Two Kinds of Waves in a Ladder of Coupled Simultaneous Oscillators

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***Abstract*— In this study, we investigate the circuit model that twenty hard oscillators with two LC resonators are coupled by inductors. We observe two kinds of waves by the change of initial states. First waves are changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase and continuously exist. We call this wave as "phase inversion waves". Second waves are the change of oscillation states from oscillation death to oscillation or from oscillation to oscillation death and exist. We name this wave as "oscillatory inversion wave".**

I. Introduction

In the natural fields, various synchronization phenomena exists. For example, firefly luminescence, swing of pendulums, cardiac heartbeat, and so on, are well known as synchronization phenomena. Oscillators containing a nonlinear resistor whose $v-i$ characteristics are described by fifth-power nonlinear characteristics are known to exhibit hard excitation [1][2]. Namely, the origin is asymptotically stable and an proper initial condition, which is larger than a critical value, is necessary to generate the oscillation. Such an oscillator is often called as hard oscillator or said to have hard nonlinearity. [3] are investigated by Datardina and Linkens. They have confirmed that nonresonant double-mode oscillations, which could not occur for the case of third-power nonlinearity, were stably excited in the coupled system. They have also confirmed that four different modes coexist for some range of parameter values; zero, two single-modes, and a double-mode. In 1954, Schaffner reported that an oscillator with two degrees of freedom could oscillate simultaneously at two different frequencies when the nonlinear characteristics are described by a fifth-power polynomial function [4]. Kuramitsu also investigated the simultaneous oscillations for three or more degrees case theoretically and confirmed the generation of simultaneous oscillation with three frequencies by circuit experiments [5]. The simultaneous oscillations are definitely one of the most common nonlinear phenomena observed in various higher-dimensional systems in the natural science fields. However, after their pioneering works, as far as the authors know, there have not been many researches clarifying the basic mechanism of the simultaneous oscillations except [6][7]. In our past study, we investigated synchronization phenomena observed from two or three coupled hard oscillators [8]-[10]. In that case, we could confirm that various synchronization phenomena including in-phase and anti-phase synchronizations, double-mode oscillation, and simultaneous oscillation are generated.

In this study, we investigate the circuit model that twenty hard oscillators with two LC resonators are coupled by inductors. We discover two kinds of waves. First waves are changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase and continuously exist. Second waves are the change of oscillation states from oscillation death to oscillation or from oscillation to oscillation death and exist.

II. Circuit Model

The circuit model is shown in Fig. 1. In the circuit, twenty hard oscillators with two LC resonators are coupled by inductors L_C and each hard oscillator consists of a nonlinear negative resistor, whose $v-i$ characteristics are described by a fifth-power polynomial function as

$$i_R(v) = g_1 v - g_3 v^3 + g_5 v^5 \qquad (g_1,\ g_3,\ g_5 > 0), \tag{1}$$

and two resonators with different natural frequencies (L_1C_1 and L_2C_2). The equations governing the coupled oscillators are described by the following differential equations including twenty nonlinear functions i_{Rj} $(j = 1, 2, ...20)$.

$$\begin{cases} C_1 \frac{dv_{j1}}{dt} = -i_{j1} - i_{Rj} - i_{Cj} + i_{C,j-1} \\ C_2 \frac{dv_{j2}}{dt} = -i_{j2} - i_{Rj} - i_{Cj} + i_{C,j-1} \\ L_1 \frac{di_{j1}}{dt} = v_{j1} \\ L_2 \frac{di_{j2}}{dt} = v_{j2} \\ (j = 1, 2, ..., 20), \end{cases} \tag{2}$$

where i_{Cn} are the currents through the coupling inductors and are given as

$$i_{Cm} = \frac{L_1(i_{m1} - i_{m+1,1}) + L_2(i_{m2} - i_{m+1,2})}{L_C} \qquad (m = 1, 2, ..., 19), \tag{3}$$

