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Propagation Waves in a Ladder of Coupled Simultaneous Oscillators

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Abstract—In this study, we investigate a ladder of simultaneous oscillators with three or four *LC* resonators coupled by inductors. By computer simulations, we observe a new type of waves where oscillation frequency or/and oscillation amplitude of each resonator changes according to the wave propagation. Further, we confirm that the oscillation amplitude of each resonator depend on its positions.

1. Introduction

In the natural fields, various synchronization phenomena exists. For example, firefly luminescence, swing of pendulums, cardiac heartbeat, and so on, are well known as synchronization phenomena. Oscillators containing a nonlinear resistor whose $v - i$ characteristics are described by fifth-power nonlinear characteristics are known to exhibit hard excitation [1][2]. Namely, the origin is asymptotically stable and an proper initial condition, which is larger than a critical value, is necessary to generate the oscillation. Such an oscillator is often called as hard oscillator or said to have hard nonlinearity. Two identical oscillators with hard nonlinearities coupled by an inductor are investigated by Datardina and Linkens [3]. They have confirmed that nonresonant double-mode oscillations, which could not occur for the case of third-power nonlinearity, were stably excited in the coupled system oscillators having hard nonlinearity. They have also confirmed that four different modes coexist for some range of parameter values; zero, two single-modes, and a double-mode. In 1954, Schaffner reported that an oscillator with two degrees of freedom could oscillate simultaneously at two different frequencies when the nonlinear characteristics are described by a fifth-power polynomial function [4]. Kuramitsu also investigated the simultaneous oscillations for three or more degrees case theoretically and confirmed the generation of simultaneous oscillation with three frequencies by circuit experiments [5]. The simultaneous oscillations are definitely one of the most common nonlinear phenomena observed in various higher-dimensional systems in the natural science fields. However, after their pioneering works, as far as the authors know, there have not been many researches clarifying the basic mechanism of the simultaneous oscillations except [6][7].

In the past studies, we investigated two or three simultaneous oscillators coupled by a resistor or inductors [8][9][10]. We could confirm that double-mode and simultaneous oscillations are generated at the same time from inductively coupled simultaneous oscillators. We also investigated the case that twenty simultaneous oscillators with two *LC* resonators are coupled by inductors in a one-dimensional array [13]. By computer simulations of the circuit model, we discovered two kinds of waves; the first wave is a propagating change of phase states between two horizontally adjacent resonators from in-phase to anti-phase or from anti-phase to in-phase, which is the same as the phase-inversion waves in [11][12]. the second wave is a propagating change of oscillation states from oscillation death to oscillation or from oscillation to oscillation death.

In this study, we investigate twenty simultaneous oscillators with three or four *LC* resonators coupled by inductors in a one-dimensional array. By computer simulation, we discover a new type of propagation waves. In this wave, oscillation frequency or/and oscillation amplitude of each resonator changes according to the wave propagation. Further, we confirm that the oscillation amplitude of each resonator depend on its positions.

2. Circuit Model

The circuit model is shown in Fig. 1. In the circuit, twenty simultaneous oscillators with three *LC* resonators are coupled by inductors L_C and each simultaneous oscillator consists of a nonlinear negative resistor, whose $v - i$ characteristics are described by a fifth-power polynomial function as

$$i_R(v) = g_1 v - g_3 v^3 + g_5 v^5 \quad (g_1,\ g_3,\ g_5 > 0), \qquad (1)$$

and three resonators with different natural frequencies ($\sqrt{L_1C_1}$, $\sqrt{L_2C_2}$ and $\sqrt{L_3C_3}$). The equations governing the coupled oscillators are described by the following differential equations including twenty nonlinear functions i_{Rj}

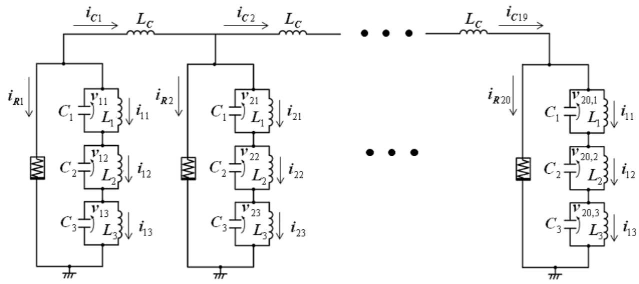


Figure 1: Twenty simultaneous oscillators with three resonators coupled by inductors in a one-dimensional array.

