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Frustrated Synchronization in Coupled Oscillator Chains with Unbalanced Parametric Distribution

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Abstract

In this study, we investigate frustrated synchronization in coupled oscillator chains when symmetry of parametric distribution is broken. When one of coupling parameters is set to different value to the others, parametric distribution of coupled oscillator chains becomes unbalance. By using computer simulations, we observe different types of frustrated synchronization states.

1. Introduction

Coupled oscillators are suitable model to analyze the natural phenomena. Because, it is easy to observe the synchronization phenomena of coupled oscillators by various elements and techniques. Also, synchronization phenomena are very basic phenomena. In particular we can observe them everywhere in nature. For example, vibration of a pendulum, firefly luminescence, gate patterns of four-leg animals, two frogs using voice religiously, periodic swinging of candle flames, and so on. Many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena [1]-[8]. The research group of the authors is also working on coupled oscillatory networks. Especially, we have been interested in coupled oscillators whose connections cause some kinds of frustrations [9]-[12].

In the previous study, we proposed coupled oscillator chains whose coupling structures are constructed to cause frustrations between oscillators and investigated synchronization phenomena observed in the circuits [13]. Interesting synchronization phenomena were observed and the observed synchronization were theoretically analyzed.

In this study, we focus on synchronization state of coupled oscillator chains when symmetry of parametric distribution is broken. In order to investigate such situation, one of the coupling parameters is set to different value to the others. Namely, coupled oscillatory chains obtain stronger frustrations by unbalanced parametric distribution. By using computer simulations, we observe different types of frustrated synchronization states.

2. Coupled Oscillator Chains [13]

In short, we explain about the coupled oscillator chains proposed and investigated in [13]. In this study, we focus on the case of three row coupled oscillator chains. Figure 1 shows the circuit model for the case of 3 oscillator chains. Each oscillator-chain consists of 3 step van der Pol oscillators weakly coupled by resistors r. In this figure, the bottom three oscillators are coupled by relatively strong resistors R_{i1} and R_{i2}, while the top three oscillators are coupled by also relatively strong resistors R_{a1} and R_{a2} via inductors. The middle oscillators are not coupled with oscillators located in the horizontal direction but weakly coupled vertically.

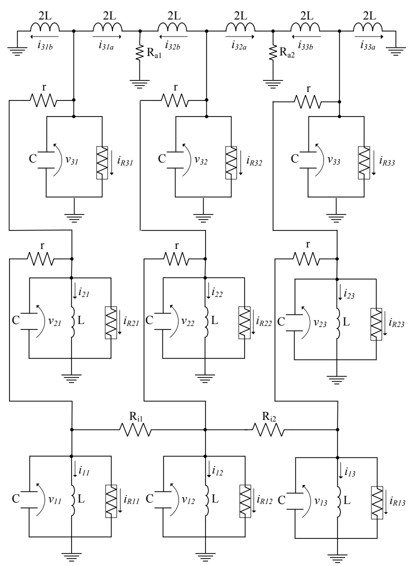

Figure 1: Circuit model for the case of 3 oscillators-chains.

The coupling structure on the bottom or in vertical couplings (by R_{i1} and R_{i2} or r) tends to make the oscillators to synchronize in in-phase. While the coupling structure on the top (by R_{a1} and R_{a2} via inductor) tends to make the oscillators to synchronize in anti-phase.

We define the bottom three oscillators as Osc_{11}, Osc_{12} and Osc_{13} from the left, those on the middle row as Osc_{21}, Osc_{22} and Osc_{23}, and the top three oscillators as Osc_{31}, Osc_{32} and Osc_{33}.

The $v - i$ characteristics of the nonlinear resistor in each oscillator is given by the following third order polynomial equation.

$$i_{R_{kj}} = -g_1 v_{kj} + g_3 {v_{kj}}^3 \qquad (1)$$

where $g_1, g_3 > 0$, $k = 1, 2, 3$, and $j = 1, 2, 3$.

