2012 International Workshop on Nonlinear Circuits, Communications and Signal Processing
NCSP'12, Honolulu, Hawaii, USA. March 4-6, 2012

Coexistence Phenomenon Observed in Coupled Chua's Circuits Involving Parameter Varying

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Abstract

In this study, we investigate behaviors of two identical coupled Chua's circuits whose parameters periodically varying in associated with each of the period of internal state values. For the periodically varying of the parameters, odd-order cycles do not exist in this system. We observed coexistence of lots of attractors which have different orders of cycles and different synchronization states.

1. Introduction

Chaotic behaviors are observed in various research field and cause interesting and complex phenomena. Coupled chaotic systems attract extensive attention as good models which describe the complimented phenomena in the natural world. The field of coupled chaotic systems has been developed since the discovering of synchronization of chaotic trajectories [1]. Studies about the coupled systems are carried out in various field, such as in physics [2], biology [3, 4] and engineering [5]. Furthermore, researchers suggest that synchronization phenomena of coupled systems have some relationships with information processing in brain. So, to investigate the coupled chaotic system is important to understanding of information processing mechanism of the brain.

By the way, it has been reported that if an autonomic neuron is influenced by external force, its characteristics are changed such as the neuron behaves periodic or chaotic [6]. How to give periodic external force is thought about in various ways, but we focus on a system whose parameter is forced into periodic varying and investigate behavior of the system and its coupling. The parametric force almost causes periodic oscillation and chaos in a simple oscillator [7, 8, 9]. Previously, we have investigated coupled logistic maps whose parameters are forced into periodic varying and observed interesting characteristic behaviors of the parametrically forced system [10].

In this study, we investigate behaviors of a two-coupled parametrically forced Chua's circuit. In the system, the linear resistor of each parametrically forced Chua's circuit is forced into periodic varying in associated with the period of its internal state value and the two circuits are coupled by a resistor in parallel. In the next section, we propose the coupled parametrically forced Chua's circuit. In section 3, we investigate bifurcations in the non-coupled parametrically forced Chua's circuit. Non-existence of odd order cycles and coexistence of different attractors are observed. In section 4, we consider the coupled parametrically forced Chua's circuits. Coexisting of many attractors whose synchronizations states are different are observed. The last section is devoted to the conclusion.

2. Parametrically Forced Chua's Circuit

In this study, we consider a coupled continuous-time system whose parameter is forced into periodic varying. Chua's circuit is used as the continuous-time system. Tho Chua's circuits are coupled by a resistor in parallel. The circuit model of the coupled system is shown in Fig. 1. For this coupling, $v_{1k}(k = 1, 2)$ affect to the other subcircuits with the coupling intensity corresponding to the coupling resistor. The linear resistors of the Chua's circuits alternately changes from R_1 to R_2 depending on v_{2k}. The switches which cause periodically varying of the resistors are controlled by v_{2k}. Namely, the switch of each of the subcircuits is controlled by the state value of each subcircuit. The switches shift when v_{2k} is equal to 0 and changes from negative value to positive value. Figure 2 shows the relationship between v_{21} and the motion of the switch. The linear resistor are changed every one period of v_{2k}.

The state equations of the parametrically forced Chua's circuit are:

$$\begin{cases} \dfrac{dv_{1k}}{dt} = \dfrac{1}{C_1}\left\{\dfrac{1}{R}(v_{2k} - v_{1k}) - f(v_{1k}) + \dfrac{1}{r}(v_{1l} - v_{1k})\right\} \\ \dfrac{dv_{2k}}{dt} = \dfrac{1}{C_2}\left\{\dfrac{1}{R}(v_{1k} - v_{2k}) + i_{3k}\right\} \\ \dfrac{di_{3k}}{dt} = -\dfrac{1}{L}v_{2k} \\ (k = 1, 2)(l = 1, 2, l \neq k) \end{cases}, \tag{1}$$