$(j = 1, 2, \cdots, 20)$.

$$\begin{cases} C_1 \frac{dv_{j1}}{dt} = -i_{j1} - i_{Rj} - i_{Cj} + i_{C,j-1} \\ C_2 \frac{dv_{j2}}{dt} = -i_{j2} - i_{Rj} - i_{Cj} + i_{C,j-1} \\ C_3 \frac{dv_{j3}}{dt} = -i_{j3} - i_{Rj} - i_{Cj} + i_{C,j-1} \\ L_1 \frac{di_{j1}}{dt} = v_{j1} \\ L_2 \frac{di_{j2}}{dt} = v_{j2} \\ L_3 \frac{di_{j3}}{dt} = v_{j3} \\ (j = 1, 2, \cdots, 20), \end{cases} \quad (2)$$

where i_{Cm} are the currents through the coupling inductors and are given as

$$i_{Cm} = \frac{1}{L_C}\{L_1(i_{m1} - i_{m+1,1}) + L_2(i_{m2} - i_{m+1,2}) + L_3(i_{m3} - i_{m+1,3})\} \quad (3)$$
$$(m = 1, 2, \cdots, 19),$$

with $i_{C0} = i_{C,20} = 0$. The currents through the nonlinear resistors i_{Rj} are given as

$$i_{Rj} = i_R(v_{j1} + v_{j2} + v_{j3}) \quad (j = 1, 2, \cdots, 20). \quad (4)$$

By using the following variables and parameters,

$$\begin{aligned} & v_{jk} = \sqrt[4]{\frac{g_1}{5g_5}} x_{jk}, \quad i_{jk} = \sqrt[4]{\frac{g_1}{5g_5}} \sqrt{\frac{C_1}{L_1}} y_{jk}, \\ & \alpha_{C1} = \frac{C_1}{C_2}, \quad \alpha_{L1} = \frac{L_1}{L_2}, \quad \gamma = \frac{L_1}{L_C}, \\ & \alpha_{C2} = \frac{C_1}{C_3}, \quad \alpha_{L2} = \frac{L_1}{L_3}, \\ & \varepsilon = g_1 \sqrt{\frac{L_1}{C_1}}, \quad \beta = \frac{3g_3}{g_1} \sqrt{\frac{g_1}{5g_5}}, \quad t = \sqrt{L_1 C_1}\tau \\ & (j = 1, 2, \cdots, 20) \quad (k = 1, 2, 3), \end{aligned} \quad (5)$$

the normalized circuit equations are given as follows.

$$\begin{cases} \frac{dx_{j1}}{d\tau} = -y_{j1} - f(x_{j1} + x_{j2} + x_{j3}) - y_{Cj} + y_{C,j-1} \\ \frac{dx_{j2}}{d\tau} = \alpha_{C1}\{-y_{j2} - f(x_{j1} + x_{j2} + x_{j3}) - y_{Cj} + y_{C,j-1}\} \\ \frac{dx_{j3}}{d\tau} = \alpha_{C2}\{-y_{j3} - f(x_{j1} + x_{j2} + x_{j3}) - y_{Cj} + y_{C,j-1}\} \\ \frac{dy_{j1}}{d\tau} = x_{j1} \\ \frac{dy_{j2}}{d\tau} = \alpha_{L1} x_{j2} \\ \frac{dy_{j3}}{d\tau} = \alpha_{L2} x_{j3} \qquad (j = 1, 2, \cdots, 20), \end{cases} \quad (6)$$

where y_{Cm} corresponding to i_{Cm} is given as

$$y_{Cm} = \gamma\left(y_{m1} - y_{m+1,1} + \frac{y_{m2} - y_{m+1,2}}{\alpha_{L1}} + \frac{y_{m3} - y_{m+1,3}}{\alpha_{L2}}\right) \quad (7)$$
$$(m = 1, 2, \cdots, 19),$$

with $y_{C0} = y_{C,20} = 0$. The nonlinear function $f(\cdot)$, which corresponds to the $v - i$ characteristics of the nonlinear resistors, is given as

$$f(x) = \varepsilon\left(x - \frac{\beta}{3}x^3 + \frac{1}{5}x^5\right). \quad (8)$$

3. Two kinds of waves [13]

In our past study, we investigated the twenty simultaneous oscillators with two LC resonators coupled by inductors and discovered coexisting two kinds of waves.

