Clustering Phenomena of Coupled Chaotic Circuits for Large Scale Networks

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Abstract— **In this study, we investigate the clustering phenomena observed from coupled chaotic circuits arranged in irregular 2-dimensional and 3-dimensional networks. The networks are fully coupled using the distance information, i.e. the coupling strength decreases with the distance between the circuits. We show that synchronized clusters can emerge, reflecting groups of neighboring circuits. By means of computer simulations, we study the clustering phenomena and confirm that the number of clusters depends on the scaling of the coupling strength. Furthermore, we study large scale and 3-dimensional networks of coupled chaotic circuits, demonstrating that clustering can also occur in more complex situations.**

I. Introduction

Nowadays, we often have to deal with huge amounts of data. To structure and analyze such data it is useful to partition the data set into clusters. The idea of clustering algorithms is to find clusters consisting of similar elements. Clustering algorithms have widespread applications in different fields, such as business data mining, image processing or the analysis of biological data. Along with the variety of applications there is a variety of different clustering algorithms. Some algorithms propose to utilize synchronization phenomena, for instance in Coupled Map Lattics (CML), for clustering [1]-[3].

Synchronization is a prominent phenomenon, widely observed and studied in the field of natural and technical sciences. For example, the synchronization of neuronal activity is an important characteristic of brain dynamics, often considered to play an important role for the brain's information processing capacity. Hence the analysis of synchronization might be a key for the realization of a brain computer. One possibility to get insight into the role of synchronization phenomena in biology is the analysis of electronic circuits. In particular, there are numerous studies about synchronization phenomena in connection with chaotic circuits, describing various interesting aspects such as chaos synchronization, space-time chaos, chaotic wandering and so on. However, there are not many studies on synchronization-based clustering in networks of coupled chaotic circuits.

In a previous study, we investigated 2-dimensional networks of complex chaotic circuits, where the coupling strength reflected the distance information [4]. We showed that the circuits arranged close to each other could synchronize in-phase, whereas coupled circuits far from each other could not be synchronized. We also observed that such networks of coupled chaotic circuits could split into different synchronized groups, revealing a clustering phenomenon.

In this study, we consider the relationship between the coupling strength and the clustering phenomenon. For this investigation, we change the scaling parameter of the coupling strength and observe the changes of the number of clusters, using a 2-dimensional irregular network of 20 circuits. Furthermore, we also carry out the analysis for a large scale network with 100 circuits and we additionally consider the example of a 3-dimensional network.

II. Circuit model

Figure 1 shows the model of the used chaotic circuit called Nishio-Inaba circuit, investigated in [5]-[7].

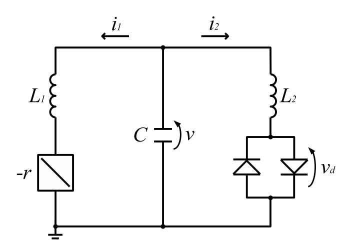


Fig. 1. Chaotic Nishio-Inaba circuit.

The circuit consists of a negative resistance, a nonlinear resistance consisting of two diodes, a capacitor and two inductors. The following equation describes the approximated $I-V$ characteristic of the nonlinear resistance, where the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \quad (1)$$

The circuit dynamics is then described by the following piecewise-linear third-order ordinary differential equation:

$$\begin{aligned} L_1 \frac{di_1}{dt} &= v + ri_1 \\ L_2 \frac{di_2}{dt} &= v - v_d(i_2) \\ C \frac{dv}{dt} &= -i_1 - i_2. \end{aligned} \tag{2}$$

Equation (2) can be normalized by changing the variables according to

$$\begin{aligned} i_1 &= \sqrt{\frac{C}{L_1}} Vx;\; i_2 = \frac{\sqrt{L_1 C}}{L_2} Vy;\; v = Vz; \\ r\sqrt{\frac{C}{L_1}} &= \alpha;\; \frac{L_1}{L_2} = \beta;\; r_d \frac{\sqrt{L_1 C}}{L_2} = \delta; \\ t &= \sqrt{L_1 C}\tau;\quad \text{"}\cdot\text{"} = \frac{d}{d\tau} \end{aligned}$$

