Double-Mode Oscillation in Chaotic Circuits Coupled by a Time-Varying Resistor

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***Abstract*—We investigate two chaotic circuits with fifth power nonlinear characteristics coupled by a time-varying resistor. In particular, we confirm some simultaneous asynchronous oscillations with two different types of chaotic phenomena in these systems.**

I. Introduction

Coupled oscillator systems with nonlinear characteristics have shown some interesting phenomena and brought improvement to natural science and fundamental research. An Oscillator with a negative resistor as the fifth-power v – i characteristics is known to exhibit hard excitation. The oscillator converges on the origin at the small initial values because the proposed fifth power characteristics near the origin are a positive resistor. The initial values beyond origin stable regions are required to generate the oscillation. Such an oscillator is called a hard oscillator or one with hard nonlinearity. Datardina and Linkens have investigated two van der Pol oscillators with hard nonlinearities coupled by an inductor [1]. They have confirmed that four different modes: Zero, in-phase synchronous mode, anti-phase synchronous mode and double-mode, coexist for some range of parameter values. These modes are zero (non-oscillation), two single-modes (in-phase and anti-phase synchronizations), and double-mode (simultaneous asynchronous oscillations with in-phase and anti-phase frequency components). In particular, the double-mode oscillations are stably excited although they do not occur in the case of common negative resistors with third-power characteristics. On the basis of this study, we have proposed two coupled chaotic circuits with hard nonlinearities for the purpose of looking for the interesting phenomena as double-mode chaotic oscillations [2][3]. We have confirmed double-mode chaos with one-periodic in-phase and chaotic anti-phase single-modes [2] and that with torus and chaos, respectively [3]. However, we have not found that with chaos and chaos in these systems. We are considering the reason that phenomena do not exist could be caused by frequency differences between two single-modes. We have to consider the double-mode oscillation with much smaller frequency differences.

On the other hand, we have investigated synchronization phenomena of two van der Pol oscillators coupled by a time-varying resistor which means positive and negative resistors switching time-periodically [4]. We have confirmed that in-phase and anti-phase single-modes are stably excited in these systems at the same parameter values. We have also confirmed that two chaotic single-modes coexist in an improved chaos version of these systems [5]. The frequency differences between two different single-modes in the systems coupled by a time-varying resistor are possibly smaller than those in the systems coupled by an inductor or capacitor such as the previous systems. Therefore, we can expect to find double-mode oscillations with chaotic both in-phase and anti-phase frequency components. In this study, we propose two chaotic circuits with hard nonlinearities coupled by a time-varying resistor and look for the expected double-mode chaos.

II. Circuit Model

Figure 1 shows the proposed circuit model. In this model, two identical chaotic circuits with hard nonlinearities are coupled by a time-varying resistor whose time waveform characteristics are shown in Fig. 2.

The equations governing the circuit in Fig. 1 are described as follows:

$$
\begin{aligned}
L_1\frac{dI_k}{dt} &= -v_r(I_k+i_k) - v_k - R_i I_k - R(t)(I_1+I_2) \\
L_2\frac{di_k}{dt} &= -v_r(I_k+i_k) - v_k - v_d(i_k) \qquad (1) \\
C\frac{dv_k}{dt} &= I_k + i_k \qquad (k=1,2)
\end{aligned}
$$

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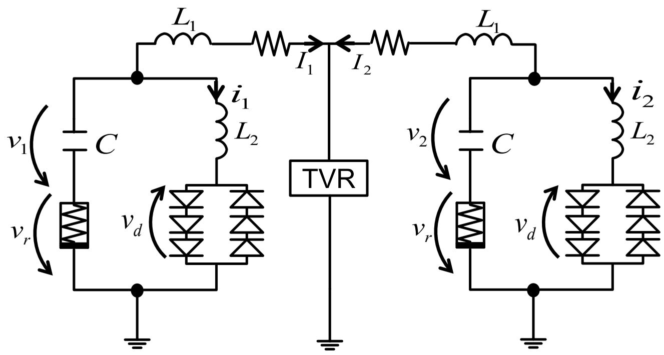


Figure 1. Circuit model. (TVR stands for Time-Varying Resistor)

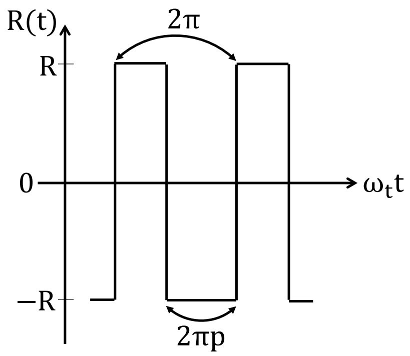


Figure 2. TVR characteristics.