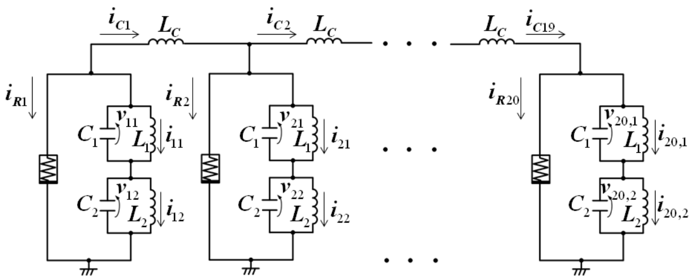


Fig. 1. Twenty coupled hard oscillators with two resonators.

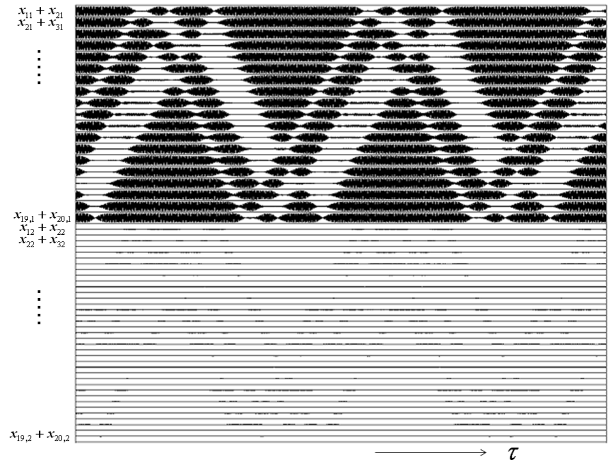


Fig. 2. Time wave of the voltage of right resonator plus left resonator.

and we assume that i_{C0} and $i_{C,20}$ and zero. The currents through the nonlinear resistors i_{Rn} are given as

$$\begin{cases} i_{Rj} = i_R(v_{j1} + v_{j2}) \\ (j = 1, 2, ..., 20). \end{cases} \tag{4}$$

By using the following variables and parameters,

$$\begin{aligned} &v_{jk} = \sqrt[4]{\frac{g_1}{5g_5}} x_{jk}, \quad i_{jk} = \sqrt[4]{\frac{g_1}{5g_5}} \sqrt{\frac{C_1}{L_1}} y_{jk}, \\ &\alpha_C = \frac{C_1}{C_2}, \quad \alpha_L = \frac{L_1}{L_2}, \quad \gamma = \frac{L_1}{L_C}, \\ &\varepsilon = g_1 \sqrt{\frac{L_1}{C_1}}, \quad \beta = \frac{3g_3}{g_1} \sqrt{\frac{g_1}{5g_5}}, \quad t = \sqrt{L_1 C_1} \tau \\ &(j = 1, 2, ..., 20) \quad (k = 1, 2) \quad , \end{aligned} \tag{5}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dfrac{dx_{j1}}{d\tau} = -y_{j1} - f(x_{j1} + x_{j2}) - y_{Cj} + y_{C,j-1} \\ \dfrac{dx_{j2}}{d\tau} = \alpha_C \{-y_{j2} - f(x_{j1} + x_{j2}) - y_{Cj} + y_{C,j-1}\} \\ \dfrac{dy_{j1}}{d\tau} = x_{j1} \\ \dfrac{dy_{j2}}{d\tau} = \alpha_L x_{j2} \qquad (j = 1, 2, ..., 20), \end{cases} \tag{6}$$

where y_C corresponds to i_C and is given as

$$y_{Cm} = \gamma \left(y_{m1} - y_{m+1,1} + \frac{y_{m2} - y_{m+1,2}}{\alpha_L} \right) \quad (m = 1, 2, ..., 19), \tag{7}$$

and y_{C0} and $y_{C,20}$ are zero; and the nonlinear function $f(\cdot)$ which corresponds to the $v-i$ characteristics of the nonlinear resistors is given as

$$f(x) = \varepsilon \left(x - \frac{\beta}{3} x^3 + \frac{1}{5} x^5 \right) \quad . \tag{8}$$