By using the following variables and parameters:

$$t = \sqrt{LC}\tau,\ v_{kj} = \sqrt{\frac{g_1}{g_3}}\, x_{kj},\ i_{kj} = \sqrt{\frac{g_1 C}{g_3 L}}\, y_{kj},$$

$$\varepsilon = \frac{1}{g_1}\sqrt{\frac{L}{C}},\ \alpha_{in} = \frac{1}{R_{in}}\sqrt{\frac{L}{C}},\ \alpha_{an} = R_{an}\sqrt{\frac{L}{C}},\ \beta = \frac{1}{r}\sqrt{\frac{L}{C}},$$

where $n = 1, 2$ and the normalized circuit equations are given as follows:

(1) Top oscillators:

$$\begin{cases} \dot{x}_{31} = \varepsilon(1 - {x_{31}}^2)x_{31} - (y_{31a} + y_{31b}) - \beta(x_{31} - x_{21}) \\ \dot{y}_{31a} = 0.5\{x_{31} - \alpha_{a1}(y_{31a} + y_{32b})\} \\ \dot{y}_{31b} = 0.5\, x_{31} \\ \dot{x}_{32} = \varepsilon(1 - {x_{32}}^2)x_{32} - (y_{32a} + y_{32b}) - \beta(x_{32} - x_{22}) \\ \dot{y}_{32a} = 0.5\{x_{32} - \alpha_{a2}(y_{32a} + y_{33b})\} \\ \dot{y}_{32b} = 0.5\{x_{32} - \alpha_{a1}(y_{31a} + y_{32b})\} \\ \dot{x}_{33} = \varepsilon(1 - {x_{33}}^2)x_{33} - (y_{33a} + y_{33b}) - \beta(x_{33} - x_{23}) \\ \dot{y}_{33a} = 0.5\, x_{33} \\ \dot{y}_{33b} = 0.5\{x_{33} - \alpha_{a2}(y_{32a} + y_{33b})\} \end{cases} \qquad (2)$$

(2) Middle oscillators:

$$\begin{cases} \dot{x}_{21} = \varepsilon(1 - {x_{21}}^2)x_{21} - y_{21} + \beta(x_{31} - 2x_{21} + x_{11}) \\ \dot{y}_{21} = x_{21} \\ \dot{x}_{22} = \varepsilon(1 - {x_{22}}^2)x_{22} - y_{22} + \beta(x_{32} - 2x_{22} + x_{12}) \\ \dot{y}_{22} = x_{22} \\ \dot{x}_{23} = \varepsilon(1 - {x_{23}}^2)x_{23} - y_{23} + \beta(x_{33} - 2x_{23} + x_{13}) \\ \dot{y}_{23} = x_{23} \end{cases} \qquad (3)$$

(3) Bottom oscillators:

$$\begin{cases} \dot{x}_{11} = \varepsilon(1 - {x_{11}}^2)x_{11} - y_{11} + \beta(x_{21} - x_{11}) - \alpha_{i1}(x_{11} - x_{12}) \\ \dot{y}_{11} = x_{11} \\ \dot{x}_{12} = \varepsilon(1 - {x_{12}}^2)x_{12} - y_{12} + \beta(x_{22} - x_{12}) \\ \qquad\qquad + \alpha_{i1}(x_{11} - x_{12}) + \alpha_{i2}(x_{13} - x_{12}) \\ \dot{y}_{12} = x_{12} \\ \dot{x}_{13} = \varepsilon(1 - {x_{13}}^2)x_{13} - y_{13} + \beta(x_{23} - x_{13}) - \alpha_{i2}(x_{13} - x_{12}) \\ \dot{y}_{13} = x_{13} \end{cases} \qquad (4)$$

where x_{kj} corresponds to the voltage across the capacitor and y_{kj}, y_{3ja}, y_{3jb} are the currents through the inductors of Osc_{kj}.

In this system, we observed the interesting synchronization phenomenon in the previous study. Figures 2 and 3 show time waveforms of the oscillators and phase shifts between the oscillators when the parameters are set as $\varepsilon = 0.10$, $\alpha_{i1} = \alpha_{i2} = \alpha_{a1} = \alpha_{a2} = 0.5$, and $\beta = 0.02$. The oscillators in the middle row have some amounts of phase shits to the bottom row, and those in the top row have twice amounts of the phase shifts between the middle row and the bottom row. The observed synchronization was explained with theoretical analysis by considering the total power consumption of the coupling resistors.

