Chaos Propagation in a Ring of Coupled Circuits Generating Chaotic and Three-Periodic Attractors

Yoko Uwate
Dept. of Electrical and Electronic Engineering,
Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email: uwate@ee.tokushima-u.ac.jp

Yoshifumi Nishio
Dept. of Electrical and Electronic Engineering,
Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email: nishio@ee.tokushima-u.ac.jp

***Abstract*— In this study, we investigate synchronization phenomena observed in coupled chaotic circuits as a ring topology when one chaotic circuit is set to generate chaos and the others are set to generate three-periodic attractors. By using computer simulations, we observe chaos propagation which is that three-periodic attractors are affected chaos and they are changing between chaotic and three-periodic attractors.**

I. Introduction

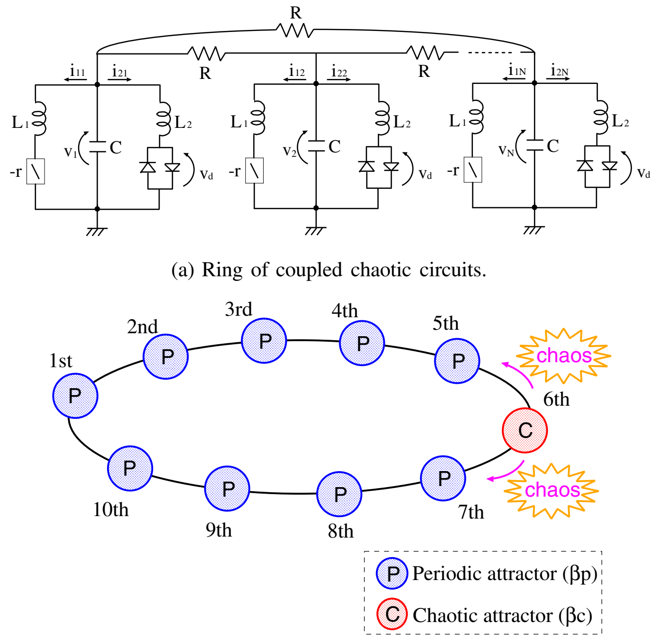
Synchronization and the related bifurcation of chaotic systems are good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. In particular, the breakdown of chaos synchronization has attracted many researchers' attentions and their mechanisms have been gradually made clear [1]-[8]. However, a lot of phenomena around chaos synchronization are still veiled as well as other nonlinear circuits problems. Hence, in order to understand and exploit such phenomena, it is important to discover them, to model them, and to investigate them.

On the other hand, it is important to investigate synchronization phenomena of coupled circuits under a difficult situation for the circuits. For examples, there are some frustrations in the circuit systems such as the coupling structure, frequency errors and external stimuli. Setou et al. have observed interesting synchronization phenomena when van der Pol oscillators with different oscillation frequencies are coupled by a resistor as a star topology [9]. We have investigated a ring consisting of van der Pol oscillators with different oscillation frequencies. By using computer simulations, oscillation death, independent oscillation and double-mode oscillation were observed [10][11]. We consider that if coupled circuits with more different characteristics are coupled, the coupled circuits feel the frustrations more strongly.

In this study, we investigate synchronization phenomena observed in coupled chaotic circuits as a ring topology when one chaotic circuit is set to generate chaos and the others are set to generate three-periodic attractors. By using computer simulations, we observe chaos propagation which is that three-periodic attractors are affected chaos and they are changing between chaotic and three-periodic attractors. Furthermore, the effect of the coupling strength between the adjacent chaotic circuits is also investigated.

II. Circuit Model

In this study, we consider a ring of the chaotic circuits as shown in Fig. 1. In the circuit system, two adjacent chaotic circuits are coupled by a resistor R. By using this coupling method, two adjacent circuits tend to synchronize with in-phase state.



(a) Ring of coupled chaotic circuits.

(b) Conceptual circuit model.

Fig. 1. Coupled chaotic circuits as a ring topology.