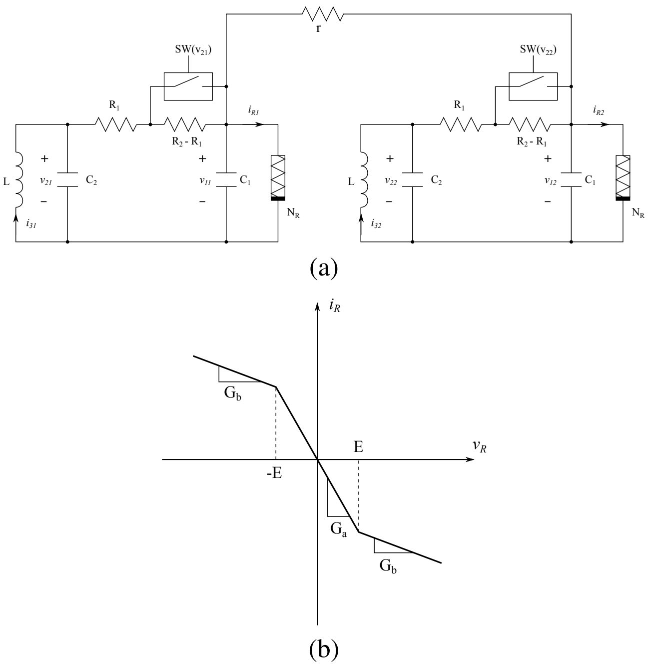


Figure 1: The circuit model of the parametrically forced Chua's circuit and the $i - v$ characteristics of the nonlinear resistor N_R.

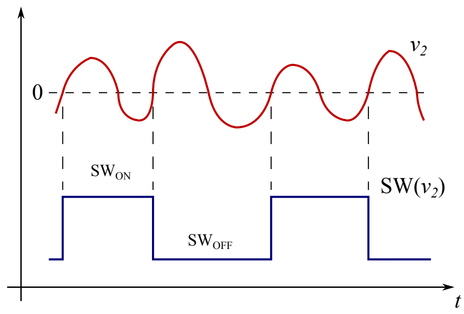


Figure 2: The motion of the switch depending on v_2.

where, R alternately changes from R_1 to R_2 depending on v_{2k}; and

$$f(v_{1k}) = G_b v_{1k} + \frac{1}{2}(G_a - G_b)\{|v_{1k+}E| - |v_{1k} - E|\} \quad (2)$$

is the $v - i$ characteristic of the nonlinear resistor N_R with a slope equal to G_a in the inner region and G_b in the outer region. A typical $v - i$ characteristic of N_R is shown in Fig. 1(b). By using following parameters and variables:

$$\left\{ \begin{array}{l} \tau = \dfrac{1}{RC_2}t, \alpha = \dfrac{C_2}{C_1}, \beta = \dfrac{C_2}{L} \\ x_k = \dfrac{v_{1k}}{E}, y = \dfrac{v_{2k}}{E}, z = \dfrac{R}{E}i_{3k}, \end{array} \right. \quad (3)$$

the normalized circuit equations are given as:

$$\left\{ \begin{array}{l} \dfrac{dx_k}{d\tau} = \alpha \left\{ (y_k - x_k) - Rf(y_k) + \dfrac{R}{r}(x_l - x_k) \right\} \\ \dfrac{dy_k}{d\tau} = x_k - y_k + z_k \\ \dfrac{dz_k}{d\tau} = -\beta R^2 y_k \end{array} \right. . \quad (4)$$

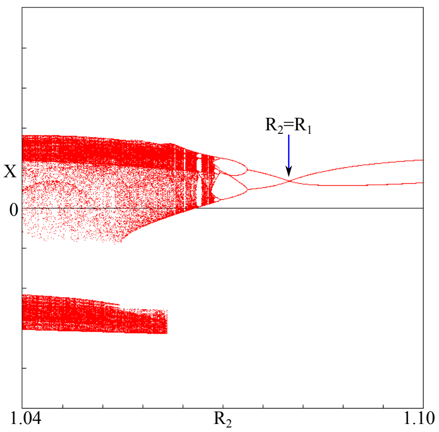
3. Bifurcation Analysis in the Non-Coupled System

We investigate bifurcations in the non-coupled parametrically forced Chua's circuit by carry out computer simulations with fourth-order Runge-Kutta method when parameters are set as $\alpha = 3.7, \beta = 4.5, G_a = -1.3$ and $G_b = -0.6$. Figure 3 shows one-parameter bifurcation diagrams when R_1 is fixed and R_2 varies. In Fig. 3(a), an order two cycle started from the right side of the figure continues to undergo flip bifurcations and the bifurcation cascade give rise to chaos. On the cascade, an order 1 periodic cycle appears at the point $R_1 = R_2$. Except that point, all cycles are even order. In Fig. 3(b), the order of an cycle started from $R_1 = R_2$ increases by both of increase and decrease of R_2 when R_2 crossing through flip bifurcations. In the basic Chua's circuit, by decreasing the resistance of the linear resistor, the orders of cycles simply increase when the cycles undergo flip bifurcations. Whereas, in the parametrically forced Chua's circuit, by decreasing R_2, the orders of cycles increase and decrease when the cycles undergo flip bifurcations.