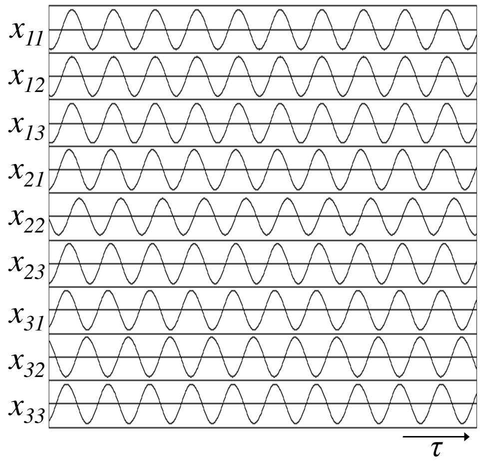

Figure 2: Time waveforms of the oscillators.

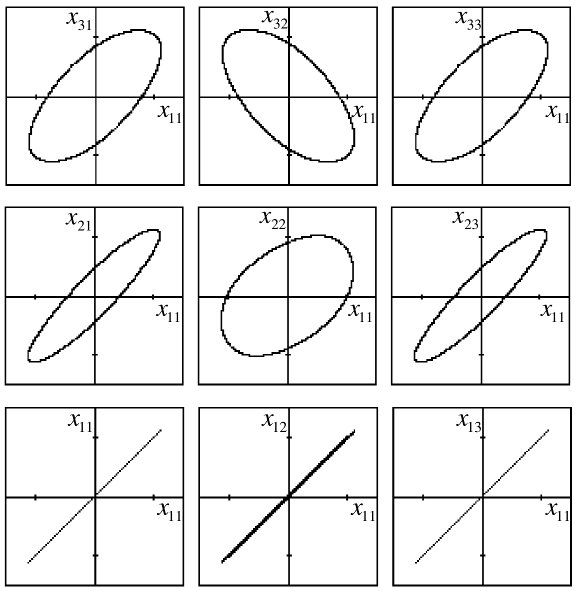

Figure 3: Phase shifts between the oscillators.

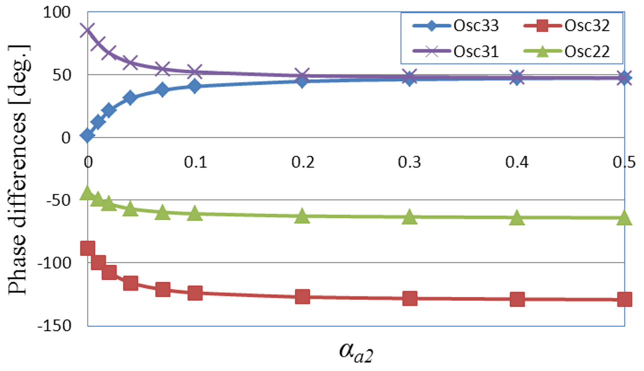

Figure 4: Phase differences between oscillators (Osc_{22}, Osc_{31}, Osc_{32} and Osc_{33}) and Osc_{11} by changing α_{a2}.

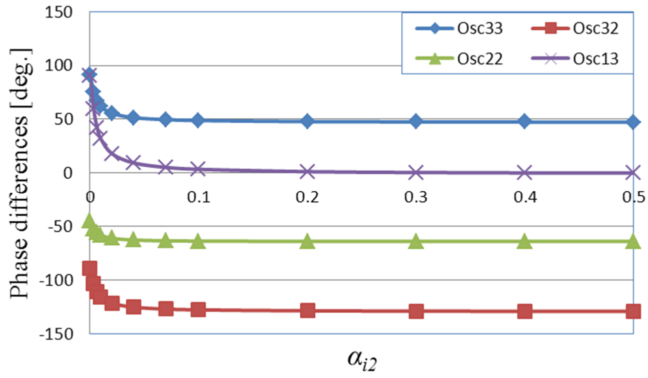

Figure 6: Phase differences between oscillators (Osc_{13}, Osc_{22}, Osc_{32} and Osc_{33}) and Osc_{11} by changing α_{i2}.

