Clustering Phenomena in Complex Networks of Chaotic Circuits

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Abstract— **In this study, we investigate synchronization phenomena in coupled chaotic circuits which are connected with distance information. We confirm that the chaotic circuits arranged in the near distance are synchronized at in-phase state, and the coupled circuits with the far distance could not be synchronized. Namely, clustering phenomena of coupled chaotic circuits is observed on two-dimensional place.**

I. Introduction

Synchronization phenomena are very typical phenomena in the field of natural science. Recently, many studies have been investigated synchronization of chaotic circuits. It is applied in the field of engineering, physics and biology and so on. Also one of the characteristic of the brain is synchronization of neuronal activity. In addition, this phenomenon is considered to play an important role in brain information processing. Then, we consider that it is very important to investigate the synchronization phenomena of coupled chaotic circuits for the future engineering applications such as analysis of the brain activity and the realization of a brain computer.

In recent years we often deal with huge amounts of data. It is useful to partition such data sets into cluster. Also, such data need to divide same group for clustering. Moreover, clustering is one of the interesting phenomena. The phenomena are also applied to many application, such as data mining, image processing and biological field. Many kinds of models and algorithms by using Coupled Map Lattics (CML) are proposed for clustering [1]-[3]. However, there are not many discussions clustering of coupled chaotic circuits.

In this study, we investigate the clustering phenomena resulting from the synchronization phenomena observed in coupled chaotic circuits when the chaotic circuits are placed on 2-dimensional space. In this circuit system, the chaotic circuits are coupled with the distance information. We observe that the chaotic circuits arranged in the near distance are synchronized at in-phase state, and the coupled circuits with the far distance could not be synchronized. From the results we clear up the relationship between clustering and synchronization phenomena. Furthermore, we also carry out the circuit experiments and compare with computer simulation results.

The paper is organized as follows. In Section II, we describe the coupled chaotic circuits. Section III, the observed interesting synchronization phenomena are explained. Finally, Section IV concludes the paper.

II. Circuit model

Figure 1 shows the circuit model, which is called Nishio-Inaba circuit investigated in [4]-[6].

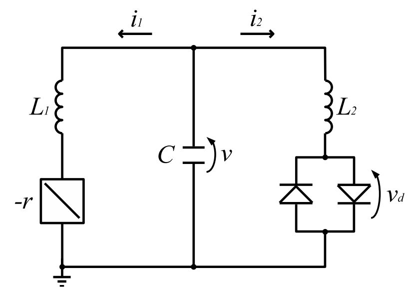

Fig. 1. Chaotic circuit.

The circuit consists of a negative resistance, a nonlinear resistance consisting of two diodes, capacitor and two inductors. The approximate the $I-V$ characteristic of the nonlinear resistance shows the following equation and the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \tag{1}$$

Then the circuit dynamics is described by the following piecewise-linear third-order ordinary differential equation:

$$\begin{aligned} L_1\frac{di_1}{dt} &= v + ri_1 \\ L_2\frac{di_2}{dt} &= v - v_d(i_2) \\ C\frac{dv}{dt} &= -i_1 - i_2. \end{aligned} \tag{2}$$

By changing the variables such that

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1C}}{L_2}Vy;\ v = Vz;$$
$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1C}}{L_2} = \delta;$$
$$t = \sqrt{L_1C}\tau;\quad \text{"}\cdot\text{"} = \frac{d}{d\tau}$$

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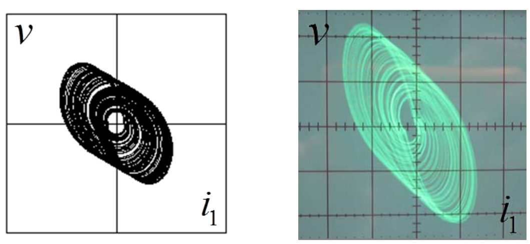
That the equation (2) is normalized as

$$\begin{aligned}\dot{x} &= \alpha x + z\\ \dot{y} &= z - f(y)\\ \dot{z} &= -x - \beta y\end{aligned} \tag{3}$$

where $f(y)$ is described as follows;

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \tag{4}$$

Figure 2 shows the chaotic attractor generated from the circuit by using computer simulation (Fig. 2 (a)) and circuit experiment (Fig. 2 (b)). For the computer simulation, we set the parameters as $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. For the circuit experiment, the parameters are fixed with $L_1 = 500[mH]$, $L_2 = 200[mH]$, $C = 0.0153[\mu F]$, and $r_d = 1.46[M\Omega]$.