First, the $i-v$ characteristics of the diodes are approximated by two-segment piecewise-linear functions as

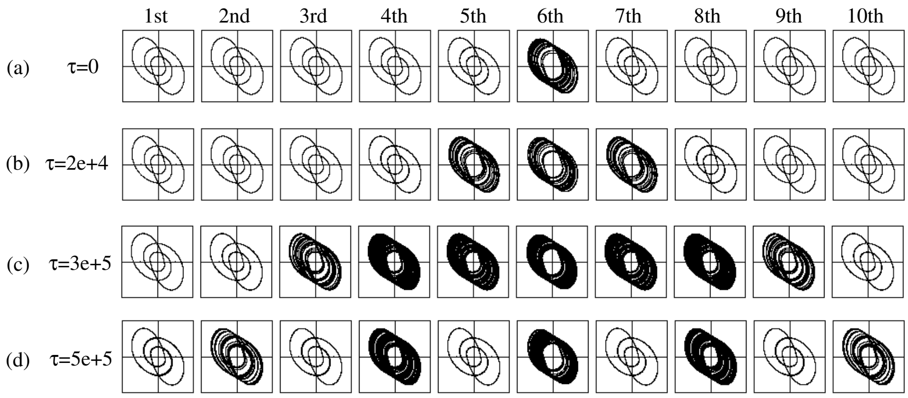
$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \quad (1)$$

By changing the variables and parameters,

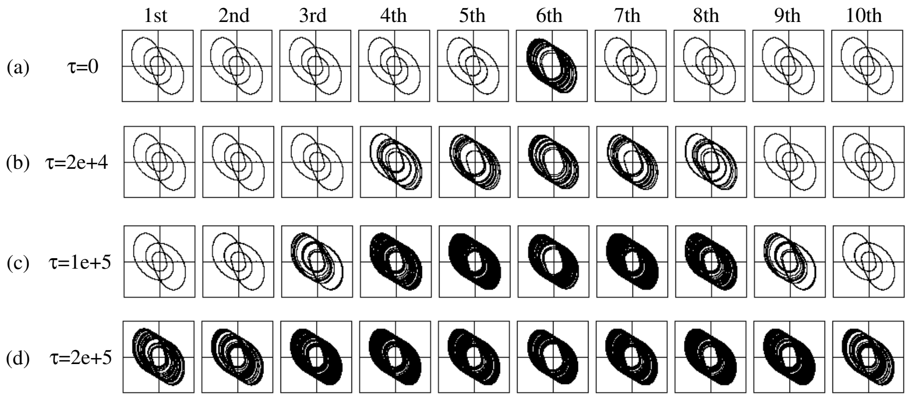
$$i_{1k} = \sqrt{\frac{C}{L_1}}Vx_k,\ \ i_{2k} = \sqrt{\frac{C}{L_1}}Vy_k,\ \ v_k = Vz_k,\ \ t = \sqrt{L_1 C}\tau$$

$$\alpha = r\sqrt{\frac{C}{L_1}},\quad \beta = \frac{L_1}{L_2},\quad \gamma = \frac{1}{R}\sqrt{\frac{L_1}{C}},\quad \delta = r_d\frac{\sqrt{L_1 C}}{L_2}, \quad (2)$$

978-1-4577-1729-1/12/$26.00 ©2012 IEEE.



(a) $\gamma = 0.002$.



(b) $\gamma = 0.008$.

Fig. 2. Examples of chaos propagation.

the normalized circuit equations are given as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \alpha x_k + z_k \\ \dfrac{dy_k}{d\tau} = z_k - f(y_k) \\ \dfrac{dz_k}{d\tau} = -x_k - \beta y_k - \gamma(z_k - z_{k+1}) \\ \qquad\qquad\qquad (k = 1, 2, \cdots, N) \end{cases} \quad (3)$$

where

$$x_{(N+1)} = 0, \quad (4)$$

$$f(y_k) = \frac{1}{2}\left(\left|y_k + \frac{1}{\delta}\right| - \left|y_k - \frac{1}{\delta}\right|\right). \quad (5)$$

In these equations, γ is the coupling strength, N is the number of coupled chaotic circuits and α denotes the bifurcation parameter. By changing α, the obtained attractors are controlled. We define α_c to generate the chaotic attractors, and α_p is defined to generate the three-periodic attractors.

For the computer simulations, we calculate Eq. (3) using the fourth-order Runge-Kutta method with step size $h = 0.005$. We set the parameters of this circuit model as follows; $\alpha_c = 0.382$, $\alpha_p = 0.413$, $\beta = 3.0$ and $\delta = 470.0$.

In this study, we consider that the total number of coupled chaotic circuits is 10. One chaotic circuit generates chaotic attractor and the other circuits generate the three-periodic attractors (see. Fig. 1 (b)).

III. Synchronization Phenomena

A. Attractors

Figure 2 shows chaos propagation phenomena when 6th chaotic circuit generates chaos at the initial states. First, the chaos attractor of 6th circuit propagates to both side of the neighbor circuits. Then, three-periodic attractors change to chaos attractors and its chaos behavior shifts to the neighbor circuits by rotation in this circuit system. Furthermore, we can confirm that the speed of chaos propagation becomes faster by setting larger value of the coupling strength γ.