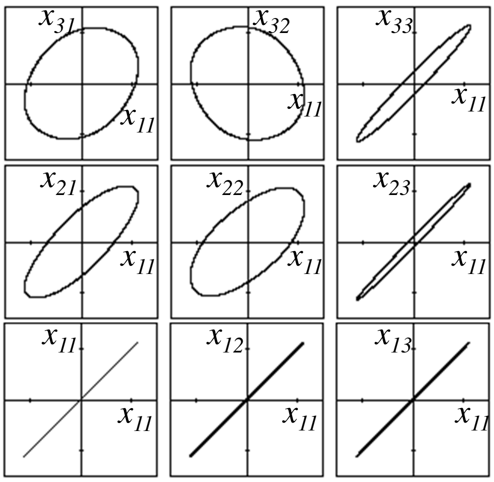

Figure 5: An example of phase shifts between oscillators ($\alpha_{a2} = 0.01$).

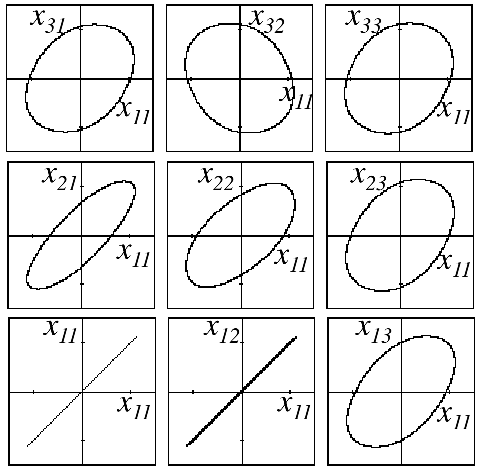

Figure 7: An example of phase shifts between oscillators ($\alpha_{i2} = 0.0033$).

3. Coupled Oscillator Chains with Unbalanced Parametric Distribution

We investigate synchronization in the coupled oscillator chains when symmetry of parametric distribution is broken. Namely, one of the coupling parameters is set to different value to the others. We consider two cases of unbalanced parametric distribution as follows:

1. Changing the coupling parameter (α_{a2}) between Osc_{32} and Osc_{33} of the top oscillators.
2. Changing the coupling parameter (α_{i2}) between Osc_{12} and Osc_{13} of the top oscillators.

(A) Synchronization in dependence on α_a

We investigate synchronization when α_{a1} is set at 0.5 and α_{a2} varies, where the other parameters are the same as the values presented in the previous section. Figure 4 shows change of phase differences between oscillators (Osc_{22}, Osc_{31}, Osc_{32} and Osc_{33}) and Osc_{11} by changing of α_{a2}. Figure 5 shows an example of phase shifts between oscillators when α_{a2} is quite small.

With decreasing α_{a2}, the coupling strength between Osc_{32} and Osc_{33} becomes weak, then, Osc_{33} tend toward in-phase synchronization with Osc_{11} via Osc_{23}. Moreover, the phase differences between oscillators (Osc_{31} and Osc_{32}) and Osc_{11} tend toward $|90|$ [*deg.*], the phase differences between oscillators (Osc_{21} and Osc_{22}) and Osc_{11} tend toward $|45|$ [*deg.*]. In the parameter region where α_{a2} is relatively large, α_{a2} is from 0.1 to 0.5, changes of the phase differences are small, whereas, in the parameter region where α_{a2} is small, α_{a2} is from 0 to 0.1, the phase differences rapidly change. The phase difference between Osc_{31} and Osc_{32} and the phase differences between Osc_{21} and Osc_{22} respectively keep 180 [*deg.*] and 90 [*deg.*] for any α_{a2}.

(B) Synchronization in dependence on α_i

We investigate synchronization when α_{i1} is set at 0.5 and α_{i2} varies, where the other parameters are the same as the values presented in the previous section. Figure 6 shows change of phase differences between oscillators (Osc_{13}, Osc_{22}, Osc_{32} and Osc_{33}) and Osc_{11} by changing of α_{i2}. Figure 7 shows an example of phase shifts between oscillators when α_{i2} is quite small.