III. SIMULATION RESULTS

In this article, we show only several computer simulated results obtained by giving different initial conditions for the fixed parameters as $\alpha_C = 0.51$, $\alpha_L = 1.0$, $\gamma = 0.01$, $\varepsilon = 0.98$ and $\beta = 2.73$. As a result, we can discover two kinds of waves. First waves are changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase and continuously exist. We call this wave as "phase inversion waves". Second waves are the change of oscillation states from oscillation death to oscillation or from oscillation to oscillation death and exist. We call this wave as "oscillatory inversion wave". In the case of the phase inversion waves, we observe various patterns are as follows. In figures 2, 3 and 4, show time waves of the voltages of right resonators plus left resonators. From these figures, we can see that the upper resonators oscillate when the lowers are oscillation death. On the other hands, when the above resonators are oscillation death, are oscillate.

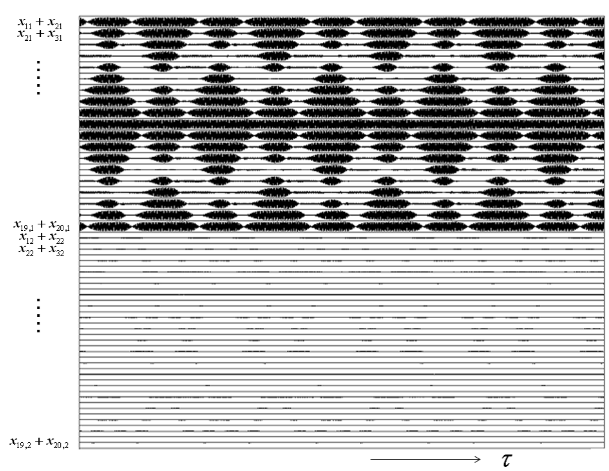


Fig. 3. Time wave of the voltage of right resonator plus left resonator.

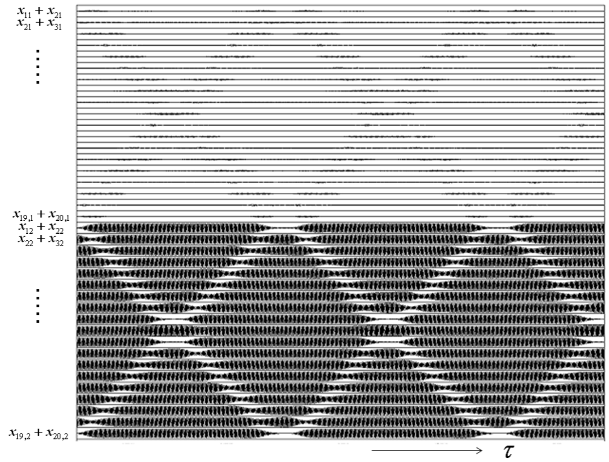
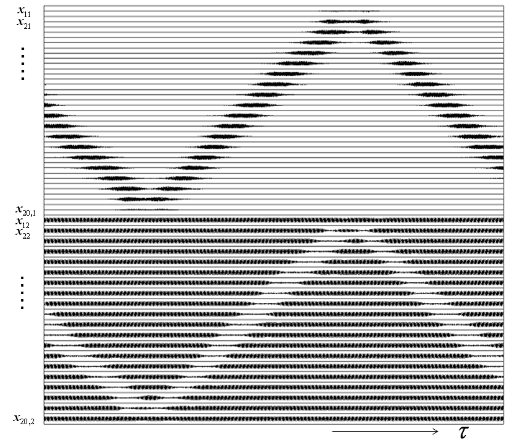
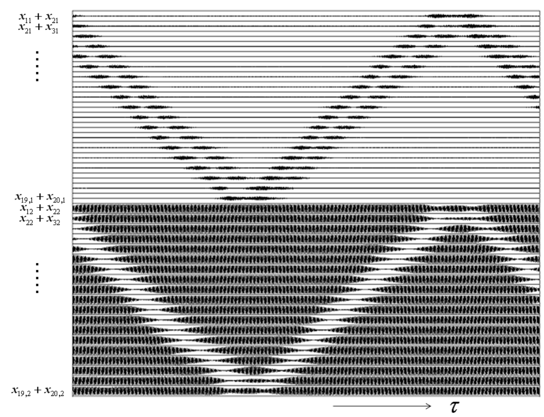