B. Phase Difference

The phase differences between 1st and 3rd (ϕ_{13}) / 1st and 6th (ϕ_{16}) chaotic circuits are calculated in Figs. 3, 4. In the case of ϕ_{13}, 1st and 3rd chaotic circuits is located far from the 6th chaotic circuits generating chaos attractor. Namely, the ϕ_{13} shows 0 degree (in-phase state) a certain time in simulations when the coupling strength is fixed with $\gamma = 0.002$ (see. Fig. 3 (a)). By increasing the value of γ, 1st and 3rd chaotic circuits are not synchronized as shown in Fig. 3 (a), (b). Finally, Fig.3 (d) shows that chaos synchronization with in-phase state is observed a certain time.

In the case of ϕ_{16}, 6th chaotic circuit generates chaotic attractor. Then, the phase difference ϕ_{16} switches from 0 to 180 degree as shown in Fig. 4 (a). Namely, 1st and 6th chaotic circuits does not synchronize. By changing the γ, we can see that the phase difference ϕ_{16} changes with simulation time.

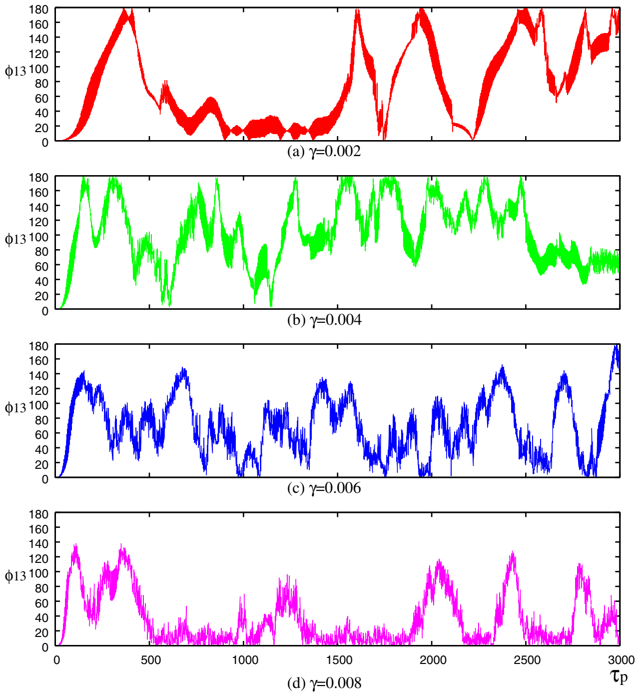


Fig. 3. Phase differences (1st and 3rd chaotic circuits).

C. Difference of Voltages between Neighbors Circuits

In order to investigate the chaos propagation, we calculate the difference of voltages between neighbors circuits. Figure 5 shows the simulation results. The horizontal axis is the computer simulation time τ and the vertical axis shows the difference of x_k and x_{k+1}. In these figures, the white region means in-phase state and the black region means that two adjacent circuits has some amount of the phase shift.

The chaotic attractor occurs the phase shift and its phase shift propagates to all circuits in this system. The speed of phase shift propagation becomes faster by setting larger value of the coupling strength γ. By increasing the value of γ, we can see the chaos synchronization of whole circuits (see. Fig.5 (d)).

D. Synchronization State

Finally, Fig. 6 shows one example of synchronization states when the coupling strength γ is changed from 0.002 to 0.008. The horizontal axis is the Poincare time and the vertical axis shows the value of x_k when the solution of the first circuit crosses to the Poincare section ($x_1 < -1.2$, $z_1 = 0$). Namely, if the circuits generates chaotic attractor, the curve of x_k