By changing the variables and parameters,

$$I_k = \sqrt{\frac{C}{L_1}}Ex_k, i_k = L_1\sqrt{\frac{C}{L_1}}Ey_k, v_k = Ez_k,$$

$$\alpha = \frac{L_1}{L_2}, \beta = r\sqrt{\frac{C}{L_1}}, \gamma(\tau) = R(t)\sqrt{\frac{C}{L_1}}, \delta = R_i\sqrt{\frac{C}{L_1}}, \quad (2)$$

$$t = \sqrt{L_1 C}\tau, \qquad \omega = \frac{1}{\sqrt{L_1 C}}\omega_t,$$

(1) is normalized as

$$\begin{aligned} \frac{dx_k}{d\tau} &= -f_r(x_k + y_k) - z_k - \delta x_k - \gamma(\tau)(x_1 + x_2) \\ \frac{dy_k}{d\tau} &= \alpha\{-f_r(x_k + y_k) - z_k - f_d(y_k)\} \\ \frac{dz_k}{d\tau} &= x_k + y_k \end{aligned} \quad (3)$$

The functions f_r and f_d correspond to v_r(fifth-power negative resistor) and v_d (six diodes), respectively, and are represented as follows:

$$f_r(x) = \begin{cases} r_2 x + a_2 - \frac{r_1 + r_2}{a_3} & \cdots \left(x > \frac{1}{a_3}\right) \\ a_2 - r_1 x & \cdots \left(\frac{1}{a_1} < x < \frac{1}{a_3}\right) \\ (a_1 a_2 - r_1)x & \cdots \left(|x| < \frac{1}{a_1}\right) \\ -a_2 - r_1 x & \cdots \left(-\frac{1}{a_3} < x < -\frac{1}{a_1}\right) \\ r_2 x - a_2 + \frac{r_1 + r_2}{a_3} & \cdots \left(x < -\frac{1}{a_3}\right) \end{cases} \quad (4)$$

$$f_d(x) = \begin{cases} 3 & \cdots \left(x > \frac{1}{b}\right) \\ 3bx & \cdots \left(|x| < \frac{1}{b}\right) \\ -3 & \cdots \left(x < -\frac{1}{b}\right) \end{cases} \quad (5)$$

Their characteristics are shown in Figs. 3 and 4, respectively.

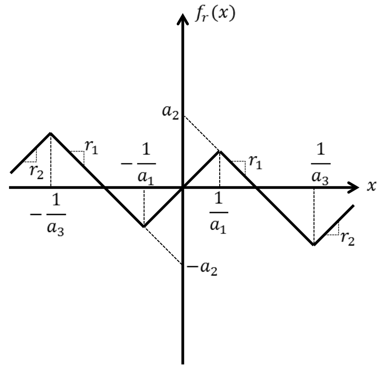


Figure 3. Normarized fifth-power characteristics of a negative resistor.

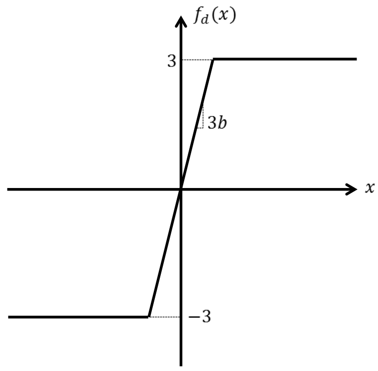


Figure 4. Normarized characteristics of six diodes.

The characteristics of the Figs. 3 and 4 realize the oscillations with hard nonlinearities and the chaotic oscillations, respectively. In this study, the parameter values of these characteristics are always fixed as $a_1 = 2.0$, $a_2 = 0.1$, $a_3 = 0.01$, $r_1 = 0.15$, $r_2 = 0.1$.