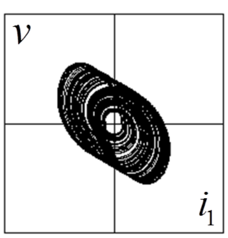
yielding

$$\begin{aligned} \dot{x} &= \alpha x + z \\ \dot{y} &= z - f(y) \\ \dot{z} &= -x - \beta y \end{aligned} \tag{3}$$

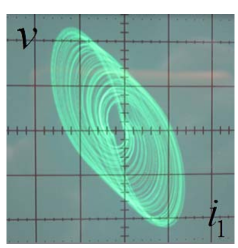
where $f(y)$ is described as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \tag{4}$$

Figure 2 shows the chaotic attractor generated by the circuit, obtained from a computer simulation (Fig. 2 (a)) and measured in a circuit experiment (Fig. 2 (b)). For the computer simulation, we set the parameters as $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. For the circuit experiment, the parameters were fixed with $L_1 = 500[mH]$, $L_2 = 200[mH]$, $C = 0.0153[\mu F]$, and $r_d = 1.46[M\Omega]$.



(a) Computer simulation.



(b) Circuit experiment.

Fig. 2. Chaotic attractor.

III. Synchronization phenomena

A. Network of 20 chaotic circuits

First, we investigate the case of a general network, where 20 chaotic circuits are placed irregularly on a 2-dimensional grid. The arrangement is shown in Fig. 3. All chaotic circuits are connected to each other by resistors.

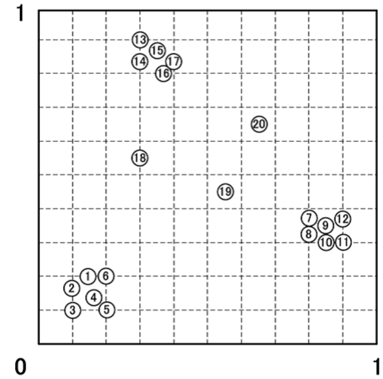


Fig. 3. Arrangement of twenty chaotic circuits.

We consider the following globally coupled chaotic circuits:

$$\begin{aligned} \frac{dx_i}{d\tau} &= \alpha x_i + z_i \\ \frac{dy_i}{d\tau} &= z_i + f(y) \\ \frac{dz_i}{d\tau} &= -x_i - \beta y_i - \sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ &(i, j = 1, 2, \cdots, N) \end{aligned} \tag{5}$$

where i in the equation represents the circuit itself, and j indicates the coupling with other circuits. The parameter γ_{ij} represents the coupling strength between the circuits. The value of γ_{ij} reflects the distance between the circuits in an inverse way, described by the following equation:

$$\gamma_{ij} = \frac{g}{(length_{ij})^2}. \tag{6}$$

$length_{ij}$ denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit. The parameter g is a weighting or scaling parameter that determines the coupling strengths. For our first study, we set the parameter to $g = 0.25 \times 10^{-4}$.

Figure 4 shows the computer simulation results of the phase differences of several circuits. We can derive the following observations, exemplified by a selection of pairwise comparisons: The first circuit and the second circuit, as well as the first circuit and the third circuit are synchronized (a-b). Actually, all the circuits within this spatial cluster or group are synchronized. However, the first circuit is not synchronized to the group of the seventh circuit (c) or to the group of the thirteenth circuit (d). All the circuits within the group of the seventh circuit are synchronized within the group (e-f), but they are not synchronized to any other circuits outside the group (g-h). All the circuits within the group of the thirteenth circuit are synchronized within the group (i-j), but again they are not synchronized to any other circuits outside the group (k). The circuits in the middle (18-20) are not synchronized with any other circuits (l). From these results, we conclude that the circuits form three clusters through chaos synchronization. Furthermore, circuits 18, 19 and 20 are isolated as they do not