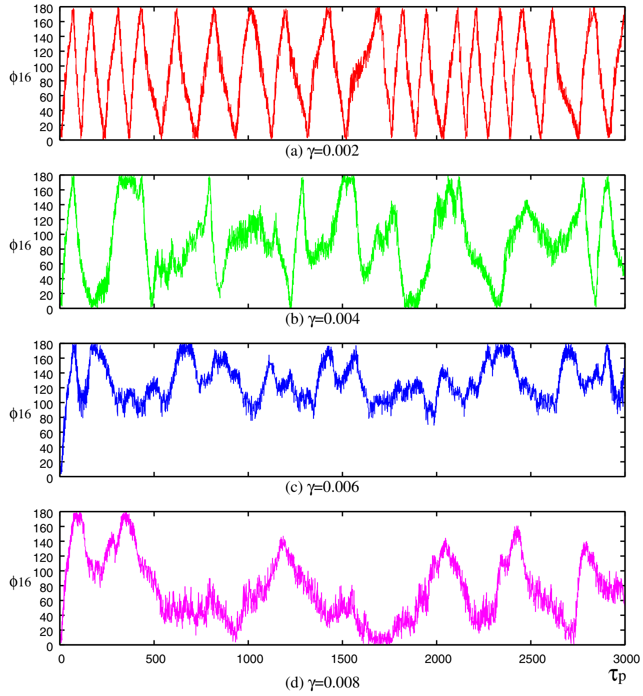


Fig. 4. Phase differences (1st and 6th chaotic circuits).

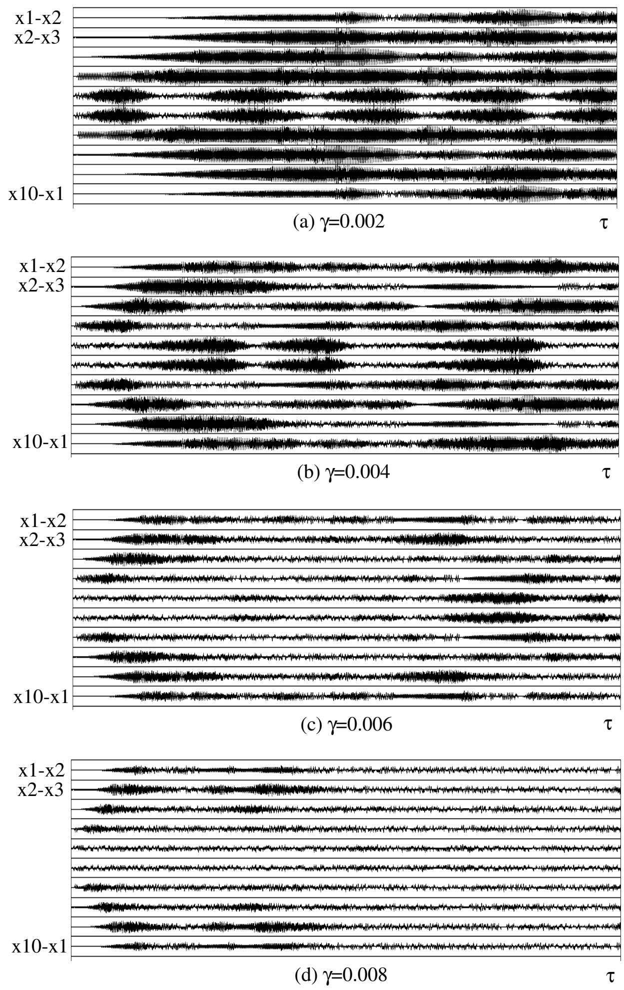
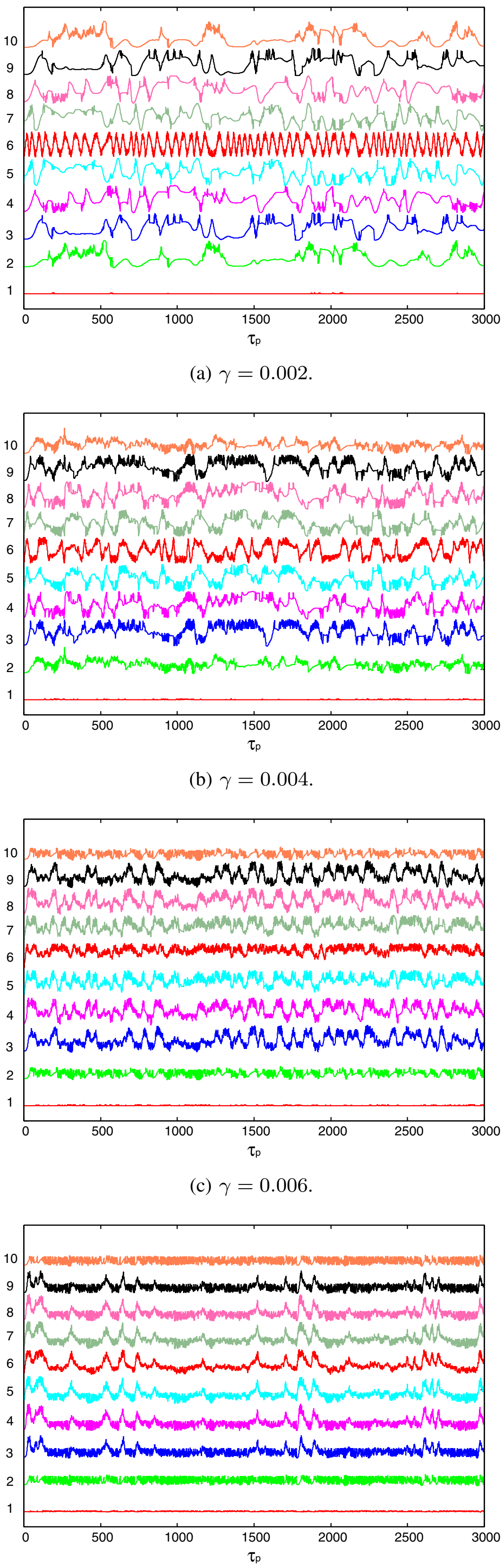