(a) Computer simulation. (b) Circuit experiment.

Fig. 2. Chaotic attractor.

III. Synchronization phenomena

A. Network of seven chaotic circuits

First, we investigate the basic clustering phenomena when seven chaotic circuits are coupled globally. The arrangement of seven chaotic circuits is shown in Fig. 3. Table 1 shows the locations of the chaotic circuits. All circuits are connected each other by resistors. Figure 4 shows the coupling method of the first chaotic circuit as an example.

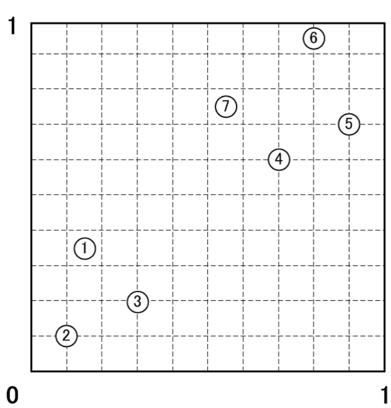


Fig. 3. Arrangement of seven chaotic circuits.

TABLE I

The location of seven circuits

location	x	y
1	0.15	0.35
2	0.10	0.10
3	0.30	0.20
4	0.70	0.60
5	0.90	0.80
6	0.80	0.95
7	0.55	0.80

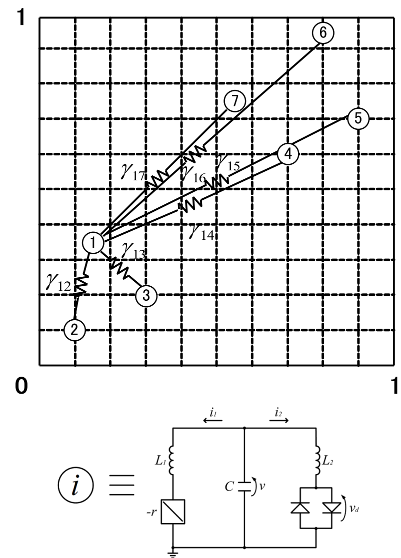


Fig. 4. Coupling between the first chaotic circuit and the others.

We consider the globally coupled chaotic circuits:

$$\begin{aligned}\frac{dx_i}{d\tau} &= \alpha x_i + z_i\\ \frac{dy_i}{d\tau} &= z_i + f(y)\\ \frac{dz_i}{d\tau} &= -x_i - \beta y_i - \sum_{i,j=1}^{N} \gamma_{ij}(z_i - z_j)\\ (i, j &= 1, 2, \cdots, N)\end{aligned} \tag{5}$$

where, i in the equation represents the circuit itself, and j is the coupling with other circuits. The parameter γ represents the coupling strength between the circuits. In this simulation, we set the coupling parameter value $\gamma_{i,j}$ to correspond the distance between the circuits by the following equation:

$$\gamma_{i,j} = \frac{g}{(length_{i,j})^2}. \tag{6}$$

The parameter g is a weight parameter that determines the coupling strengths. In this case, we set the parameter $g = 0.00327$. And $length_{i,j}$ denotes the Euclidean distance between the $i-th$ and the $j-th$ circuits.

Figure 5 shows the computer simulated results obtained from the seven chaotic circuits arranged as shown in Fig. 3. We also confirm the same synchronization and clustering phenomena by the circuit experiments as shown in Fig. 6. From these results, we confirm that the first, the second and the third chaotic circuits are synchronized at in-phase state, and also the fourth, fifth, sixth and seventh chaotic circuits synchronize with in-phase state. However, the first and the fourth chaotic circuits are not synchronized. Namely, the circuits form two clusters defined by chaos synchronization. Figure 7 shows the clustering results of seven chaotic circuits.

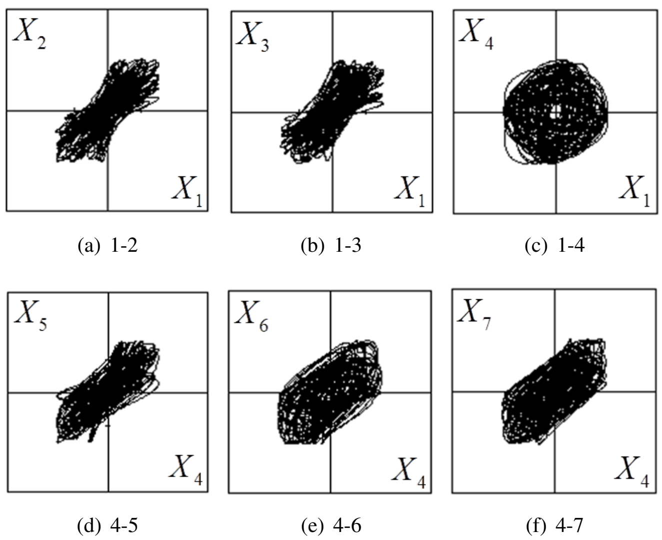


Fig. 5. Phase difference between two circuits (Computer simulations).