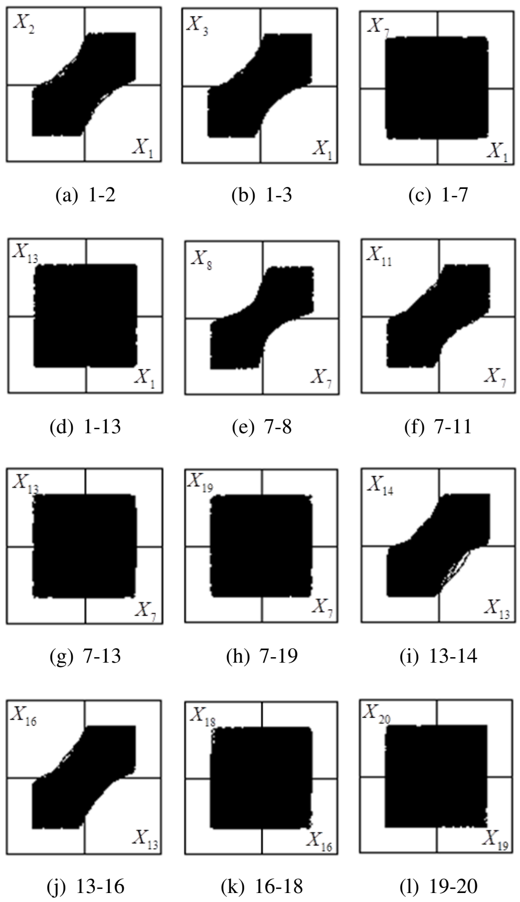


Fig. 4. Phase difference between two circuits (computer simulation).

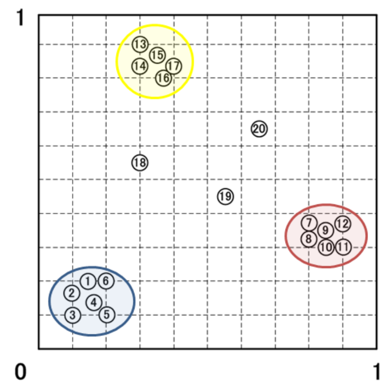


Fig. 5. The clustering result of twenty chaotic circuits.

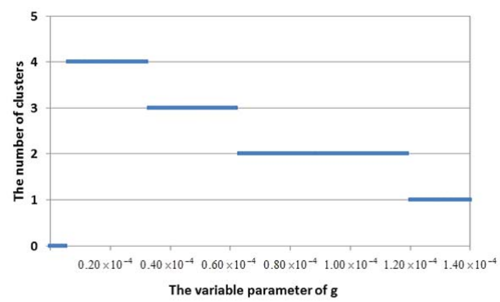


Fig. 6. The relationship between the coupling strength and the number of distinguishable groups.

synchronize to any other circuit. Figure 5 summarizes these clustering results.

In a next step, we consider the relationship between the coupling strength, controlled by the parameter g, and the number of clusters that occur. For the analysis we introduce the following group numbering: Group 1 consists of the circuits 1-6, group 2 consists of the circuits 7-12, group 3 consists of the circuits 13-17 and the residual circuits are group 4. Figure 6 shows the number of synchronized clusters that occur depending on the value of g. For small values $g \leq 0.05 \times 10^{-4}$ no synchronization occurs (no distinguishable groups) In the region between $g = 0.06 \times 10^{-4}$ and $g = 0.32 \times 10^{-4}$ clustering can be observed. The clustering in this region corresponds to the result shown in Fig. 4 and Fig. 5. So, we have 4 distinguishable groups. For values between $g = 0.33 \times 10^{-4}$ and $g = 0.62 \times 10^{-4}$ group 1 and 2 are synchronized in one cluster. Hence we have two synchronized clusters and one group of isolated circuits (3 distinguishable groups). Between $g = 0.63 \times 10^{-4}$ and $g = 1.10 \times 10^{-4}$ all the circuits of the groups 1, 2 and 3 are synchronized. Hence we have one synchronized cluster and still one group of isolated circuits (2 distinguishable groups). Finally, all the circuits are synchronized in one cluster if $g \geq 1.20 \times 10^{-4}$.

B. Network of 100 chaotic circuits

Next, we investigate the case of large scale networks, by studying a 2-dimensional irregular network of 100 chaotic circuits. The arrangement of the circuits is shown in Fig. 7. Again, all the circuits are connected to each other by resistors and the coupling strength between two circuits is determined by Eq. (6). In this case, we set the parameter $g = 0.85 \times 10^{-6}$.

