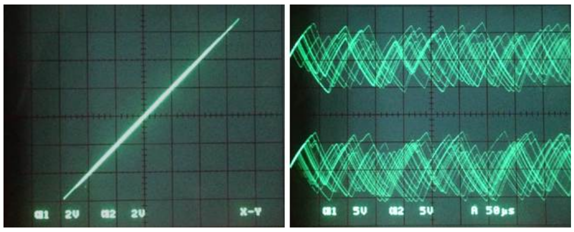
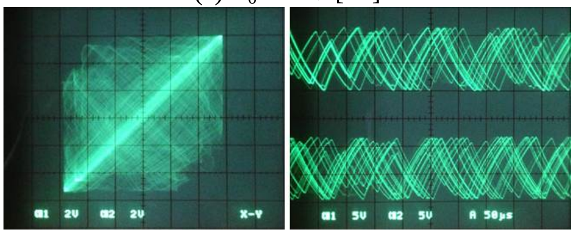
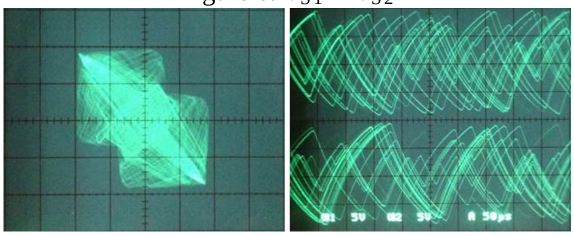


(a) C_0 = 67.9[nF]

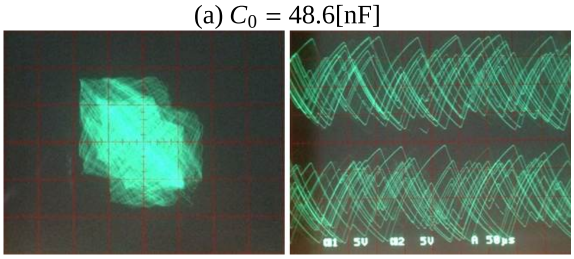


(b) C_0 = 1.544[nF]

Figure 6: $V_{S1} = V_{S2}$



(a) C_0 = 48.6[nF]



(b) C_0 = 10.0[nF]

Figure 7: $V_{S1} = -V_{S2}$

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