Fig. 4. Time wave of the voltage of right resonator plus left resonator.

The case of the oscillatory inversion wave, we observe various patterns of the waves. In figures 5, 6 and 7, (a) shows time waves of forty x_{mn}; (b) shows time waves of the voltages of right resonators plus left resonators. In this case, oscillation of the resonators continuously propagate. To make it more clearly, figure 8 shows the enlargement of a part of time waves on the oscillatory inversion wave. This figure is separated into four groups of lines. The top four lines shows time waves of the voltage of right resonator plus left resonator among the above resonators. The second group of lines shows time wave of the voltage of right resonator plus left resonator among the blow resonators. The third group of lines shows time wave of four x_{m1} of the upper resonators. The fourth group of lines shows time wave of four x_{m2} of the lower resonators. From two lower groups of lines in the figure, we can see propagation of oscillation from a resonator to adjacent resonators. Here we focus on the relationship between the upper resonators and the lower resonators. When an upper resonator is oscillation death, the lower resonator blow the upper resonators is oscillation death. Whereas, when an upper resonator is oscillation death, the lower resonator blow the upper resonator oscillate. Moreover, upper and lower resonators oscillate with different frequencies. The two upper groups of lines show the sum of amplitudes of adjacent resonators. Thus, if adjacent resonators are synchronized in in-phase, the sum is the double of the amplitude of the resonator. Whereas, if adjacent resonators are synchronized in anti-phase, the sum is zero. From the top group of lines in the figure, we can see that amplitudes tend toward zero. Namely, adjacent resonators among the upper resonators tend toward anti-phase synchronization. On the other hand, from the second group of lines in the figure, we can see that amplitudes of the sums of adjacent resonators are double of the amplitude of the resonator. Namely, adjacent resonators among the lower resonators tend toward in-phase synchronization.



(a)



(b)

Fig. 5. (a) Time waves of forty x_{mn}. (b) Time wave of the voltage of right resonator plus left resonator.

IV. Conclusion

In this study, we have investigated the circuit model that twenty hard oscillators with two resonators are coupled by inductors L_C. We could observe two kinds of waves. First waves are changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase and continuously exist. We call this wave as "phase inversion waves". Second waves are the change of oscillation states from oscillation death to oscillation or from oscillation to oscillation death and exist. We call this wave as "oscillatory inversion wave". In the oscillatory inversion wave, adjacent resonators among the upper resonators tend toward anti-phase synchronization. On the other hand, adjacent resonators among the lower resonators tend toward in-phase synchronization.

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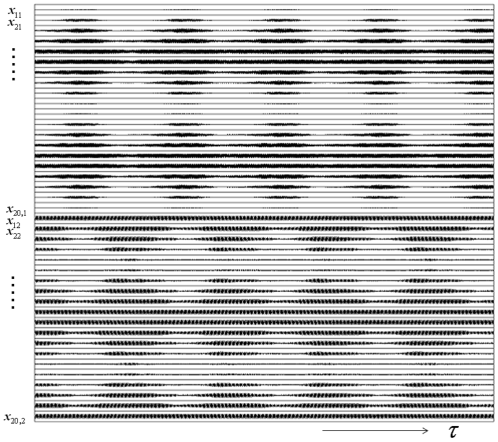
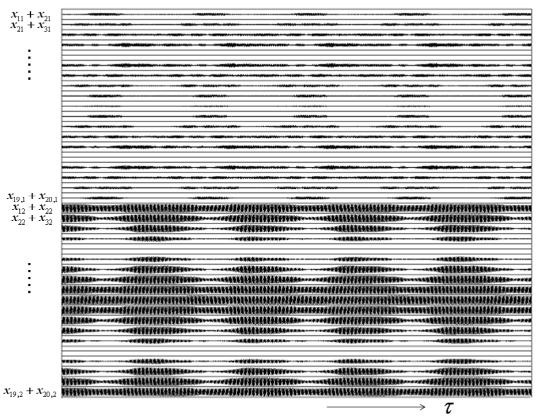