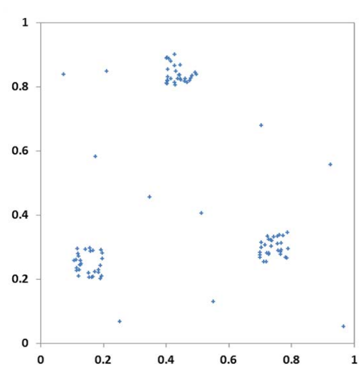


Fig. 7. Arrangement of 100 chaotic circuits.

Figure 8 shows the corresponding clustering results. A cluster consists of 30 chaotic circuits. The networks can divide 3 cluster from synchronization.

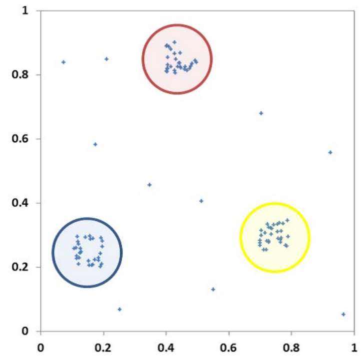


Fig. 8. The clustering result of 100 chaotic circuits.

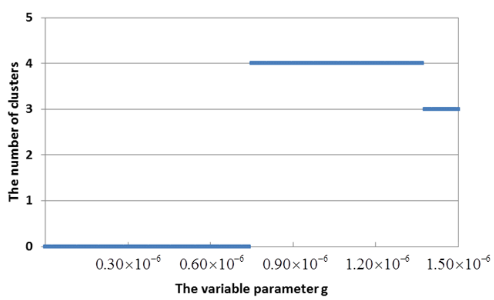


Fig. 9. The relationship between the coupling strength and the number of distinguishable groups.

Figure 9 shows the relationship between the coupling strength and the number of occurring groups in the network. We define the blue group (in Fig. 8) as group 1, the red group as group 2 and the yellow group as group 3. All the other circuits belong to group 4. There is again no synchronization for small values $g \leq 0.74 \times 10^{-6}$. Between $g = 0.75 \times 10^{-6}$ and $g = 1.37 \times 10^{-6}$ clustering with 3 synchronized clusters and one group of isolated circuits can be observed (see example). Between $g = 1.38 \times 10^{-6}$ and $g = 1.57 \times 10^{-6}$ group 1 and 2 are synchronized. Thus the network can be divided into 3 distinguishable groups. If $g \geq 1.58 \times 10^{-6}$ this network becomes divergent.

From these examples we can make the following observations. The networks are not synchronized if the coupling strength is weak. For intermediate values of the coupling strength we can observe synchronized clusters. Finally, for strong couplings the networks are fully synchronized or divergent.

IV. Chaotic circuits in 3-dimensional network

Finally, we study a 3-dimensional network of coupled chaotic circuits. The arrangement of the circuits is shown in Fig. 10. It consists of a plane ring with 50 circuits and a sphere of radius 0.1 placed in the center, containing another 50 circuits. Again, the circuits are globally coupled according to equation (6). The result for this network is shown in Fig. 11. We see that clustering can also occur in 3D for clusters with a nontrivial shape such as a ring .

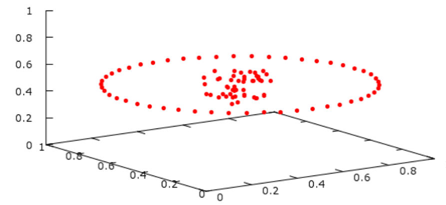


Fig. 10. Irregular 3-dimensional network of 100 chaotic circuits.

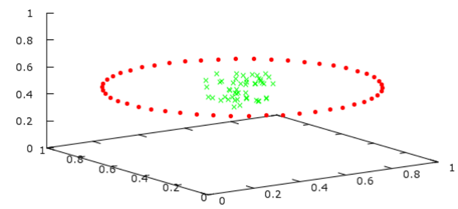


Fig. 11. Corresponding clustering result.

V. Conclusion

In this study, we examined clustering phenomena in 2- and 3-dimensional irregular networks of coupled chaotic circuits in dependence on the coupling strength. Based on computer simulations, we showed that the circuits could cluster into several groups of synchronized units. We found that the number of clusters changed depending on the coupling strength. Interesting clustering results only occurred for intermediate values of the coupling strength, whereas for weak and strong couplings no clustering could be observed.

In our future work, we would like to develop a more efficient method for determining the phase synchronization. This will allow us to examine more complex networks or to use the method for data clustering.

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