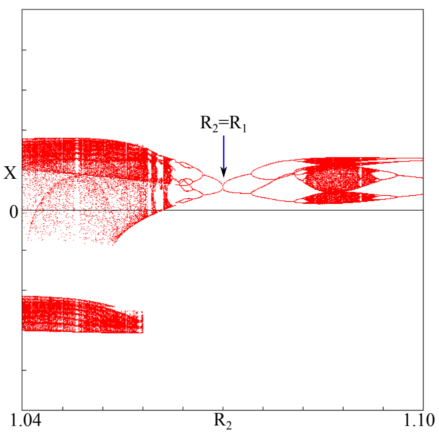
Figure 4 shows attractors and their Poincaré maps on a cascade when R_1 is fixed and at 1.10 and R_2 decreases. In Fig. 4(a) and (a'), the parameters are the same, whereas initial values are different. Two order 2 cycles coexist. By decreasing R_2, the cycles undergo a flip bifurcation and then become order 4 cycles as shown in Fig. 4(b) and (b'). The cycles continue to undergo flip bifurcations and then give rise to chaos. For some parameter regions, different order cycles coexist, for instance, an order 6 cycle and a chaos coexist as shown in Fig. 4(c) and (c'). More decreasing R_2, one of the coexisting attractors disappears and a chaotic attractor remains as shown in Fig. 4(d). Finally, a double-scroll chaotic attractor is observed as shown in Fig. 4(e).

4. Synchronization in the Coupled System

In this section, we investigate coexistence of attractors observed in two coupled parametrically forced Chua's circuits when the parameters are set as $\alpha = 3.7, \beta = 4.5, G_a = -1.3, G_b = -0.6, R_1 = 1.07$, $R_2 = 1.0676$ and $r = 4.0$. For the parameters of the subcircuits, order 2 cycle and order



(a)



(b)

Figure 3: One-parameter bifurcation diagrams for the fixed parameters $\alpha = 3.7, \beta = 4.5, G_a = -1.3$ and $G_b = -0.6$ and variant R_2. (a) $R_1 = 1.10$. (b) $R_1 = 1.07$

4 cycle coexist in the non-coupled case.

Figure 5 shows attractors and phase differences observed in computer simulations. Four attractors coexist as shown in Fig. 5. Three of the four coexisting attractors are order 2 cycles, and the remain coexisting attractor is order 4 cycle. In any case of the coexisting attractors, the two subcircuits are synchronized at the in-phase. Here, we focus on the motions of the switches. We can see that there are two pattern of the motion of the switch that the two switches shift at the in-phase and the opposite-phase. By the coupling, the motions of the switches are attracted at the in-phase or the opposite-phase. For the motions of the switches, lots of stable states are constructed.

Figure 6 shows basins on (x_1, z_1) phase plane when $y_1 = 0$ and $(x_2, y_2, z_2) = (0.3828, 0, 0.485)$ which is a coordinate of the orbit shown in Fig. 5(2). In the figure, red, blue, yellow and green colored regions correspond that the trajectories started from those regions converge the attractors shown in Fig. 5(1), (2), (3) and (4), respectively. Geometric patterns are presented fractal structures are confirmed.