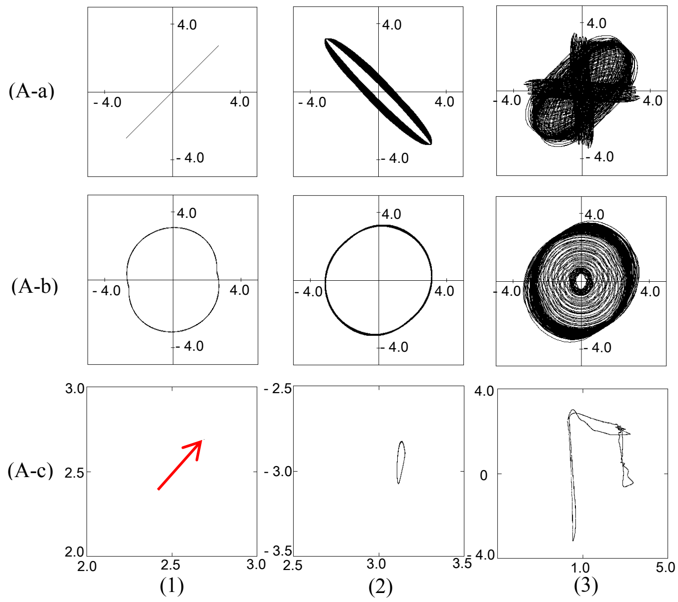
III. MULTI-MODE CHAOS

Four different oscillation modes (zero, in-phase single-mode, anti-phase single-mode and double-mode) coexist in the circuit model of the Fig. 1. Each mode is determined by the initial values. In this section, we show some simulation results of some attractors and Poincare maps on each mode to investigate its behavior when the parameter α increases. The other parameters are fixed as $\beta = 0.15$, $\gamma = 0.1$, $\delta = 0.05$, $d\tau = 0.001$, $\omega = 1.88$. (Fig. 5)

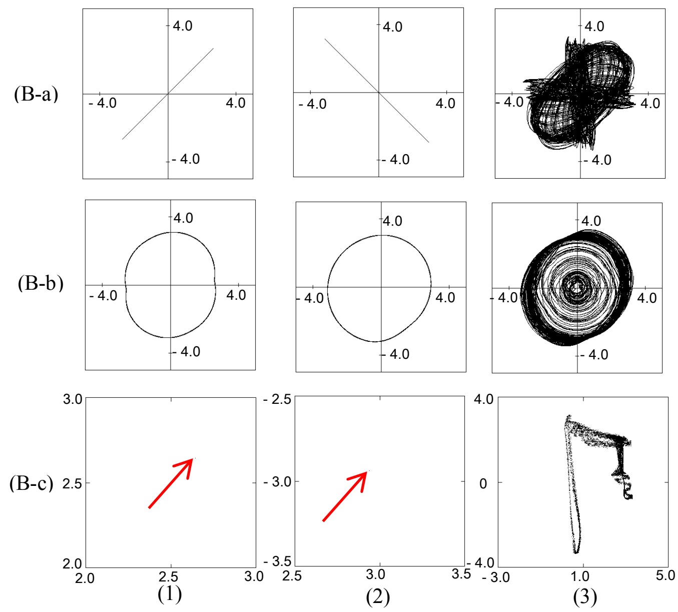
Zero means that neither of the two oscillators is excited. This state is always stable in some range of small initial values. We do not mention this state in the rest of this manuscript because it is trivial.

In-phase single-mode means that the two oscillators are synchronized with the in-phase. The in-phase single-mode is one-periodic oscillation at $\alpha = 4.0$, 8.0 and 12.0, chaotic oscillation at $\alpha = 16.0$ by the attractors and Poincare maps.

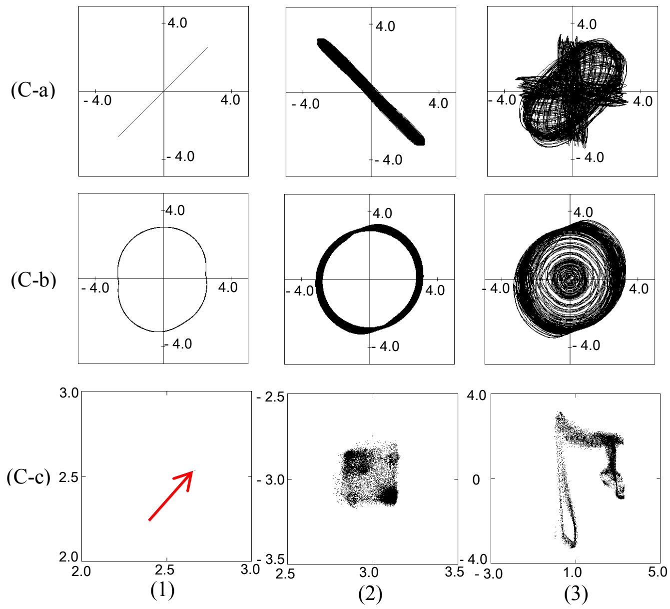
Anti-phase single-mode means that the two oscillators are synchronized with π phase difference. The anti-phase single-mode is quasi-periodic oscillation at $\alpha = 4.0$, one-periodic oscillation at $\alpha = 8.0$, chaotic oscillation at $\alpha = 12.0$ and 16.0 by the attractors and Poincare maps.



