Clustering Phenomena Considering the Density of Coupled Chaotic Circuits Networks

Yuji Takamaru†, Yoko Uwate†, Thomas Ott‡ and Yoshifumi Nishio†
†Dept. of Electrical and Electronic Engineering, Tokushima University,
2-1 Minami-Josanjima, Tokushima, 770-8506, Japan
phone:+81-88-656-7470, fax: +81-88-656-7471
Email:{takamaru, uwate, nishio}@ee.tokushima-u.ac.jp
‡Institute of Applied Simulation, Zurich University of Applied Sciences,
Einsiedlerstrasse 31a, 8820 Waedenswil, Switzerland
phone:+41-058-934-56-84
Email:thomas.ott@zhaw.ch

***Abstract*— We have investigated the clustering phenomena observed from coupled chaotic circuits networks. The networks are globally coupled with each other by resistor using the distance information. The coupling strength is depended on the distance between the circuits.**

In this study, we consider the relationship between density and distance of each cluster that configured by chaotic circuits. For this investigation, we study the clustering phenomena when we change the number of circuits in a cluster. Furthermore, we consider the clustering phenomena by changing the distance between each cluster composed of chaotic circuits using 2-dimensional place.

I. Introduction

Nowadays, we often have to deal with huge amounts of data. To structure and analyze such data it is useful to partition the data set into clusters. The idea of clustering algorithms is to find clusters consisting of similar elements. Clustering algorithms have widespread applications in different fields, such as business data mining, image processing or the analysis of biological data. Clustering of data is helpful high-speed information processing and observing complex relationship. Along with the variety of applications there is a variety of different clustering algorithms. Some algorithms propose to utilize synchronization phenomena, for instance in Coupled Map Lattics (CML), for clustering [1]-[3]. Previously, these studies are used discrete time model for clustering, however analysis of using a continuous time model has not almost studies. Continuous time model can treat high-speed information processing more than discrete time model.

Synchronization is a prominent phenomenon, widely observed and studied in the field of natural and technical sciences. One possibility to get insight into the role of synchronization phenomena is the analysis of electronic circuits. However, there are not many studies on synchronization-based clustering in networks of coupled chaotic circuits. We can see the chaos synchronization when we connect chaos circuits each other. We applied to chaos synchronization for clustering.

In a previous study, we investigated 2-dimensional networks of complex chaotic circuits, where the coupling strength reflected the distance information [4]-[5]. We showed that the circuits arranged close to each other could synchronize in-phase, whereas coupled circuits far from each other could not be synchronized. We also observed that such networks of coupled chaotic circuits could split into different synchronized groups, revealing a clustering phenomenon.

In this study, we consider the relationship between density and distance of each cluster that configured by chaotic circuits. For this investigation, we study the clustering phenomena when we change the number of circuits in a cluster. Furthermore, we consider the clustering phenomena affect the distance between each cluster composed of chaotic circuits using 2-dimensional place in continuous time model. Also, we investigate the relation of density and distance of each cluster.

II. Circuit model

Figure 1 shows the model of the used chaotic circuit called Nishio-Inaba circuit, investigated in [6]-[8].

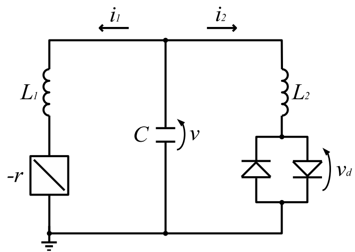

Fig. 1. Chaotic Nishio-Inaba circuit.

The circuit consists of a negative resistance, a nonlinear resistance consisting of two diodes, a capacitor and two inductors.

The normalized equations of this circuit is described as

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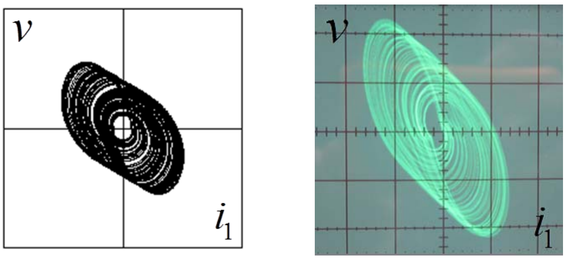
follows:

$$\begin{aligned} \dot{x} &= \alpha x + z \\ \dot{y} &= z - f(y) \\ \dot{z} &= -x - \beta y \end{aligned} \tag{1}$$

where $f(y)$ is described as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \tag{2}$$

Figure 2 shows the chaotic attractor generated by the circuit, obtained from a computer simulation (Fig. 2 (a)) and measured in a circuit experiment (Fig. 2 (b)). For the computer simulation, we set the parameters as $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. For the circuit experiment, the parameters were fixed with $L_1 = 500[mH]$, $L_2 = 200[mH]$, $C = 0.0153[\mu F]$, and $r_d = 1.46[M\Omega]$.