With decreasing α_{i2}, the coupling strength between Osc_{12} and Osc_{13} becomes weak, then, (Osc_{13} and Osc_{23}) are synchronized with Osc_{33} in the in-phase. Moreover, the phase differences between oscillators (Osc_{31} and Osc_{32}) and Osc_{11} tend toward $|90|$ $[deg.]$, the phase differences between oscillators (Osc_{21} and Osc_{22}) and Osc_{11} tend toward $|45|$ $[deg.]$. In the parameter region where α_{i2} is relatively large, α_{i2} is from 0.1 to 0.5, changes of the phase differences are small, whereas, in the parameter region where α_{i2} is small, α_{i2} is from 0 to 0.1, the phase differences rapidly change. The phase difference between Osc_{31} and Osc_{32} and the phase differences between Osc_{21} and Osc_{22} respectively keep 180 $[deg.]$ and 90 $[deg.]$ for any α_{a2}.

We could confirm different synchronization phenomena which are caused by asymmetric couplings.

4. Conclusions

In this study, we investigated synchronization state of coupled oscillator chains with unbalanced parametric distribution. By using computer simulations, we observed different types of frustrated synchronization states.

The results in this study would be a good model of various natural and artificial systems. For example, stone-paved square of an old town (e.g. we observed one in Evora, Portugal) starts from one edge of the square to line up stones in a regular way like in-phase. However, sometimes the other edge of the square has a different constraint like anti-phase. In that case, frustrations occur somewhere in the square and they are not compensated at one or some particular points but over a wide area of the square. That is one example of synchronization phenomena observed from oscillator networks with frustration.

Acknowledgment

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References

[1] T. Endo and S. Mori, "Mode analysis of a multimode ladder oscillator," IEEE Trans. Circuits Syst., vol. 23, pp. 100-113, Feb. 1976.

[2] T. Endo and S. Mori, "Mode analysis of two-dimensional low-pass multimode oscillator," IEEE Trans. Circuits Syst., vol. 23, pp. 517-530, Sep. 1976.

[3] T. Endo and S. Mori, "Mode analysis of a ring of a large number of mutually coupled van der Pol oscillators," IEEE Trans. Circuits Syst., vol. 25, no. 1, pp. 7-18, Jan. 1978.

[4] Y. Nishio and S. Mori, "Mutually coupled oscillators with an extremely large number of steady states," Proc. ISCAS'92, vol. 2, pp. 819-822, May 1992.

[5] S. Moro, Y. Nishio and S. Mori, "Synchronization phenomena in oscillators coupled by one resistor," IEICE Trans. Fundamentals, vol. E78-A, no. 2, pp. 244-253, Feb. 1995.

[6] W. Wei, W. Zhou and T. Chen, "Cluster synchronization of linearly coupled complex networks under pinning control," IEEE Trans. Circuits Syst. I, vol. 56, no. 4, pp. 829-839, Apr. 2009.

[7] Y. Uwate, Y. Nishio and R. Stoop, "Synchronization in three coupled van der Pol oscillators with different coupling strength," Proc. NCSP'10, pp. 109-112, Mar. 2010.

[8] J. Lu, G. Chen and M. Bernardo, "On some recent advances in synchronization and control of complex networks," Proc. ISCAS'10, pp. 3773-3776, May 2010.

[9] Y. Setou, Y. Nishio and A. Ushida, "Synchronization phenomena in many oscillators coupled by resistors as a ring," Proc. APCCAS'94, pp. 570-575, Dec. 1994.

[10] Y. Setou, Y. Nishio and A. Ushida, "Synchronization phenomena in resistively coupled oscillators with different frequencies," IEICE Trans. Fundamentals, vol. E79-A, no. 10, pp. 1575-1580, Oct. 1996.

[11] Y. Uwate, Y. Nishio and R. Stoop, "Synchronization in two polygonal oscillatory networks sharing a branch," Proc. NDES'10, pp. 62-65, May 2010.

[12] Y. Uwate and Y. Nishio, "Synchronizing Coupled Oscillators in Polygonal Networks with Frustration," Proc. ISCAS'11, pp. 745-748, May 2011.

[13] K. Matsumura, T. Nagai, Y. Uwate and Y. Nishio, "Analysis of Synchronization Phenomenon in Coupled Oscillator Chains," Proc. ISCAS'12, May 2012.