The first wave is a propagating change of phase states between two horizontally adjacent resonators from in-phase to anti-phase or from anti-phase to in-phase. An example of this type of waves is shown in Fig. 2. The vertical axes of this diagram are the sum of the voltages of two horizontally adjacent resonators. Hence, black area of the diagram shows in-phase synchronization states, while white area shows anti-phase synchronization states or oscillation death. From this figure, we can see that two phase-inversion waves propagates in the upper resonators, while all the lower resonators stop the oscillation.

The second wave is a propagating change of oscillation states from oscillation death to oscillation or from oscillation to oscillation death. An example of this type of waves is shown in Fig. 3. The vertical axes of this diagram correspond to the voltages of the resonators. Hence, white area simply shows the oscillation death. From this figure, we can confirm that the upper resonators oscillate only when the lower resonators stop to oscillate, and vice versa.

4. Propagation wave in three LC resonators

In this study, we investigate the twenty simultaneous oscillators with three LC resonators coupled by inductors in

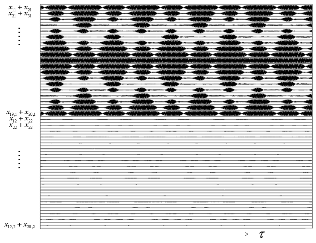


Figure 2: Propagating change of phase states between two horizontally adjacent resonators. $\alpha_C = 0.57$, $\alpha_L = 1.0$, $\gamma = 0.01$, $\varepsilon = 0.98$ and $\beta = 2.73$.

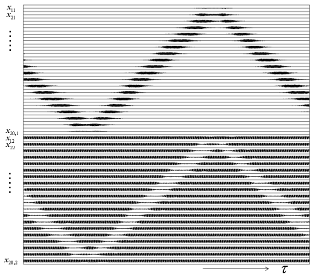


Figure 3: Propagating change of oscillation states. $\alpha_C = 0.57$, $\alpha_L = 1.0$, $\gamma = 0.01$, $\varepsilon = 0.98$ and $\beta = 2.73$.

a one-dimensional array. We can observe a new type of wave propagation phenomenon which is different from the waves observed from the twenty simultaneous oscillators with two *LC* resonators.

Figure 4 shows an example of the new propagation wave. The vertical axes of this diagram correspond to the voltages of the resonators; the top 20 graphs are the voltages of the upper resonators, the middle 20 graphs are the middle resonators, and the lower 20 graphs are the lower resonators.

There are two characteristics of this wave. First, all the resonators always oscillate, although the oscillation amplitude becomes smaller for the area without the new wave. This means that oscillation death cannot be observed. The second characteristic is that the propagation waves in the three region (upper, middle and lower) look like the same.

Furthermore, we can observe that the propagation speed of the waves depends on the coupling strength γ. Figure 5 shows how the propagation speed changes for different values of γ. The values of the other parameters of Fig. 5 are $\alpha_{C1} = 0.81$, $\alpha_{L1} = 1.0$, $\alpha_{C2} = 0.9$, $\alpha_{L2} = 1.0$, $\varepsilon = 0.35$ and $\beta = 3.3$. Moreover, if we change the value of parameter of α_C corresponding to the ratio of the capacitors, namely if we change the ratio of the natural frequencies of the resonators, the amplitude of some of the new waves becomes smaller as shown in Fig. 6. The values of the other parameters of Fig. 6 are $\alpha_{L1} = 1.0$, $\alpha_{L2} = 1.0$, $\gamma = 0.2$, $\varepsilon = 0.35$ and $\beta = 3.3$.

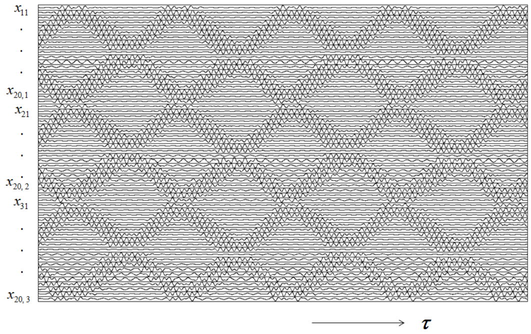
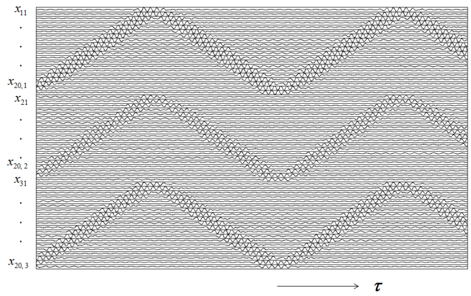