(a) Computer simulation. (b) Circuit experiment.

Fig. 2. Chaotic attractor.

We can consider the following equations when each chaotic circuits coupled globally with each other.

$$\begin{aligned} \frac{dx_i}{d\tau} &= \alpha x_i + z_i \\ \frac{dy_i}{d\tau} &= z_i + f(y) \\ \frac{dz_i}{d\tau} &= -x_i - \beta y_i - \sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ &(i, j = 1, 2, \cdots, N) \end{aligned} \tag{3}$$

where i in the equation represents the circuit itself, and j indicates the coupling with other circuits. The parameter γ_{ij} represents the coupling strength between the circuits. The value of γ_{ij} reflects the distance between the circuits in an inverse way, described by the following equation:

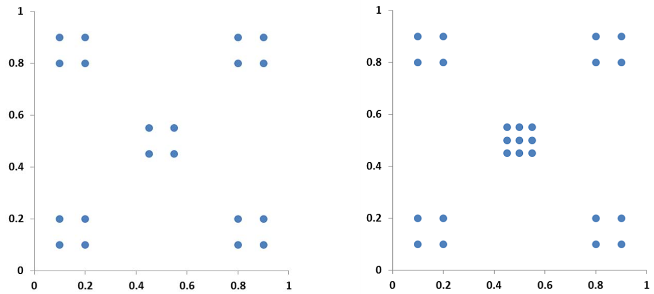
$$\gamma_{ij} = \frac{g}{(length_{ij})^2}. \tag{4}$$

$length_{ij}$ denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit. The parameter g is a weighting or scaling parameter that determines the coupling strengths. For our study, we set the parameter as $g = 1.0 \times 10^{-4}$.

III. Clustering Phenomena

A. Considering the density of the cluster

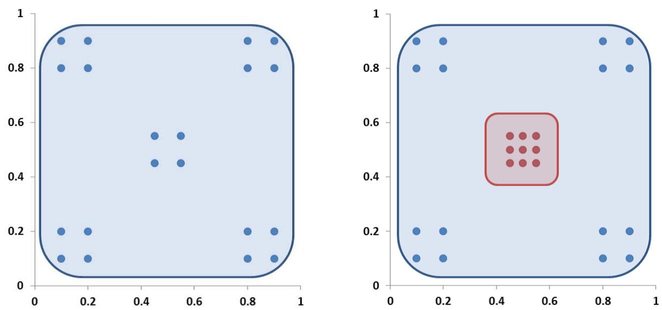
First, we investigate the case of the relationship between clustering and density of a cluster. In a previous study, we have observed that the coupled chaotic circuits arranged close to each other could synchronize in-phase, whereas coupled chaotic circuits far from each other could not be synchronized. We study the clustering phenomena when we change the number of circuits in a cluster. We set 2 patterns for arrangement of the circuits. The arrangement of the circuits are shown in Fig. 3. Arrangement of Fig. 3 (a) is composed the same number of circuits in all groups, Fig. 3 (b) is changed the number of circuits in the middle group. Also, all circuits are connected with each other by resistors and the coupling strength between two circuits is determined by Eq. (4).



(a) Same number of groups. (b) Different number of groups.

Fig. 3. Arrangement of chaotic circuits.

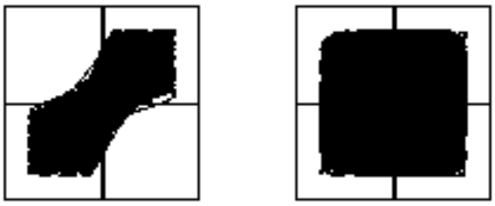
Figure 4 shows the clustering results. All circuits group are synchronized in one cluster from the result of Fig. 4 (a), however we can see the 2 cluster from synchronization between the middle group and the other groups from the result of Fig. 4 (b). Figure 5 shows the phase difference between the middle group and the other groups.