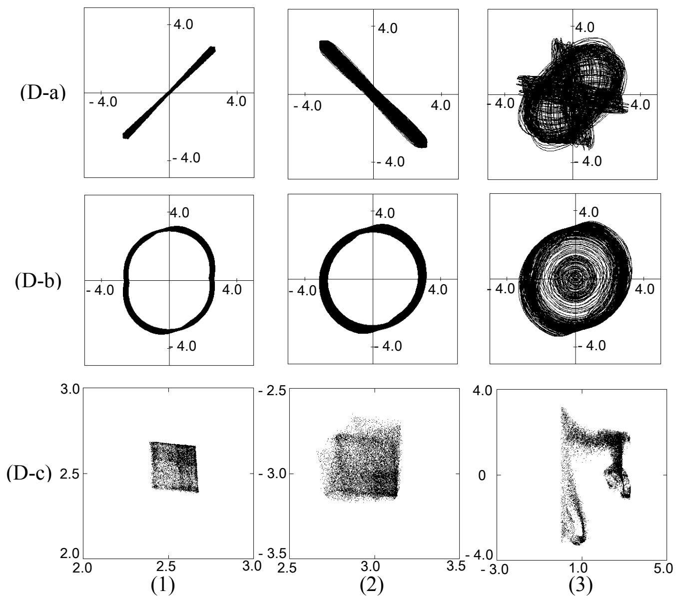
A: α = 4.0



B: α = 8.0



C: α = 12.0

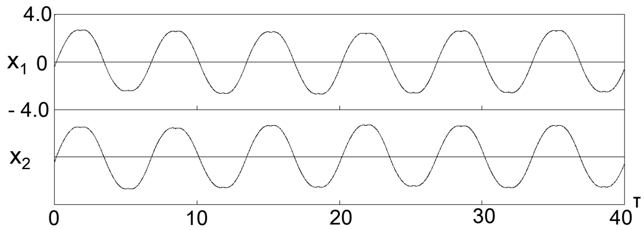


D: α = 16.0

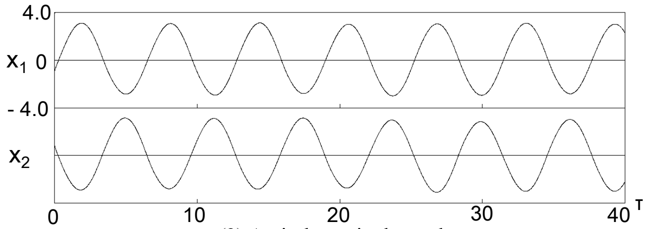
Figure 5. Attractors and Poincare maps of three different oscillation modes. (a) $x_1 - x_2$ Attractors. (b) $x_1 - z_1$ Attractors. (c) $x_1 - x_2$ Poincare maps. (A) $\alpha = 4.0$. (B) $\alpha = 8.0$. (C) $\alpha = 12.0$. (D) $\alpha = 16.0$.
(1) In-phase sigle-mode. (2) Anti-phase single-mode. (3) Double-mode.
Red arrows point to the one-periodic Poincare maps: (A-c)(1), (B-c)(1)(2) and (C-c)(1).

Double-mode means that the above two single-mode oscillations (in-phase and anti-phase) are excited, simultaneously and asynchronously. The double-mode is quasi-periodic oscillation at $\alpha = 4.0$, chaotic oscillation at $\alpha = 8.0$, 12.0 and 16.0 by the attractors and Poincare maps. Although the oscillations of double-mode chaos in the system coupled by a capacitor have been reported [2][3], these oscillations coupled by a time-varying resistor have not been reported yet. Especially, the double-mode chaos with chaos (in-phase) and chaos (anti-phase) such as the results at $\alpha = 16.0$ could not be found in the previous studies [2][3].

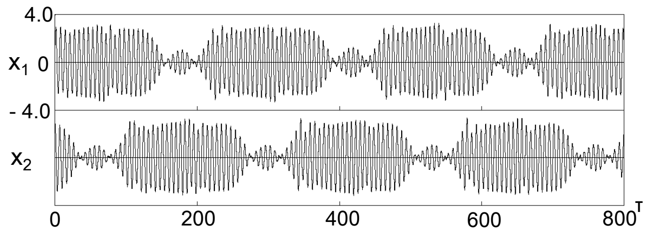
Figure 6 shows time waveforms of three different oscillation modes. The frequency differences between two single-modes are smaller than those of the previous studies [2][3]. We consider that this possibly affects the bifurcation phenomena of each mode.



(1) In-phase single-mode



(2) Anti-phase single-mode



(3) Double-mode

Figure 6. Time waveforms vs amplitude ($\alpha = 16.0$).

IV. Conclusions

We proposed two identical chaotic circuits with hard nonlinearities coupled by a time-varying resistor and investigated the double-mode chaos in those systems by looking at some attractors and Poincare maps. As a result, we confirmed the interesting phenomena as double-mode chaos with chaos (in-phase) and chaos (anti-phase) by computer simulations. We will analyze the bifurcation of this double-mode chaos in the near future.

Acknowledgment

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