(a)



(b)

Fig. 6. (a) Time waves of forty x_{mn}. (b) Time wave of the voltage of right resonator plus left resonator.

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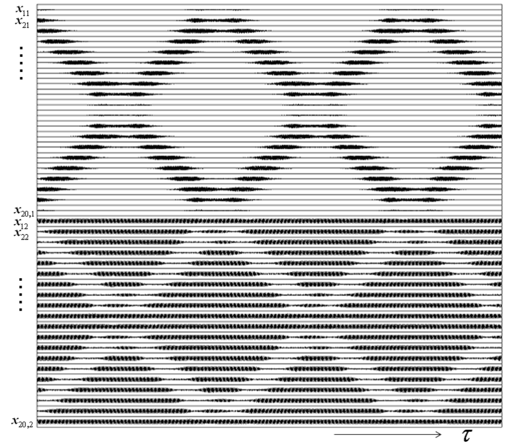
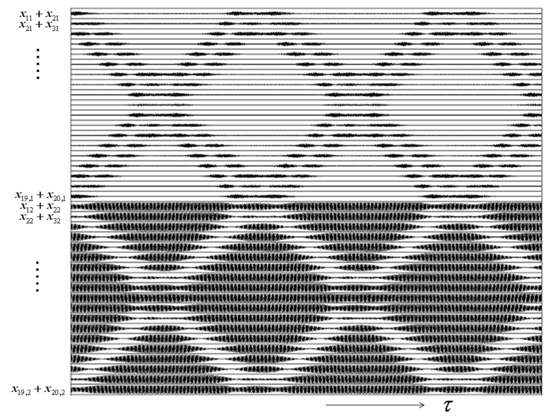


(a)



(b)

Fig. 7. (a) Time waves of forty x_{mn}. (b) Time wave of the voltage of right resonator plus left resonator.

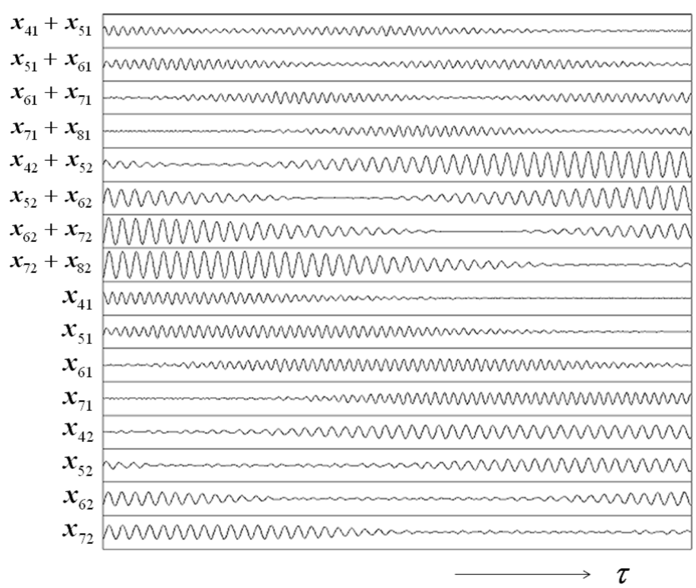


Fig. 8. Enlargement of a part of time waves of Fig.7