(a) One cluster (b) Two clusters

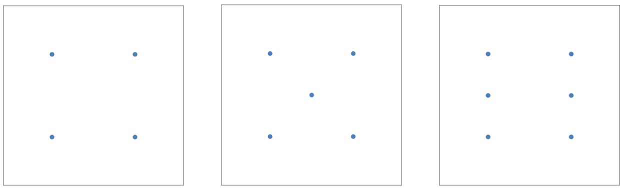
Fig. 4. The clustering results

We consider the relationship between the cluster and density in a group. We change the density in the middle circuits group from 4 to 9 as shown in Fig. 6. The results of phase difference between the middle group and the other groups arrangement in Fig. 6 correspond to Fig. 7.

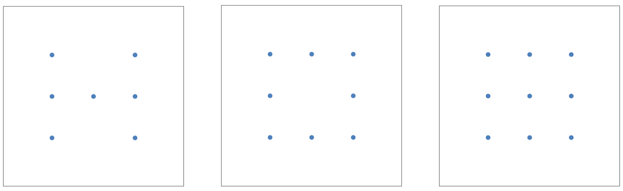


(a) Synchronization (b) Not synchronization

Fig. 5. Phase difference between middle group and other groups.



(a) 4 chaotic circuits (b) 5 chaotic circuits (c) 6 chaotic circuits



(d) 7 chaotic circuits (e) 8 chaotic circuits (f) 9 chaotic circuits

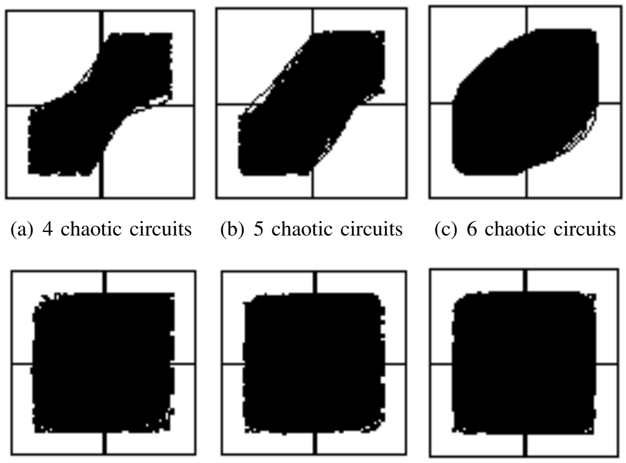
Fig. 6. Change the number of circuits in the middle cluster.

From these results in Fig. 7, we can make the following observations. We can obtain the synchronization phenomena between the middle group and the other groups when the middle circuits is up to 6. A circuit in the middle group grow in density from 7 to 9, the middle group is not synchronized from other groups. That is the middle group of isolated circuits from other groups more than 9. From the above results, the clustering phenomena for chaotic circuit network is related to density of the circuits in a cluster.

B. Considering the distance for each cluster

In a next step, we investigate the clustering phenomena obtained by changing the distance between each cluster. Each groups consist of 4 circuits are gradually coupled with each other and each groups close to the middle group consist of 9 chaos circuits. Figure 8 shows the example for close to the middle group. Now this state is clustered into 2 groups between the middle group and the other groups from chaos synchronization. We consider the form clustering by synchronization when arrangement the number of circuits can be obtained clustering phenomena (the middle group consist of 7 to 9 circuits). Now, the distance d between the middle group and each groups is $d = 3.54$. The method for measuring the distance is shown in Fig. 9.

Figure 10 shows the computer simulation results of the phase differences between the middle group and the other groups when the middle group consist of 9 chaotic circuits. We



(a) 4 chaotic circuits (b) 5 chaotic circuits (c) 6 chaotic circuits

(d) 7 chaotic circuits (e) 8 chaotic circuits (f) 9 chaotic circuits

Fig. 7. Phase difference between middle group and other groups.

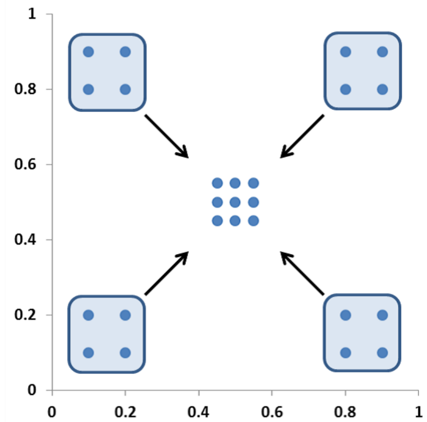


Fig. 8. Close to the middle group each cluster.

set the each distance from the middle group in Fig. 10. We can derive the following observations, exemplified by a selection pairwise comparison. Clustering can be observed for values $d \geq 0.325$. This clustering corresponds to the result shown in Fig. 4 (b). Thus, we have 2 distinguishable groups. All circuits are synchronized in one cluster if $d \leq 0.297$. Synchronization up to $d = 0.240$ is strongly. We can see from the above that it is related to the density and distance of the chaotic circuits for clustering.