Fig. 5. Time wave forms.



(a) $\gamma = 0.002$.

(b) $\gamma = 0.004$.

(c) $\gamma = 0.006$.

(d) $\gamma = 0.008$.

Fig. 6. Synchronization state.

oscillates. While if the circuits generates the periodic attractor, the curve of x_k becomes constant.

From these results, we can see that the region of chaotic behavior increases with the value of the coupling strength γ.

IV. Conclusion

In this study, we have investigated synchronization phenomena observed in coupled chaotic circuits as a ring topology when one chaotic circuit is set to generate chaos and the others are set to generate three-periodic attractors. By using computer simulations, we have observed chaos propagation which is that three-periodic attractors are affected chaos and they are changing between chaotic and three-periodic attractors. Furthermore, the effect of the coupling strength between the adjacent chaotic circuits was also investigated.

As our future works, we develop an efficient method to analyze the obtained synchronization phenomena of the proposed system. Considering the other types of chaotic circuits and the other coupling methods are also important subjects for us.

References

[1] P. Ashwin, J. Buescu and I. Stewart, *Bubbling of Attractors and Synchronisation of Chaotic Oscillators.* Phys. Lett., vol. A193, pp. 126-139, 1994.

[2] T. Kapitaniak and L.O. Chua, *Locally-Intermingled Basins of Attraction in Coupled Chua's Circuits.* Int. J. Bifurcation and Chaos, vol. 6, no. 2, pp. 357-366, 1996.

[3] M. Wada, Y. Nishio and A. Ushida, *Analysis of Bifurcation Phenomena on Two Chaotic Circuits Coupled by an Inductor.* IEICE Trans. Fundamentals, vol. E80-A, no. 5, pp. 869-875, May 1997.

[4] J. Chubb, E. Barreto, P. So and B.J. Gluckman, *The Breakdown of Synchronization in Systems of Nonidentical Chaotic Oscillators: Theory and Experiment.* Int. J. Bifurcation and Chaos, vol. 11, no. 10, pp. 2705-2713, 2001.

[5] R.L. Viana, C. Grebogi, S.E. de S. Pinto, S.R. Lopes, A.M. Batista and J. Kurths, *Bubbling Bifurcation: Loss of Synchronization and Shadowing Breakdown in Complex Systems.* Physica D, vol. 206, pp. 94-108, 2005.

[6] C.P. Li, W.H. Deng and D. Xu, *Chaos Synchronization of the Chua System with a Fractional Order.* Physica A, vol. 360, pp. 171-185, 2006.

[7] M. Chen and J. Kurths, *Chaos Synchronization and Parameter Estimation from a Scalar Output Signal.* Phys. Rev. E, vol. 76, 027203, 2007.

[8] H.T. Yau and J.J. Yan, *Chaos Synchronization of Different Chaotic Systems Subjected to Input Nonlinearity.* Applied Math. and Comp., vol. 197, pp. 775-788, 2008.

[9] Y. Setou, Y. Nishio and A. Ushida, *Synchronization Phenomena in Resistively Coupled Oscillators with Different Frequencies* IEICE Trans. Fundamentals, vol. E79-A, no. 10, pp. 1575-1580, 1996.

[10] Y. Uwate, Y. Nishio and R. Stoop, *Group Synchronization of van der Pol Oscillators with Different Frequencies* Proc. of NOLTA'08, pp. 612-615, 2008.

[11] Y. Uwate, Y. Nishio and R. Stoop, *Synchronization of Distant Neighbors Oscillators in a Ring Network* Proc. of NOLTA'09, pp. 439-442, 2009.