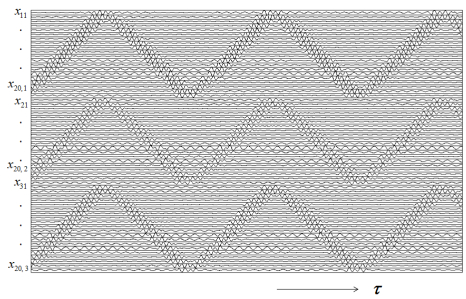
Figure 4: Propagation wave in three *LC* resonators. $\alpha_{C1} = 0.81$, $\alpha_{L1} = 1.0$, $\alpha_{C2} = 0.9$, $\alpha_{L2} = 1.0$, $\gamma = 0.3$, $\varepsilon = 0.35$ and $\beta = 3.3$.

5. Propagation wave in four *LC* resonators

We also investigate the twenty simultaneous oscillators with four *LC* resonators coupled by inductors in a one-dimensional array. We can confirm the generation of the same propagation waves as those observed from the case of three *LC* resonators. Figure 7 shows an example of the Propagation wave in four *LC* resonators.



(a)

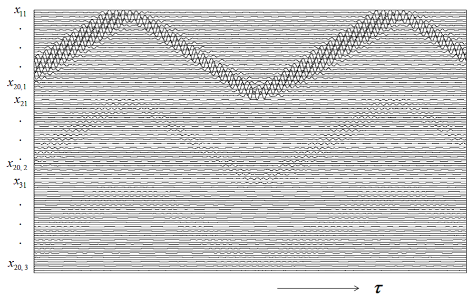


(b)

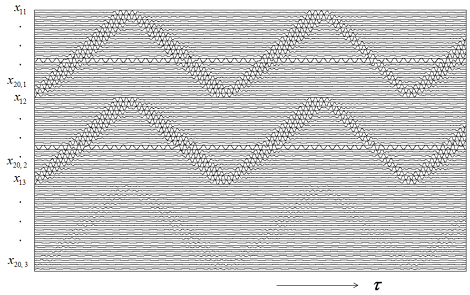
Figure 5: Propagation waves in three *LC* resonators for different coupling strengths. (a) $\gamma = 0.2$. (b) $\gamma = 0.38$.

6. Conclusions

In this study, we investigated twenty simultaneous oscillators with three or four *LC* resonators coupled by in-



(a)



(b)

Figure 6: Propagation waves in three *LC* resonators for different sets of natural frequencies. (a) $\alpha_{C1} = 0.1$ and $\alpha_{C2} = 0.3$. (b) $\alpha_{C1} = 0.2$ and $\alpha_{C2} = 0.9$.

ductors in a one-dimensional array. By computer simulation, we discovered a new type of propagation waves where oscillation frequency or/and oscillation amplitude of each resonator changed according to the wave propagation. Further, we confirmed that the oscillation amplitude of each resonator depended on its positions.

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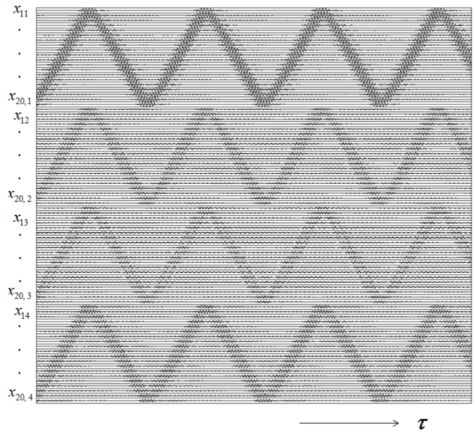

Figure 7: Propagation wave in four *LC* resonators. $\alpha_{C1} = 0.65$, $\alpha_{L1} = 1.0$, $\alpha_{C2} = 0.71$, $\alpha_{L2} = 1.0$, $\alpha_{C3} = 0.81$, $\alpha_{L3} = 1.0$, $\gamma = 0.2$, $\varepsilon = 0.08$ and $\beta = 3.3$.

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