Next, we change the number of circuits in the middle group when the middle group consist of 7 chaotic circuits in case of clustering phenomena obtained. Similarly, we simulate the phase differences between the middle group and the other groups shown in Fig. 11. The results shown in Fig. 11 is similar to the results when the middle group consist of 9 chaotic circuits. Thus, we found that clustering phenomena occurs depending on the distance of the cluster and density of the circuits. Furthermore, all groups are synchronized when the number of chaotic circuits in the middle group less to 7.

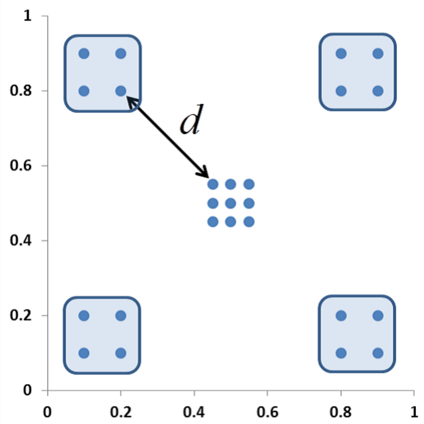
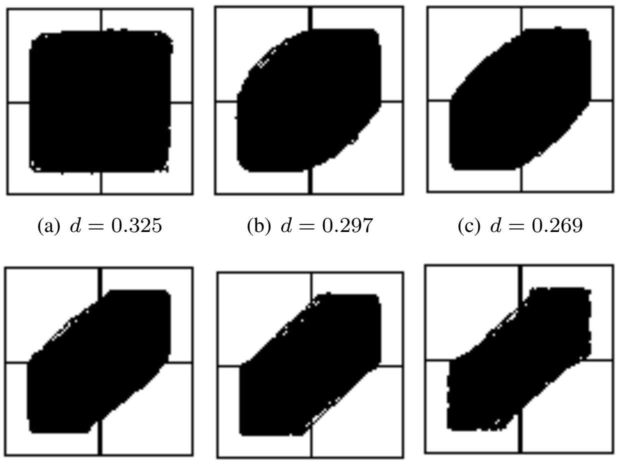


Fig. 9. Definition of the distance d.



(a) $d = 0.325$ (b) $d = 0.297$ (c) $d = 0.269$

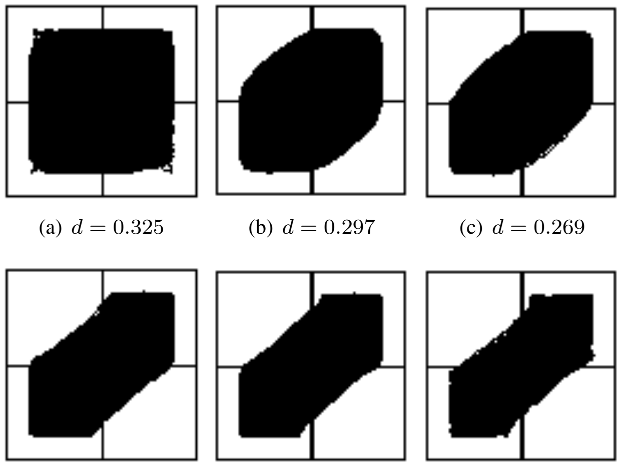
(d) $d = 0.240$ (e) $d = 0.212$ (f) $d = 0.184$

Fig. 10. Phase difference when the middle group consist of 9 circuits.

IV. Conclusion

In this study, we have examined the relationship between density and distance of each cluster that configured by chaotic circuits. For this investigation, we have studied the clustering phenomena when we change the number of circuits in a cluster. Based on computer simulation, we have obtained the clustering phenomena from coupled chaotic circuits using 2-dimensional place in continuous time model. We showed the clustering phenomena affected other cluster when density in the chaotic circuits is high. Furthermore, we found that clustering phenomena occurs depending on the distance of the cluster and density of the circuits.

In our future work, we would like to develop a more efficient method for determining the phase synchronization. In this case we will research detail the relationship between the clustering and density of coupled chaotic circuits. Besides that we apply this method for data mining, image processing and other many applications.



(a) $d = 0.325$ (b) $d = 0.297$ (c) $d = 0.269$

(d) $d = 0.240$ (e) $d = 0.212$ (f) $d = 0.184$

Fig. 11. Phase difference when the middle group consist of 7 circuits.

Acknowledgment

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