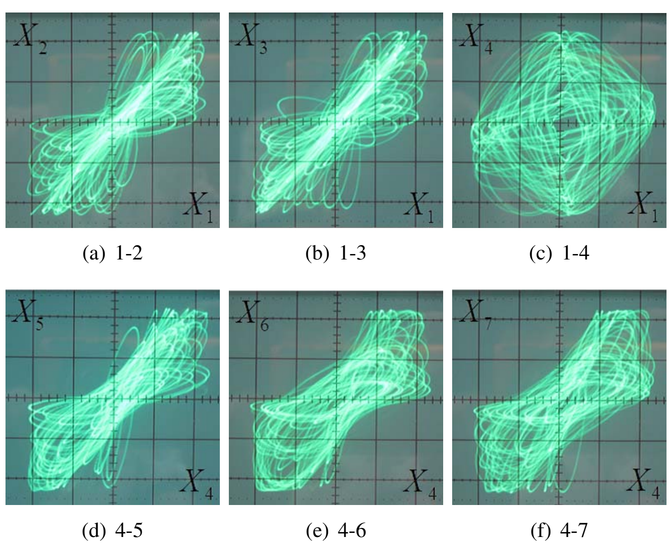


Fig. 6. Phase difference between two circuits (Circuit experiments).

B. Network of 20 chaotic circuits

Next, we investigate the case of general networks. Twenty chaotic circuits are placed on 2-dimensional including the positional information. The arrangement of twenty chaotic circuits is shown in Fig. 8. Also, Table 2 shows the locations

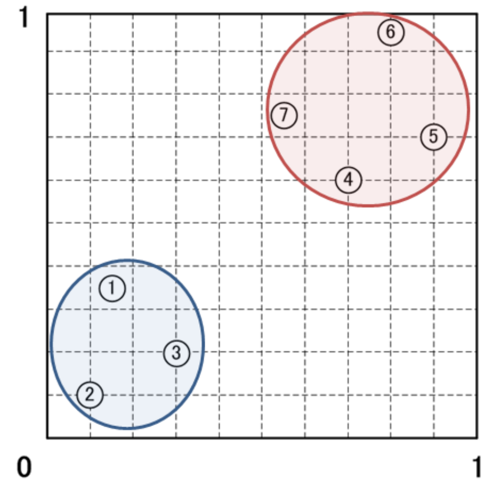


Fig. 7. Clustering result for seven chaotic circuits.

of the chaotic circuits. Similarly, all the chaotic circuits are connected each other by resistors, and the coupling strength between two circuits is determined by Eq. (6). In this case, we set the parameter g = 0.000025.

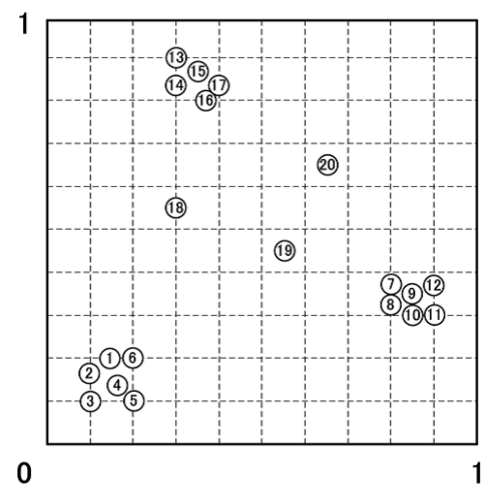


Fig. 8. Arrangement of twenty chaotic circuits.