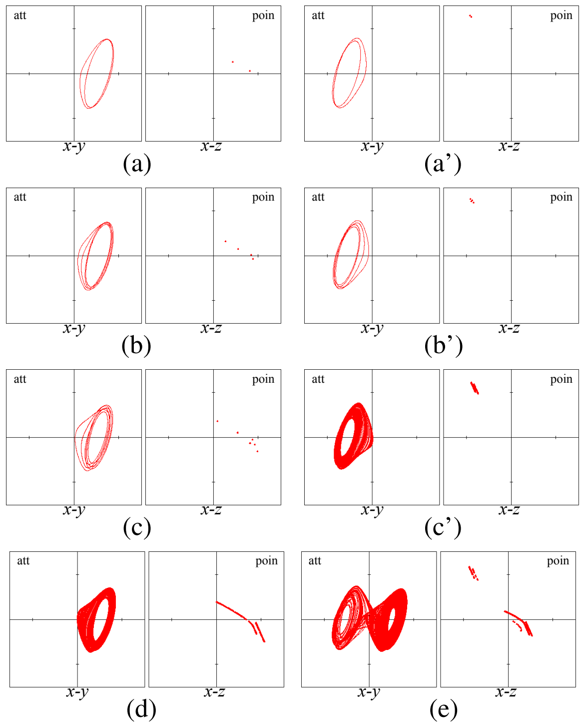


Figure 4: Attractors observed in the computer simulations for $\alpha = 3.7, \beta = 4.5, G_a = -1.3, G_b = -0.6$ and $R_1 = 1.07$. (a)$R_2 = 1.08$. (b)$R_2 = 1.07$. (c)$R_2 = 1.056$. (d)$R_2 = 1.05$. (e)$R_2 = 1.04$.

5. Conclusions

In this study, we investigated synchronization in two identical coupled Chua's circuits whose parameters periodically varying in associated with each of the period of internal state values. For the periodically varying of the parameters, odd-order cycles do not exist in this system. We have confirmed coexistence of lots of attractors which have different orders of cycles and different synchronization states. By the coupling, the motions of the switches are attracted at the in-phase or the opposite-phase. Then, the motions of the switches cause lots of stable states.

Acknowledgment

This work was partly supported by JSPS Grant-in-Aid for Scientific Research 22500203.

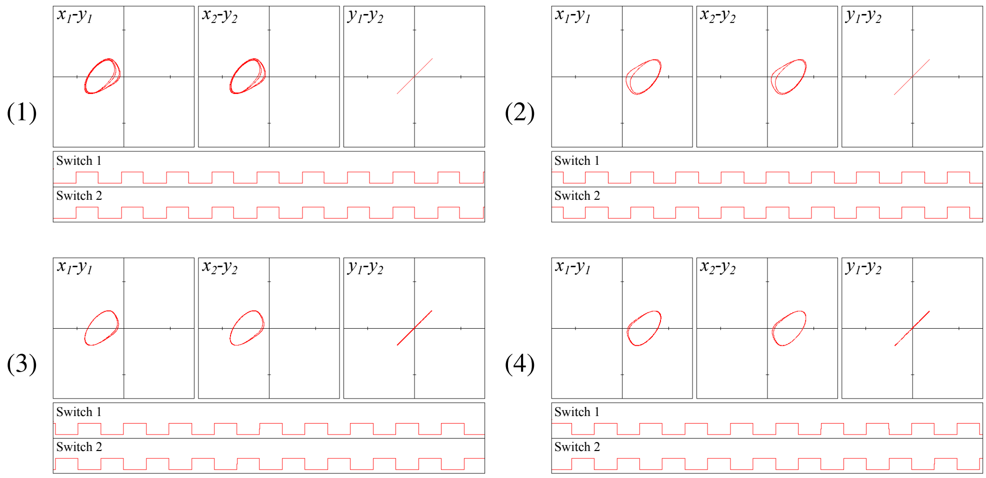


Figure 5: Attractors and phase differences in the coupled parametrically forced Chua's circuit.

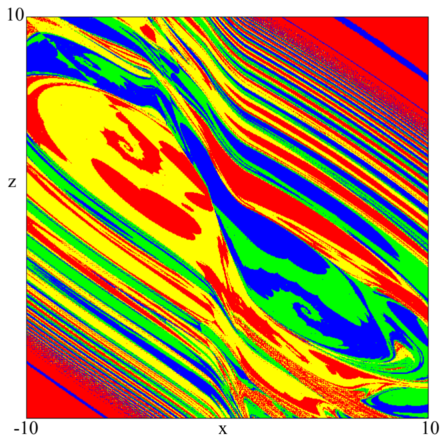


Figure 6: Basins on (x_1, z_1) phase plane when $y_1 = 0$ and $(x_2, y_2, z_2) = (0.3828, 0, 0.485)$

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