Figure 9 shows the computer simulation results of the phase differences. From these results, we confirm that the first and the second chaotic circuits are synchronized. However, the first and the group of seventh chaotic circuit are not synchronized. Also the first and the group of thirteenth circuit are note synchronized. Similarly, between seventh circuit and group of eight circuit are synchronized, between thirteen circuit and group of fourteen circuit are synchronized, however between group of seventh circuit and group of thirteen circuit is asynchronous, From these results, the circuits can form three clusters defined by chaos synchronization. Further, circuits 18, 19 and 20 are isolated, namely they do not synchronize to any of the others. Figure 10 shows the clustering results of twenty

TABLE II

The location of 20 circuits

location	x	y
1	0.13	0.20
2	0.10	0.16
3	0.10	0.10
4	0.16	0.13
5	0.20	0.10
6	0.20	0.20
7	0.80	0.36
8	0.80	0.33
9	0.85	0.36
10	0.85	0.30
11	0.90	0.30
12	0.90	0.36
13	0.30	0.90
14	0.30	0.83
15	0.35	0.85
16	0.36	0.80
17	0.40	0.85
18	0.30	0.55
19	0.55	0.45
20	0.65	0.65

chaotic circuits.

IV. Conclusion

In this study, we have investigated the clustering phenomena resulting from the synchronization phenomena observed in coupled chaotic circuits when placed on 2-dimensional. The chaotic circuits are coupled with the distance information. By using the computer simulations and the circuit experiments, we have confirmed that the chaotic circuits arranged in the near distance are synchronized at in-phase state, and the coupled circuits with the far distance could not be synchronized. We could cluster the coupled chaotic circuits into several groups depending on the circuit arrangement.

For the future works, we would like to develop the efficient method to distinguish cluster. Applying this proposed system to the real data is also important project to us.

Acknowledgment

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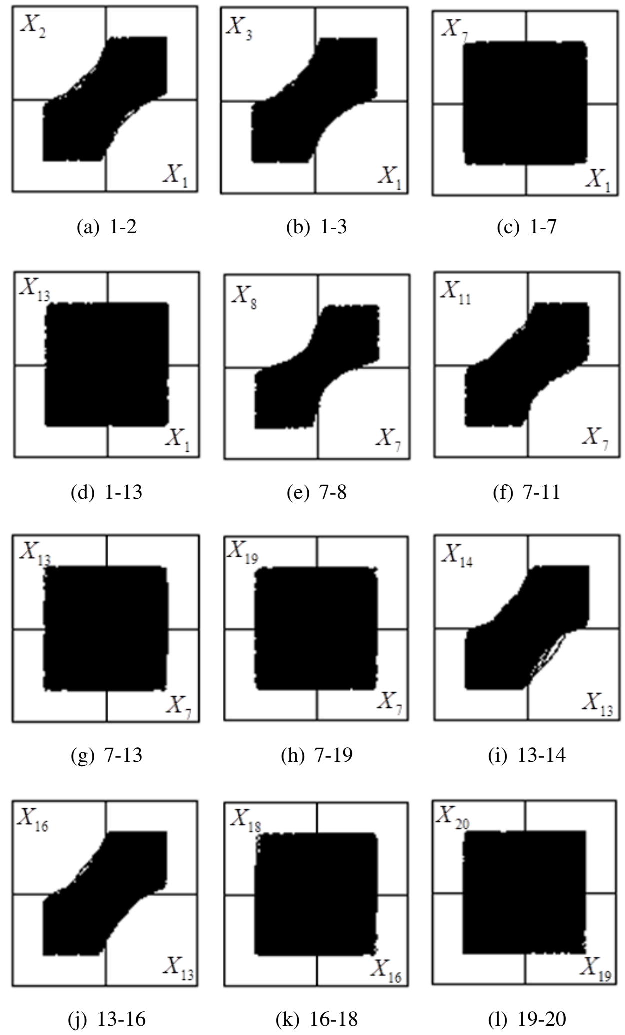

Fig. 9. Phase difference between two circuits (Computer simulations).

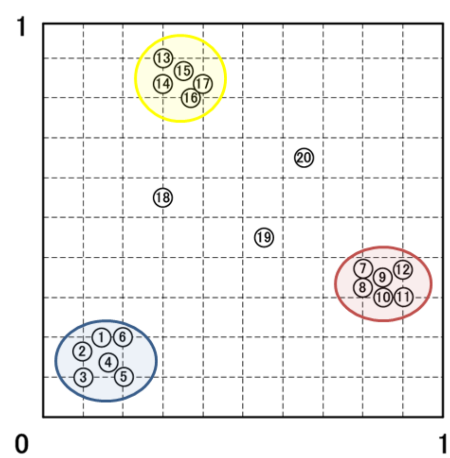

Fig. 10. The clustering result of twenty chaotic circuits.